Advanced Micro I Andrei Gomberg

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EXERCISE 1: Math Review

1. Consider the following potential properties of a binary relation R on a set X

(i) Reflexivity
(ii) Completeness
(iii) Transitivity

Which of the above properties is satisfied by each of the following preference relations?
a) $X = \mathbb{R}^2_+$, xRy iff (iff = if and only if) $x \geq y$
b) $X = \mathbb{R}^n_+$, xRy iff $\min\{x_1, x_2, ..., x_n\} \geq \min\{y_1, y_2, ..., y_n\}$
c) $X = \mathbb{R}_+$, xRy iff $x > y$
d) $X = \mathbb{R}^2_+$, xRy for all $x, y \in X$
e) $X = \mathbb{R}_+$, xRy iff $\frac{x}{5} - \left[\frac{x}{5}\right] \geq \frac{y}{5} - \left[\frac{y}{5}\right]$, where $[z]$ (integer part of z) is the largest integer smaller than z.

2.. (JR A1.11) Let $D, R \subset \mathbb{R}$ and consider the function $f : D \to R$ given by $f(x) = x^2$ Determine whether this function is injective, surjective, or bijective for
a) $D = \mathbb{R}, R = \mathbb{R}$
b) $D = \mathbb{R}, R = \mathbb{R}_+$
c) $D = \mathbb{R}_+, R = \mathbb{R}$
d) $D = \mathbb{R}_+, R = \mathbb{R}_+$

3. What conditions on parameters α and β are required to insure that the utility function $u(x_1, x_2) = x_1^\alpha x_2^\beta$ is a strictly concave function on $\mathbb{R}^2_{++}$. What if we consider as its domain the set $\mathbb{R}^2_+$?

4. (JR A 1.47) Let $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ both be concave functions and g also strictly increasing. Show (without using calculus) that the composite function $h(x) = g(f(x))$ is also concave.

5. (Just make sure you know how to do all of this. You DO NOT have to submit it). Plot a couple of level curves and indicate direction of increase for each of the following functions $f : \mathbb{R}^2_+ \to \mathbb{R}$

$$f(x_1 x_2) = \sqrt{x_1} + 2\sqrt{x_2}$$

$$f(x_1 x_2) = -\min\{3x_1, x_2\}$$

$$f(x_1 x_2) = x_1 - 2x_2$$

$$f(x_1, x_2) = \sqrt{x_1 + 4x_2}$$

$$f(x_1, x_2) = x_1 + \ln x_2$$

Are these functions q-concave? q-convex? strictly q-concave? strictly q-convex? both? neither? Explain. Without solving the maximization problem, indicate in the graph where can the argmaximum and argminimum of each of these functions subject to the usual constraint $p \cdot x \leq w$ $(p >> 0, w > 0)$ be.