

## Exercise 4

Solve ALL FOUR questions. All questions have equal weight. Show all relevant work, but once you get an equation that determines the answer you do not have to solve it. You have 3 hours to do this. Be sure to write your name on the exam. GOOD LUCK!

1. Consider an economy with two individuals. Each individual  $i$  lives for 2 periods and in each period consumes a single perishable consumer good (denote the consumption of individual  $i$  in period  $t$  as  $c_t^i$ ). In each period  $t$  individual  $i$  receives an income of  $y^i$  units of the consumption good. The individuals can lend/borrow to/from each other using perfectly competitive credit markets (denote the market interest rate as  $\rho$ ). Individual preferences are identical and may be represented by the utility function

$$u(c_0^i, c_1^i) = \frac{(c_0^i)^{1-\sigma}}{1-\sigma} + \beta \frac{(c_1^i)^{1-\sigma}}{1-\sigma}, 0 < \beta < 1, \sigma > 0$$

- a) calculate individual demands as a function of the interest rate  $\rho$ .
- b) for which values of  $\alpha, \rho$  and  $\sigma$  would individuals choose to consume mostly in the first period?
- c) assuming the first individual's endowment stream is  $(y_0^1, y_1^1) = (1, 0)$  and the second individual's income stream is  $(y_0^2, y_1^2) = (0, 1)$  what would be the equilibrium interest rate and how much would each individual consume in each period?
- d) suppose now there is a production technology that transforms a unit of period 0 consumption good into a unit of period 1 consumption good. Let this technology be given by the production function  $f(z_0) = \sqrt{z_0}$  How does your answer change?

2. (*possible problems with demand aggregation*) Two individuals, Anna and Boris, consume two goods, honey and milk and each has an income of \$1000. When the price vector is  $(10, 10)$ , then Anna consumes 25 units of honey and 75 units of milk, while Boris consumes 75 units of honey and 25 units of milk. When

the price vector is  $(15,5)$ , Anna's consumption bundle is  $(40,80)$ , while Boris's is  $(64,8)$ .

a) do individual demands obey WARP?

b) does the aggregate demand (*i.e.*, the sum of their individual demands) obey WARP?