Consumer and Producer Theory — Andrei Gomberg
Fall 2016

EXERCISE 1: Math Review: Binary Relations (mostly)

Due August 22

1. Consider the following potential properties of a binary relation R on a set X

(i) Reflexivity: for any $x \in X, xRx$
(ii) Completeness: for any $x, y \in X, xRy$ or yRx (or both)
(iii) Transitivity: for any $x, y, z \in X$, if xRy, yRz it follows that xRz

Which of the above properties is satisfied by each of the following binary relations?
a) $X = \mathbb{R}_+^n$, xRy iff (iff = if and only if) $x \geq y$
b) $X = \mathbb{R}_+^n$, xRy iff $\min\{x_1, x_2, ..., x_n\} \geq \min\{y_1, y_2, ..., y_n\}$
c) $X = \mathbb{R}_+^2$, xRy iff either $x_1 > y_1$, or if $x_1 = y_1$ then $x_2 \geq y_2$
d) $X = \mathbb{R}_+$, xRy iff $x > y$
e) $X = \mathbb{R}_+$, xRy for all $x, y \in X$
f) $X = \mathbb{R}_+$, xRy iff $\frac{x}{5} - \left[\frac{x}{5}\right] \geq \frac{y}{5} - \left[\frac{y}{5}\right]$, where $[z]$ (integer part of z) is the largest integer smaller than z.

2. A binary relation R over some set X is said to be **negatively transitive** if for every $x, y, z \in X$, $\urcorner(xRy)$ and $\urcorner(yRz)$ imply $\urcorner(xRz)$. A binary relation R over X is said to be **asymmetric** if for every $x, y \in X$, xRy implies $\urcorner(yRx)$. Show that any R that is both asymmetric and negatively transitive is transitive.

3 A binary relation R on X is called **asymmetric** if for every $x, y \in X$ xRy implies$\urcorner$ (yRx) and it is called **irreflexive** if for every $x \in X$ we have $\urcorner(xRx)$. Show that every preference relation that is both irreflexive and transitive is asymmetric.

4. Show that if a binary relation is transitive then so are its symmetric and asymmetric parts.

5. (Kreps, exercise 2.2) A binary relation R is **symmetric** if xRy implies yRx for any x, y. Consider the following proof that transitivity and symmetry imply reflexivity (i.e. that xRx for any x). For any x take y such that xRy. Clearly, symmetry implies yRx and, therefore transitivity then implies that

xRx. Is this proof correct? Why, or why not? Is the statement itself (that symmetry and transitivity together imply reflexivity) correct?

6. Show that the set $\mathbb{R}_+$ is connected (i.e., it cannot be partitioned into two clopen sets).