Advanced Microeconomics Andrei M. Gomberg

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EXERCISE 3: Production

1. Consider a firm with the production function

$$f(x_1, x_2) = \sqrt[3]{x_1 + 3x_2}$$

a) What are the returns to scale that this firm faces?

b) Calculate the conditional factor demand and the cost function for the firm.

c) What is the optimal level of output (as a function of the price vector) for this firm?

d) Calculate the profit function.

e) Repeat parts (a) - (d) assuming the production function is

$$f(x_1, x_2) = x_1 + 3x_2$$

f) Repeat parts (a) - (d) assumign the production function is

$$f(x_1, x_2) = \sqrt[3]{x_1} + 3\sqrt[3]{x_2}$$

g) *(this one is hard)* Repeat parts (a) - (d) assumign the production function is

$$f(x_1, x_2) = x_1 + 3\sqrt[3]{x_2}$$

2. A road-construction company operates for two months, January and February. Each month it uses a single input, labor, and produces a single output, meters of new road. It can only hire labor by month, so that every individual hired has to work either 1 or 2 months. Because of crowding on the construction site, there are decreasing marginal returns to labor, so that if it hires x_i men for a given month it will build $\sqrt{x_i}$ meters of road during the same month ($i = January, February$). The government pays the company p pesos for each meter of the road built (it doesn't matter when).

a) Suppose it is known that labor is 25% more expensive in January than in February. Find how much road will this company be willing to build and how much labor will it hire each day.

b) How would your answer change if there were no crowding problems on the site (i.e., the returns to labor were constant)?

c) Suppose the labor wage in January is unknown now, but both January and February prices will be known on Dec 20, before the company actually has to hire any labor. It is known, however, that the January wage w_j can be either high $\left(w^H\right)$ with probability p or low $\left(w^L < w^H\right)$ with probability $(1-p)$. (For simplicity, you may assume that the February wage is known for sure). If the company can sign labor contracts for both January and February today fixing the wages at their expected levels, would the risk-neutral owners of the company want to do it, or would they prefer to wait for the January wage to be known?