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EXERCISE 1:

1. For the following utility functions $u : \mathbb{R}^2_+ \to \mathbb{R}$ find the following:

Marshallian demand
Indirect utility function

$$\begin{aligned} u(x) &= (\alpha x_1^\rho + \beta x_2^\rho)^{\frac{1}{\rho}}, \rho \leq 1 \\ u(x) &= x_1^\alpha x_2^{1-\alpha} \\ u(x) &= \min\{\alpha x_1, \beta x_2\} \\ u(x) &= x_1 + \sqrt{x_2} \end{aligned}$$

Make sure to check that the properties we defined in class hold. Fixing the price of the first good $p_1 = 1$, plot the demands as functions of the price of the second good p_2.

2.John lives on ale and chips, that he buys at prices $(p_a, p_c) = (2, 2)$, where p_a is the price of a pint of ale and p_c the price of one tray of chips (all prices in dollars). In particular we observe that, at current prices, John buys one pint of ale and two trays of chips. This morning John received good news and bad news: his income is now \$10 dollars, but the price of chips has changed to $p_c' = 4$. Assuming that John's choices satisfy the weak axiom of revealed preference analyze how these changes will affect his consumption? Will he be better or worse off? Illustrate in a diagram.

3. Mr. A. lives 3 days and each day consumes a single good, apples, which he picks of a tree he owns. At the beginning of his lifetime, there are 20 kilos of apples on the tree. Each morning the total mass of apples on the tree increases by $\gamma\%$ compared with what was there at nightfall. Mr. A. is an impatient and discounts his utility from period to period at a constant rate β. If he consumes x kilos of apples today, he gets a utility of $u(x) = \ln x$ (you may assume intertemporal additive separability).

a) write a utility function representing Mr. A.'s preferences

b) assuming he can freely borrow and/or lend apples against his future output at the same $\gamma\%$ rate at which the apples on his tree grow, what would be his optimal consumption stream

c) supposing he cannot borrow or lend, for which values of β and γ would your answer to (b) change and how?