Consumer and Producer Theory

Andrei Gomberg Spring 2015

FINAL EXAMINATION May 26, 2015

INSTRUCTIONS: Every question has the same weight. You may use Mas-Colell, Whinston and Green and/or Rubinstein and *nothing else* as a reference. Answer **ALL THREE** questions (the fourth is the take-home). Show all relevant work. You have up to **2.5 hours** to do this. Don't forget to write your name on the exam. GOOD LUCK!

1. Consider a choice structure $(\mathcal{B}, C(.))$ such that \mathcal{B} is *closed under unions* (that is if both A and B belong to \mathcal{B} than so does $A \cup B$)

Recall that Sen's α axiom states that if $x \in A \subset B$ and $x \in C(B)$ then $x \in C(A)$.

a) Consider the following version of what is sometimes known as the *expansion* axiom: if $x \in C(A) \cap C(B)$ then $x \in C(A \cup B)$. Are the two properties related?

b) Consider the following choice procedure from a finite consumption space X (# $X = n < \infty$). An individual has a rational preference over X but chooses all alternatives in B that are strictly preferred to by no more than $\frac{1}{4}$ (#B) elements of B Would a choice structure thus generated have to satisfy α ? Expansion? WARP?

2. Let $X = \mathbb{Z}^n_+$ (the set of non-negative **integer** vectors). A function $u : X \to \mathbb{R}$ is called *additively separable* if for every $x = (x_1, x_2, ..., x_n) \in X$

$$u(x) = u_1(x_1) + u_2(x_2)$$

where $u_i : \mathbb{Z} \to \mathbb{R}$ and it is called *additive* if for any $x, y \in X$

$$u(x+y) = u(x) + u(y)$$

a) Are these two properties of functions logically related?

b) Consider a lexicographic preference relation on $X = \mathbb{Z}_{+}^{n}$ (for simplicity, you can do it for n = 2, as long as it is clear how to generalize it). Can you construct an additively separable utility function representing it? If no, explain why not. If yes, provide an example of such a function.

c) Provide an example of a rational preference relation on \mathbb{Z}^{n}_{+} that is not representable by an additively separable utility.

3. Consider a strictly risk-averse expected utility maximizing individual who has a wealth of w can invest in one of m independently and identically distributed risky assets that are priced the same in the market (there is no riskless asset he can keep his wealth in - he is in Zimbabwe). How should he allocate his wealth? Show all work.