Microeconomics Andrei M. Gomberg
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EXERCISE 2: Choice Under Uncertainty

NOTE: You have 2 weeks to solve all of these.

1. Anna has a \$20 dollars in initial wealth. She also owns a stock that will pay her either \$16 or \$80 tomorrow with probability 0.5 each. She is an expected utility maximizer, whose preferences are represented by the Bernoulli utility function of final wealth

$$u(x) = \sqrt{x}$$

What is the smallest price p she would accept to sell the stock?

2. Mr. Smith is a farmer and an expected utility maximizer with Bernoulli utility function of final wealth

$$u(x) = \sqrt{x}$$

He has no assets other than the corn he planted. If the weather is good he will earn $\$10,000$, but if there is a drought his harvest will loose 36% of its value, and if there is a flood he will earn just \$100 for the entire season. He estimates, there is a $\frac{1}{3}$ chance of a drought and a $\frac{1}{6}$ chance of a flood. He is offered an insurance policy which will pay 90% of his damages in either case. What is the maximal price (premium) he is willing to pay for the policy *(just write the equation, you don't have solve it)*?

3. Mr. Fernandez is a risk-averse resident of a seismically active area whose preferences are represented by a differentiable Bernoulli utility function $u(x)$. His initial wealth is $\$W$. He thinks there is a 50% chance of a major earthquake, in which case he will suffer a loss of $\$L$. The government offers him earthquake insurance at an actuarially fair rate (why wouldn't a private insurance company offer such a policy?). Mr. Fernandez has the right to buy any amount of insurance (i.e., if he buys $\$I$ of insurance, he will get $\$I$ in case of the earthquake).

a) How much insurance will Mr. Fernandez buy?

b) Suppose now that the government introduces a new program which, in case Mr. Fernandez has bought no insurance at all, would still compensate 50% of his losses. The government will continue to provide actuarially fair insurance, but those who buy any insurance will not be eligible for the new program. How much insurance will Mr. Fernandez be willing to buy in this case?

4 A street vendor has to decide whether to invest in umbrellas or in icecream. He has a total of \$1000 to invest. Each umbrella costs him \$5 at the factory,

but could be sold for \$10 if it rains. If it is sunny, however, he can only sell them back to the factory for \$1 each. The icecream costs the vendor \$10 a kilo, and if it is sunny he could sell it all for \$40 a kilo, but he will sell none of it if it rains (so he will lose all of his icecream investment). For simplicity, you may assume that fractional quantities of both goods are possible. The vendor is a strictly risk-averse expected utility maximizer with the Bernoulli utility function of his wealth

$$u(x) = \sqrt{x}$$

a) If he knows that it will rain with probability 1/3 and with probability 2/3 the it will be sunny, how much should he invest in umbrellas and how much in icecream (For simplicity, you may assume that fractional quantities of both goods are possible).

b) if you find out that he invests \$500 in icecream and the same amount into umbrellas, what does it imply about his subjective beliefs about the probability of rain.

5 A consumer has a total wealth $w = 20$ units of money and a Bernoulli utility function of money $u(x) = \sqrt{x}$. There is a $\frac{1}{3}$ probability that because of a fire his wealth will drop to 5 units of money (e.g., his house burns down, but he still owns the plot).

(a) What would be the actuarially fair full insurance policy?

(b) What would be the price charged for full insurance by a monopolistic insurer who can extract all surplus from the consumer?

(c) Suppose now there is no insurance available, but the consumer has access to a fire-prevention technology, which reduces the probability of fire to $\frac{1}{10}$. What is the maximal price the consumer is willing to pay for the technology?

6 Consider a strictly risk-averse expected utility maximizing individual who owns x units of an asset that with probability p will be worth $\$a$ per unit and with probability $(1-p)$ will be worth nothing. He is offered instead a safe asset that will pay him $\$pa$ per unit with probability one. He can exchange any part of his risky asset holdings for the safe asset on a 1-to-1 basis (one unit of the risky asset for one unit of the safe one). How many units of the safe asset will he acquire? Show all work. You may assume that the individual's preferences may be represented by a twice continuously differentiable Bernoulli utility function ($u' > 0, u'' < 0$) and that any fractional quantity of any asset may be acquired or exchanged.

7. In 1905 Koba was robbing banks to get money for the Russian Communist Party. He could rob one of two banks. Each bank had \$3600, which Koba could get in case of success (he'd get \$0 in case of failure). He knew that one of the banks had been warned about the robbery, but he did not know, which bank, so he thought each bank had equal probability of being warned. Without a warning, Koba estimated his chance of success to be $\frac{1}{2}$, but with a warning it

would fall to $\frac{1}{4}$. He is an expected utility maximizer with the CRRA Bernoulli utility function of his personal wealth

$$u(x) = \log_{10} x$$

and the Party pays him a fixed stipend of \$100 and allows him to keep 25% of the robbery proceeds for himself (the rest goes to the Party).

a) draw a tree representing the compound lottery Koba was considering. What is the simple lottery that this compound lottery is equivalent to?

b) if Koba could pay a fixed amount π to learn which bank has been warned, what would be the maximal amount of money he'd be willing to pay?