Consumer and Producer Theory

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Fall 2016

# EXERCISE 7: Choice Under Uncertainty II

**1.** Mr. A is an expected utility maximizer, consuming two goods, apples $a$ and carrots $c$. His preferences over the two goods are represented by a Bernoulli utility function

$$u(a,c) = f(a) \times f(c)$$

where $f$ is a strictly increasing function. The prices of the two goods are uncertain, so that the price vector $\binom{p_a}{p_c}$ can be equal either to $\binom{3}{1}$ or $\binom{1}{3}$ with equal probability. He is given an option to purchase both goods at prices $\binom{p_a}{p_c} = \binom{2}{2}$. Will he take such an option, or will he wait for the price realization? Show work.

**2.** A street vendor in Mongolia has to decide whether to invest in umbrellas or in icecream. He has a total of 900 Mongolian *tugriks* to invest. Each umbrella costs him 5 *tugriks* at the factory, but could be sold for 10 tugriks if it rains. If it is sunny, however, he can only sell them back to the factory for 1 *tugrik* each. The icecream costs the vendor 10 *tugriks* a kilo, and if it is sunny he could sell it all for 40 *tugriks* a kilo, but he will sell none of it if it rains (so he will lose all of his icecream investment). For simplicity, you may assume that fractional quantities of both goods are possible. The vendor is a strictly risk-averse expected utility maximizer with the CRRA Bernoulli utility function of his wealth (risk-aversion coefficient equal to $\frac{1}{3}$).

a) Suppose he has to invest all his assets in either one of two goods (cash cannot be kept at all). If he knows that it will rain with probability 1/3 and with probability 2/3 the it will be sunny, how much should he invest in umbrellas and how much in icecream (For simplicity, you may assume that fractional quantities of both goods are possible).

b) If you find out that he invests 450 *tugriks* in icecream and the same amount into umbrellas, what does it imply about his subjective beliefs about the probability of rain.

c) How does your answer change, if he is allowed to keep some cash?

**3.** (Kreps, based on Kahneman and Tversky 1979). Consider an vNM expected utility maximizer who owns wealth $W$ of which $L$ may be lost in an accident which occurs with probability $p$. You know that this individual is indifferent between buying and not buying the full insurance at a given premium $\pi$ if this is the only insurance contract available. However, this instead he is offered a contract in which he only has to pay half the premium $\pi$ upfront but, conditional on the accident occruing, with probability 1/2 he will have to pay the

remaining half before the insurance company reimburses, while with probability 1/2 he will be refunded his initial payment and not reimbursed anything.Suppose you observe (as Kahneman and Tversky did) that this individual rejects this new insurance contract. Could this individual be a von Neumann-Morgenstern expected utility maximizer? If yes, could this individual be risk-averse?

**4.** (*Adverse Selection*) Consider a society of individuals who receive an income of$\$W$ pesos if they are healthy and $\$W - L$ pesos if they are sick. There are two types of individuals: some who get sick with a probability of $p$ and others who are sick with probability of $p' > p$. There are equal numbers of individuals of each type in the population. All individuals are CRRA expected utility maximizers with the relative risk-aversion coefficient equal to 1/2

a) supposing insurance companies can observe the agents' types, what are the actuarially fair rates they would charge each type for insurance?

b) supposing insurance companies can't observe the types, what would be the actuarially fair rate they would charge if every individual were obliged to buy insurance ?

c) if only full insurance is available, which agents would insure at the rate in (b)?

d) given your answer in (c), would the insurance companies be willing to offer such a rate if individuals were free to choose, whether to buy insurance or not?

e) if an insurance company could offer people who choose not to be insured in (e) a partial insurance contract, can it profitably do so? What is the most insurance that can be offered.

**5.** (*Moral Hazard*). Consider a society of individuals who receive an income of$\$W$ pesos if they are healthy and $\$W - L$ pesos if they are sick. Their probability of being sick is affected by a risky but enjoyable behavior: e.g.., they could choose to smoke or not. if they don't smoke, their probability of sickness is $p$, otherwise it is $p' > p$. All individuals are expected utility maximizers with the same utility function s in the previous example. In addition, they get a lump sum "utility gain" of $\varepsilon > 0$ "utils" if they smoke (it is not essencial that utility is additively separable in wealth and smoking, it is enough for this example that every individual prefers smoking to non-smoking for any given wealth).

a) supposing insurance companies can observe the agents' behavior and write contracts contingent on it.. What are the actuarially fair rates they would charge smokers and non-smokers for insurance?

b) supposing insurance companies can't observe the behavior (or for some other reason can't write contingent contracts), what would be the actuarially fair rate they would charge?

c) what would the agents do, if they all bought full insurance (at any rate)?

d) if $\varepsilon$ is very small, would the agents get insurance?