FINAL EXAM
December 19

Solve ALL FOUR questions. All questions have equal weight. Show all relevant work, but once you get an equation that determines the answer you do not have to solve it.   You have 3 hours to do this.   Be sure to write your name on the exam. GOOD LUCK!

1. Consider an economy with two individuals. Each individual i lives for 2 periods and in each period consumes a single consumer good (denote the consumption of individual i in period t as c_t^i). In each period t individual i receives an income of y^i units of the consumption good. The individuals can lend/borrow to/from each other using perfectly competitive credit markets (denote the market interest rate as ρ). Individual preferences are identical and may be represented by the utility function

$$u\left(c_1^i, c_2^i\right) = \frac{\left(c_1^i\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(c_2^i\right)^{1-\sigma}}{1-\sigma}, 0 < \beta < 1, \sigma > 0$$

a) calculate individual demands as a function of the interest rate ρ.

b) for which values of α, ρ and σ would individuals choose to consume mostly in the first period?

b) assuming the first individual's income stream is $(y_1^1, y_2^1) = (1, 1)$ and the second individual's income stream is $(y_1^2, y_2^2) = (1, 0)$ what would be the equilibrium interest rate and how much would each individual consume in each period?

2. Two individuals, Anna and Boris, consume two goods, honey and milk and each has an income of \$1000. When the price vector is (10,10), then Anna consumes 25 units of honey and 75 units of milk, while Boris consumes 75 units of honey and 25 units of milk. When the price vector is (15,5), Anna's consumption bundle is (40,80), while Boris's is (64,8).

a) do individual demands obey WARP?

b) does the aggregate demand (*i.e.*, the sum of their individual demands) obey WARP?

3. Mr. R. is a strictly risk-averse resident of a seismically active area whose preferences are represented by a continuously differentiable Bernoulli utility function $u(x)$. His initial wealth is W. He thinks there is a 50% chance of a major earthquake, in which case he will suffer a loss of L. The government offers him earthquake insurance at an actuarially fair rate (that is, at the price that would make the expected profit of the insurer equal to zero). Mr. R. has the right to buy any amount of insurance (i.e., if he buys I units of insurance, he will get I in case of the earthquake).

a) How much insurance will Mr. R. buy?

b) What if the insurance is more expensive than actuarially fair?

c) Suppose now that the government introduces a new program which, in case Mr. R. has bought no insurance at all, would still compensate 50% of his losses. The government will continue to provide actuarially fair insurance, but those who buy any amount of insurance will not be eligible for the new program. How much insurance will Mr. R. be willing to buy in this case?

4. Consider a firm with the production function

$$f(z_1, z_2) = z_1^\alpha z_2^\beta, \ \alpha, \beta > 0$$

a) For each α and β what are the returns to scale that this firm faces?

b) Calculate the conditional factor demand, the cost function, supply and the profit function for this firm when $\alpha = \beta = \frac{1}{5}$.

c) How does your answer to (b) change when $\alpha = \beta = \frac{1}{2}$