

# Final Exam

May 30

Solve ALL FIVE questions. All questions have equal weight. Show relevant work. You have 90 minutes to do this. You are not allowed to use any books or notes, but you can use a calculator or dictionary, if that helps. The answers can be written in either English or Spanish. Be sure to write your name on the exam. GOOD LUCK!

1. Compute the Hicksian demand and the expenditure function for an individual with preferences represented by the utility function on  $\mathbb{R}_+^2$

$$u(x) = \sqrt{x_1 + 2 \ln(x_2 + 1)}$$

Given your answer, can you find the Marshallian demand and indirect utility function WITHOUT solving the utility maximization problem? If yes, how? If no, why not?

2. A road-construction company operates for two periods, “Sunday” and “Monday”. On Sunday the road surface is covered with gravel and on Monday with asphalt. Unless the road surface has been covered with gravel Sunday, asphalt cannot be applied Monday. In either production process only one input (labor) is used. Because of crowding on the construction site, there are decreasing marginal returns to labor, so that if it hires  $x$  men-hours on any day it covers  $\sqrt{x}$  meters of road on the same day. On Sunday it has to pay overtime, so that the wage it pays workers is 25% higher than the Monday wage  $w$ . The government pays the company  $p$  pesos for each meter of the completed (asphalted) road. Find how much road will this company be willing to build and how much labor will it hire each day.

3. A contractor is in charge of building a rail line between points A and B. According to the provisions of his contract he will be paid a fixed amount for the project to cover both his costs and his profits (the contractor is the owner of his construction company). He has to hire labor and purchase equipment

for the project. The construction will only start in 6 months and he can do labor hiring and equipment purchasing either now or right before the start of the construction (for some reason no hiring/purchasing can be done in between). Both the labor and equipment owners will be paid the market wage/price at the time the transaction is done. The prices today are known, but they may change in 6 months. The contractor, however, expects that, on average, today's prices are the best predictor for the future ones. When should the risk-neutral contractor do the hiring?

*(see page 2)*

4. (in answering this question it is enough to write the equations determining the answer - you don't have to solve them, but you have to explain how you derive them; you may assume that everything is differentiable as many times as you need it).

Ivan Petrov, a farmer at the "Lenin's Road" collective farm can work either on his own plot of land or at the collective field. The collective farm pays him a guaranteed wage of 4 rubles for each day of work if the output target is met and 3 rubles for each day of work if it is not met; his own plot may bring him either 11 rubles/day, if at the end of the season he is allowed to freely sell his output, or 1 ruble/day if the collective farm decides to confiscate his output for nominal compensation. It is known that the probability of confiscation is equal to  $\frac{1}{2}$  and the probability of meeting the target is equal to  $\frac{1}{4}$ . There is a total of 90 working days in the farming season. Assume Ivan is a strictly risk-averse expected utility maximizer.

a) if he can choose to work either full time on the collective farm or full time on his home plot, where would he want to work?

b) if he can split his work time, so that he would work some days on his plot and the rest of his time on the collective plot, how much should he work on his own plot?

c) if instead you observe that he works 30 days on the collective farm, and, furthermore, you find out that he believes that the collective farm's output target can't be fulfilled, what does it imply about his subjective beliefs about the probability of confiscation?

5.

a) In a competitive economy with three goods Mr. A makes the following choices. When the price vector is  $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$  he chooses to consume the bundle

$\begin{pmatrix} 5 \\ 19 \\ 9 \end{pmatrix}$ . When prices are  $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$  the chosen bundle is  $\begin{pmatrix} 12 \\ 12 \\ 12 \end{pmatrix}$ . When prices

are  $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$  the consumed bundle is  $\begin{pmatrix} 27 \\ 11 \\ 1 \end{pmatrix}$ . Do these choices satisfy WARP.

Can they be rationalized?

b) A consumer has preferences on  $\mathbb{R}_+$  defined by

$$x \succsim y \text{ iff } \sin x \geq \sin y$$

Are these preferences rational? convex? Can they be represented by a utility function? If yes, provide a utility function representation; if no, explain your reasoning.