Consumer and Producer Theory — Andrei Gomberg, Fall 2016

EXERCISE 2: Math Review, Choices and Preferences

Part I

Exercises 1.B.1, 1.B.2, 1.C.1, 1.C.2, 1.C.3, 1.D.1, 1.D.2, 1.D.3, 3.B.1, 1.D.4 from Mas-Colell. Exercises 1.1,1.2, and 1.4 from Rubinstein.(also, check out 1.3 and 1.5 - they are fun).

Part II

1. Consider a binary relation R on some connected space X. As usual, let P be its asymmetric part and I be its symmetric part. The relation R is called **continuous** on X if for any $x \in X$ the sets $A = \{y \in X : xRy\}$ and $B = \{y \in X : yRx\}$ are both closed in X.

a) Which of the binary relations in the first problem in Exercise 1 are continuous, and which are not?

For the rest of the problem assume R is a complete preorder (i.e., it is both complete and transtitive).

b) Suppose there exists a pair $x, y \in X$ s.t. xPy. Show that if R is continuous there must exist a $z \in X$ such that xPz and zPy.

c) Would the above statements still hold if R were not complete?

2.

a) Does the correspondence $\Phi : \mathbb{R}_{++} \to \mathbb{R}$ given by $\Phi(x) = \{y \in \mathbb{R} : \ln x \leq y \leq e^x\}$ have a closed graph? is it non-empty-valued? closed-valued? convex-valued? compact-valued? (if compact-valued) upper hemi-continuous? lower hemi-continuous? continuous? Graph this correspondenc.

b) show that the correspondence $B : \mathbb{R}^3_{++} \to \mathbb{R}^2_+$ given by $B(x) = \left\{y \in \mathbb{R}^2 : x_1 y_1 + x_2 y_2 \leq x_3\right\}$ is continuous.