Revealed votes

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Abstract

In this paper I consider choice correspondences defined on a novel domain: the decisions are assumed to be taken not by individuals, but by committees, whose membership is observable and variable. In particular, for the case of two alternatives I provide a full characterization of committee choice structures that may be rationalized with two common decision rules: unanimity with a default and weighted majority.

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1 Introduction

Ever since Houthakker (1950) it has been known that a simple consistency condition on choices (the Strong Axiom of Revealed Preference, SARP) is necessary and sufficient for being able to explain individual choices with rational preference maximization. Of course, this approach has long been a basis for the formal decision theory used by political scientists, as well as economists. That observations of group decisions themselves may be used to uncover both individual preferences and group decision rules is, however, frequently ignored. Even less studied is the potential for exploiting group membership data for such purposes.

In this paper I apply the methodology of revealed preference to data consisting of choices made by a number of committees with overlapping membership. This allows me to define empirical consequences of two group-decision rules. In the first of these, one of the alternatives is marked as the default, which can only be overcome by a unanimous vote of all the group members. The other rule is the weighted majority (with individual group member weights unknown to the observer). I provide a complete characterization of each of these rules on my conjectured data set, by formulating the necessary and sufficient conditions for these rules to be consistent with the outcomes observed. I then expand the choice set facing the committees to more than two alternatives and extend the characterization of the weighted majority rule to this environment, while reinterpreting it as a more general *scoring* rule.

It is well-known that not much could be said from a single observation of a committee choice or even of a series of choices made by a group of people, whose membership is unobserved. Formally, it has been known since McGarvey (1953) that, unless something is known about committee membership, *any* choice structure is consistent with even the most unsophisticated theory, such as, for instance, that the group members decide by simple majority, while voting sincerely and independently from each other, based on some unobserved individual preferences. However, as I try to establish in this paper, if observations of decisions taken by committees with *variable, but overlapping, membership* are available, one can use such data to reject, at least, this "naive" theory of committee decision-making.

Group, rather than individual, decisions may be all one can go by in a variety of settings, such as legislatures, monetary policy committees in central banks, shareholder meetings, or courts. We may know more or less about what happens inside the doors and inside the minds of the various individuals forming a committee, the actions of which we observe. Crucially, even when we know the formal decision rules a committee must act by, the true votes of committee members may be unknown, either because these are kept secret for policy reasons[1] or because they are not actually recorded[2], or because in many cases individual members might are free to manipulate the formal record without impacting the actual decision. What we do usually know, though, is who was participating and what decision they collectively took. As outside observers, we may want to have some questions answered from the observed data. Were all members of a group treated equally, or did some of them possess greater weight? Were the various alternatives treated symmetrically, or were some of them privileged (as, for instance, a status quo could be in comparison with a reform proposal)? Did the group members vote strategically or sincerely? Did they take into account preferences of and/or information possessed by their fellow committee members? If only committee decisions are made public, with votes and deliberations remaining secret, could we still test theories about the functioning of the committee?

Committees with overlapping membership having to repeatedly decide over the same issue are not so rare in practice. Think, for instance, of the route that a bill or a nomination must pass to obtain approval by a legislative chamber, where it would have to be considered by one or many committees and subcommittees, all before coming up to the floor for the final vote. Similarly, in a number of Latin American countries, where a judicial decision does not always constitute legal precedent, courts have to repeatedly decide on constitutionality of the same law, as applied to different plaintiffs (such cases are known as *amparo* in Mexico[3] and *tutela* in Colombia) In the particular case of Colombia, the nine-member Constitutional Court has adopted a peculiar system of case designation, in which *tutela* decisions are *randomly* assigned to overlapping three-member panels of justices, with every member potentially having to participate in a ruling on the issue (in certain cases the entire court may also decide *en banc,* with members of the multiple previous

[1] Whether voting records of central bank decision-makers should be public has been a subject of substantial controversy and research in recent years and the international practice has varied (see, for instance, Sibert 2003, or Gersbach and Hahn 2008).

[2] Thus, in a legislature no formal vote may be taken on an issue since the parliamentary leaders know that it would fail anyway.

[3] See, for instance, Vargas (1996) for a discussion of the role - and lack of precedential authority - of the *writ of amparo* in Mexican legal system.

panels joining in a single decision).[4] A dataset of court decisions emerging from this institution would be remarkably similar to the one proposed in this paper.

Recently, Degan and Merlo (2009) have explored empirical implications of sincere (versus strategic) voting. In fact, if the formal decision rule is known, this work may be reinterpreted precisely as the test of voter sincerity: if I know how the votes are counted, violations of the conditions established here could only be interpreted as indications that the scores do not directly reflect rational individual preference. Thus, to the extent one maintains the assumption that voters are rational, sincere voting would be falsified in this case. Another closely related work is that by Apesteguia *et al.* (2014), who characterize observable outcomes of sincere and strategic application of various agenda rules employed in legislatures. In contrast with those papers, however, I do not assume observability of individual votes or preferences, but try to infer votes from observing the group choice data. I review the rest of the related literature in Section 5 of the paper.

The rest of this paper is organized as follows. In section two I introduce the basic ideas, while characterizing choices that could be generated by a common voting rule (unanimity with default) in a binary choice setting. In section three I introduce the weighted majority rule and provide a characterization of restrictions they impose on committee choices In section four I extend the analysis to the case of three or more alternatives, while reinterpreting the weighted majority as scoring. Section five explores relation of this paper to the previous literature, while section six concludes.

2 Unanimity with a Default

In order to fix the ideas, I shall start by proposing a simple characterization of choice patterns that may be generated by a fixed choice rule in a setting with just two alternatives. The rule I characterize is, in fact, quite common: unanimity with a default (this could, for instance, be a rule used in a criminal jury trial: unless all members of the jury vote to convict the defendant is declared innocent). Suppose we observe a collection of decisions of different committees with varying membership on the same binary issue and believe

[4] The details of the case assignment are described in Articles 49 and 50 of the *Reglamento Interno* (Internal Rules) of the Colombian Constitutional Court. The author thanks Juan Bertomeu for drawing his attention to this arrangement.

that all of these committees used this rule. Would we be able to test this theory using this data, without observing individual votes and without knowing, what is the default choice? In other words, are there restrictions on the committee choices such that if these are violated, the theory is conclusively falsified?

In order to answer this question, it shall be useful to define the notion of a *committee choice structure*. Let $X = \{a, b\}$ be the set of alternatives that committees face (it could be the Guilty versus Innocent, if the committee is a jury, or approve versus reject, if we are dealing with congressional committees considering nominations, etc.). Let a finite set $N = \{1, 2, ...n\}$ denote the general pool of agents, from which the committees are drawn and let $\mathcal{E} \subset 2^N \setminus \{\varnothing\}$ be the set of committees, that are observed and let the function $C : \mathcal{E} \rightarrow X$ denote the committee choice.[5] The *committee choice structure* is defined as a pair $(\mathcal{E}, C(.))$ - a record of observed committee choices. I shall view it as a data set, which may be used by an observer to deduce the preference profiles and the preference aggregation rules the committee uses.

I shall assume that each individual $i \in N$ has a well-defined complete preference relation over X, $\succcurlyeq^i$(its asymmetric part being denoted as $\succ^i$). One can define the committee choice rule to be unanimity with default $y \in X$ if $C(S) = y$ unless for $x \neq y$ we have $x \succ^i y$ for all $i \in S$. Note, that though, in general, individuals' true preferences might not be directly observable and asking committee members to declare them might lead to a strategic behavior, in a binary choice environment under the unanimity with a default rule it is a weakly dominant strategy for all players to simply declare their true preference, irrespective of who else is in the committee.[6]

Suppose that we observe that the committee choice structure satisfies the following two properties:

$U1$ (expansion): for any $S, T \in \mathcal{E}$ such that $(S \cup T) \in \mathcal{E}$ and $C(S) = C(T) = x \in X$ it must be that $C(S \cup T) = x$.

$U2$ (no more than one reversal): let $S \subset T \subset Q$ and $C(S) \neq C(T)$ then $C(Q) = C(T)$.

[5]The reason I am only considering choice functions, rather than choice correspondences in this section - an assumption that will be relaxed later - is that the unanimity rule, as here defined, always produces a unique choice.

[6]Note, that this would not be the case if the point of the vote were to aggregate information, rather than preferences, as shown in, for instance, Feddersen and Pesendorfer (1997).

It is pretty straightforward to see that the two properties here defined are necessary consequences of the unanimity decision-making with the default being $y \neq x^*$. Indeed, if two committees both came to the same decision, then either it is the default (in which case at least one member of the joint committee must be in favor of sticking to it), or it isn't (in which case the joint committee must be unanimous in its desire to overturn the default). In both cases, the joint committee must agree with the decision of its subcommittees, satisfying expansion. Furthermore, if a larger committee disagrees with the smaller one, it must be the case that one of its members - and none of the members of the subcommittee - must be in favor of sticking to the default. Once the default is thus imposed, further inclusion of additional members can no longer affect committee choice.

A less immediate observation is that, if the set $\mathcal{E}$ is sufficiently rich (*i.e.*, if it is closed under unions), the above two properties completely characterize the committee choice structures over it that can be *rationalized* with this group decision rule. In this case, by *rationalizability* I mean existence of an actual preference profile$\{\succsim^i\}_{i \in N}$ such that application of the unanimity with a default would produce choices identical to those observed in the data. In fact, the following simple proposition can be easily proved:

Proposition 1 *Let $\mathcal{E}$ be closed under unions.*[7] *Then a committee choice structure over $\mathcal{E}$ may be rationalized by unanimity with default if and only if it satisfies $U1$ and $U2$.*

Proof: *The only if part I have demonstrated above, so it remains to show that for every such committee choice structure I can find a profile of individual preferences that would generate me the original choice structure. I shall do this by construction. If $C(S) = x \in X$ for every $S \in \mathcal{E}$, then we can obtain rationalization trivially. Indeed, taking, without loss of generality, $x = a$ we may define aP^ib for all $i \in N$, which will immediately generate the original choice. Hence, the interesting case is when there exist $Q, W \in \mathcal{E}$ such that $C(Q) \neq C(W)$. Since $\mathcal{E}$ is closed under unions we may, without loss of generality take $C(Q \cup W) = C(Q) = a \neq b = C(W)$. For each $i \in S$ such that $C(S) = b$ I shall define bP^ia. I shall define aP^ib whenever it is not bP^ia. Clearly, P^i is an asymmetric binary relation on $X = \{a, b\}$, which can be completed to an anti-symmetric relation by assuming reflexivity. I will now show that it, indeed, generates the original committee choice structure,*

[7] That is, if $S, T \in \mathcal{E}$, then $S \cup T \in \mathcal{E}$.

i.e. that $b = C(S)$ *if and only if* bP^ia *for all* $i \in S$ *(note, that showing this would also imply that* $a = C(S)$ *if and only if there exists at least one* $j \in S$ *such that* aP^ib*). Clearly, if* $b = C(S)$, bP^ia *follows by construction for all* $i \in S$*. Suppose* bP^ia *for all* $i \in S$ *but* $C(S) = a$*. From the construction of* P^i *we know that for each* $i \in S$ *there exists* $i \in S_i \in \mathcal{E}$ *such that* $C(S_i) = b$*. Since* $\mathcal{E}$ *is closed under unions, by* $U1$ *it follows that* $C(\cup_{i \in S} S_i) = b$*. Furthermore, by* $U1$ *we know that since* $C(Q \cup W) = C(S) = a$*, so we must have* $C(S \cup Q \cup W) = a$*. Since* $S \subset (S \cup W) \subset (S \cup W \cup Q)$*, it follows by* $U2$ *that* $C(S \cup W) = a$*. But by* $U1$ *we have* $C((\cup_{i \in S} S_i) \cup W) = b$ *and* $W \subset (S \cup W) \subset ((\cup_{i \in S} S_i) \cup W)$*, so that by* $U2$ $C(S \cup W) = b$ *- a contradiction. Q.E.D.* □

Though simple, this proposition provides us with a characterization, that allows us to test a hypothesis that the groups were making their choices using a common voting rule - and, in fact, it allows us to determine from data what the default is even if we do not know it.[8] Of course, this assumes that each voter was taking a decision without regard to the presence or votes of the other committee members: an assumption plausible if we are dealing with preference aggregation in a legislature (in which case sincerely voting one's preference would be a weakly dominant strategy), but less likely if we are dealing with information aggregation in a jury setting, where strategic voting might be important.[9] Hence, the condition developed here could be used as a test of voting sincerity - and, consequently, of preference versus information aggregation.

The assumption that $\mathcal{E}$ is closed under unions appears restrictive, as it would fail in many realistic circumstances. Thus most *tutela* decisions in Colombia are adopted by 3-member court panels. Likewise, in many experimental papers analyzing group decision-making report data on decisions by fixed-size overlapping committees.[10] Clearly, the fixed committee size would preclude $\mathcal{E}$ being closed under unions. Fortunately, this assumption can be easily relaxed. To do this, it will be convenient to divide $\mathcal{E}$ into the collections of committees deciding for a and for b. Thus, let $\mathcal{A} = \{S \in \mathcal{E} : C(S) = a\}$

[8] It easily follows from the proof of the proposition above that, except in the trivial case where the choice is the same for all committees, the choice of the default is uniquely implied by the data.

[9] See, for instance, Feddersen and Pesendorfer (1996).

[10] For a recent survey of such studies see, for instance, Kugler *et al.* (2012). The committee overlap typically arises there from rematching committee members in order to avoid repeated interaction effects between experimental subjects.

and $\mathcal{B} = \{S \in \mathcal{E} : C(S) = a\}$. Even if a particular committee T is never observed in the data, if it can be decomposed into a union of elements of either $\mathcal{A}$ or $\mathcal{B}$, *expansion* would allow us to impute its decision. Hence, I shall define $\mathcal{A}^*$ and $\mathcal{B}^*$ to be the smallest supersets of, respectively, $\mathcal{A}$ and $\mathcal{B}$ that are closed under unions. With this in mind one can modify assumptions $U1$ and $U2$ as follows

$W1$ (expansion): $\mathcal{A}^* \cap \mathcal{B}^* = \varnothing$.
$W2$ (no reversal from a): let $S \subset T$ then if $S \in \mathcal{A}^*$ then $T \in \mathcal{A}^*$.

It is straightforward to check that the proof of the preceding proposition still follows through, with minor modifications. Indeed, the necessity of the two conditions is obvious. Let $\mathcal{E}^*$ be the smallest superset of $\mathcal{E}$ that is closed under unions. We may define an *extension* of C as follows

$$C^*(S) = \begin{array}{c} b, \text{ if } S \in \mathcal{B}^* \\ a, \text{ if } S \in \mathcal{E}^* \backslash \mathcal{B}^* \end{array}$$

Clearly, under assumptions $W1$ and $W2$ C^* coincides with C on $\mathcal{E}$ and satisfies assumptions $U1$ and $U2$ on $\mathcal{E}^*$. As the necessity of the two assumptions is straightforward, we obtain the following corollary:

Corollary 1 *A committee choice structure over $\mathcal{E}$ may be rationalized by unanimity with default a if and only if it satisfies $W1$ and $W2$.*

Example 1 *Consider $N = \{1, 2, 3, 4\}$. Suppose the following choices are observed: $C(\{1, 2\}) = C(\{2, 3\}) = C(\{3, 4\}) = b$; $C(\{1, 3\}) = C(1, 4) = a$. Though we do not observe choices of all of these committees,$\{1, 3\} \in \mathcal{B}^*$, $\{1, 3, 4\} \in \mathcal{A}^*$ and $\{1, 2, 3, 4\} \in \mathcal{B}^*$, which, of course, would contradict $W2$. Hence this choice structure cannot be rationalized by unanimity with either default.*

3 Weighted majority

Suppose now that we conjecture that, instead of the unanimity with a default, the group is using a weighted majority rule, in which each agent votes for his favorite alternative, but, potentially, different voter's decisions are accounted with different weight (the weights themselves being unknown). For the moment, I shall continue working with the binary choice space, $X = \{a, b\}$. In order to participate, each agent would have to submit a vote $v_i : X \rightarrow \{0, v_i\}$, where $v_i > 0$ stands for the voting weight of individual i. As in the previous section, I shall assume that the votes (and, hence, the individual weights) do not depend on who else is in the committee.

Given a vote from each of its members a committee S chooses an alternative that gets the highest score

$$C^{wm}(S) = \arg\max_{x \in X} \sum_{i \in S} v_i(x)$$

where $\sum_{i \in S} v_i(x)$ is called the *score* received by an alternative $x \in X$ in voting by committee S. Such a choice structure is said to be generated by a weighted majority rule. Note, that, since there could be voting ties, in this section I shall allow the choices to be multivalued, so that, in general, $C^{wm}(S)$ would be a (non-empty-valued) correspondence.

It should be noted (following Myerson 1995), that one could allow agents to submit votes that are distinct from reporting their preference orderings, whatever these may be. In fact, for the purposes of defining the weighted majority rule one does not need to assume that the votes themselves derive from rational (complete and transitive) preferences. All the weighted majority rule requires agents to do, is to report a ranking of alternatives in X by means of their votes $v_i \in \mathbb{R}^2$. In general such a ranking may not necessarily represent a rational preference. However, it may be convenient to view the votes as representing some underlying preference relations, so that $v_i(x) \geq v_i(y)$ if and only if $x \succsim^i y$.

It is important to observe, that an individual assigning the same score to each of the alternatives would be equivalent to an abstention, and is allowed here. However, if an individual abstains in a decision by one committee, we shall assume s/he abstains in every committee of which s/he is a member. If the votes reflect an underlying preference, this may be viewed as a direct consequence of sincere voting. In that case, the characterization of this section may be taken as a test for the hypothesis of "sincere voting". As in the case of the unanimity with a default rule, the weighted majority rule in the case of two alternatives does not create any incentives for strategic voting: voting sincerely is, in fact, a weakly dominant strategy.

If a committee choice structure $(\mathcal{E}, C(.))$, where $C : \mathcal{E} \rightrightarrows X$ is a non-empty-valued: correspondence, is such that for any $S \in \mathcal{E}$

$$C(S) = C^{wm}(S)$$

where the votes are consistent with preferences for some rational preference profile $\succsim$. I shall say that $\succsim$ rationalizes $(\mathcal{E}, C(.))$ via a weighted majority rule.

It is clear that not every such committee choice structure would be rationalizable with weighted majority. Crucially, the notion of weighted majority studied here implies that each individual's votes are independent of the committee composition. Hence, if we ever observe that for two disjoint committees $S \cap T = \varnothing$ we have $C(S) = C(T) = \{x\}$ it must, indeed, follow that $C(S \cup T) = \{x\}$. This, property, introduced, for instance, in characterizations of scoring rules by Smith (1973) and Young (1975) is usually known as the *reinforcement* axiom and, at least in the single-valued choice case, is, clearly, implied by the *expansion* condition of the previous section. Clearly, reinforcement must be a necessary condition for the rationalizability here desired. But the weighted majority has an even stronger implication for the actual scores that committees assign to alternatives: the score difference between the alternatives must be added up if two disjoint committees are joined.

In fact, if weighted majority is the rule used, the difference w between the scores assigned to x_1 and to x_2 by the committee S

$$w(S) = \sum_{i \in S} v_i(a) - \sum_{i \in S} v_i(b)$$

will define a (signed) measure on the finite measurable space $\left(N, 2^N\right)$, as long as one naturally sets $w(\varnothing) = 0$, since $w(S \cup T) = w(S) + w(T) - w(S \cup T)$ for any two committees $S, T \in 2^N$.

However, we do not observe the actual scores or their differences, but only choices, which correspond to the sign of w. Defining $\mathcal{E}^* = \mathcal{E} \cup \varnothing$ it may be convenient to summarize our observations with a function $f : \mathcal{E}^* \to \{-1, 0, 1\}$ defined by the

$$f(S) = sign(w(S)) = \begin{cases} -1, \text{ if } C(S) = \{b\} \\ 0, \text{if } C(S) = \{a, b\} \text{ or } S = \varnothing \\ 1, \text{if } C(S) = \{a\} \end{cases}$$

This function f is, of course, non-additive. If, however, we can, consistently with it, assign individual vote differences w_j to each individual in such a way that

$$sign\left(w\left(S\right)\right) = sign\left(\sum_{j\in S} w_j\right) = f\left(S\right)$$

, we shall obtain a weighted–majority-based theory that would explain how the observed choice structure arose!

Fortunately, it turns out that this problem is closely related to well-established problems in utility theory. In fact, a very similar mathematical problem emerges if one considers the question of when could a binary relation "at least as likely as" over a finite states space be represented by a probability measure, which has been posed and solved by Kraft et al. (1959). The following example, which is, essentially, a reinterpretation for the present setting of the one they constructed, implies that the reinforcement alone, though necessary, is not sufficient for such a theory to be possible.

Example 2 *Suppose* $N = \{1, 2, 3, 4, 5\}$ *and* $f(\{4\}) = f(\{2, 3\}) = f(\{1, 5\}) = f(\{1, 3, 4\}) = 1$ *whereas* $f(\{1, 3\}) = f(\{1, 4\}) = f(\{3, 4\}) = f(\{2, 5\}) = -1$. *The example does not violate reinforcement. However, it is not hard to see that this set of choices is not consistent with weighted majority, as it would imply that* $2w_1 + w_2 + 2w_3 + 2w_4 + w_5$ *is simultaneously positive and negative!*

Consequently, a stronger condition, which I shall call *strong reinforcement,* is required, which is analogous to strong additivity of Kraft et al. (1959). Following Fishburn (1986) it can be presented as follows. Consider two collections (of equal cardinality) of committees $\mathbf{S} = (S_1, S_2, ..., S_m)$ and $\mathbf{T} = (T_1, T_2, ..., T_m)$. Note, that an empty set is taken here as a possible committee and that a committee might be repeated several times within a collection. Denote as $n_j(\mathbf{S})$ the number of committees in the collection $\mathbf{S}$ that individual j is included in. We say that $\mathbf{S} \approxeq \mathbf{T}$ if for each individual $j \in N$ $n_j(\mathbf{S}) = n_j(\mathbf{T})$.

I am now ready to define the following condition of choice structures.

$W1$. The choice correspondence C satisfies *strong reinforcement* if for each pair of committee collections $\mathbf{S}, \mathbf{T}$ such that $\mathbf{S} \approxeq \mathbf{T}$ if $f(S_i) > f(T_i)$ or $f(S_i) = f(T_i) = 0$ for $i = 1, 2, ..., m-1$ then not $f(S_m) > f(T_m)$.

It should be noted that strong reinforcement is indeed a strong property, which implies a number of desirable conditions of the choice structures. Thus, it can be easily seen to imply the reinforcement property itself. It turns out that, in fact, this condition characterizes choice structures that can be explained with a weighted majority vote.

Theorem 1 *A committee choice structure* $(\mathcal{E}, C(.))$ *may be generated by a weighted majority rule strictly consistent with rational preferences if and only if the choice structure satisfies strong reinforcement.*

Proof. The necessity part is straightforward, since if it were not the case, there would exist a pair of committee collections $\mathbf{S} \approxeq \mathbf{T}$ such that $f(S_i) > f(T_i)$ or $f(S_i) = f(T_i) = 0$ for all $i = 1, 2, ..., m-1$ and $f(S_m) > f(T_m)$ However, as $f(S_i) = sign(w(S_i)) = sign\left(\sum_{j \in S_i} w(\{j\})\right)$ it follows that $\sum_{j \in S_i} w_j > \sum_{j \in T_i} w_j$ or $\sum_{j \in S_i} w_j = \sum_{j \in T_i} w_j = 0$ for $i = 1, 2, ...m-1$ and $\sum_{j \in S_m} w_j > \sum_{j \in T_m} w_j$, which, if we some across the committees in each collection, in turn would imply that $\sum_{j \in N} n_j(\mathbf{S}) w_j > \sum_{j \in N} n_j(\mathbf{T}) w_j$.

The proof of sufficiency closely follows that of Theorem 4.1 in Fishburn (1970). If all committees make the same choice, the theorem is trivially true, therefore, I shall henceforth assume that there exists at least one pair of committees $(S, T) \in \mathcal{E}^* \times \mathcal{E}^*$ such that $f(S) > f(T)$ Let $K \in \mathbb{N}$ be equal to the number of distinct committee pairs $S, T \in \mathcal{E}$ $(S \neq T)$ such that $f(S) > f(T)$ and $M \in \mathbb{Z}_+$ be equal to one half of the number of committee pairs $(S, T) \in \mathcal{E}^* \times \mathcal{E}^*$ such that $f(S) = f(T) = 0$. Note that the latter includes committee pairs of the form $(S, \varnothing)$ and $(\varnothing, T)$. Clearly, $K + M \leq 2^{2n} < \infty$.

For each committee S let the indicator function

$$1_S(j) = \begin{cases} 1 \text{ if } j \in S \\ 0 \text{ if } j \notin S \end{cases}$$

Clearly, if for each of the first $k = 1, 2..., K$ committee pairs S^k, T^k defined above we may write

$$\sum_{j=1}^{n} w_j a_j^k > 0$$

and for each of the following $k = K+1, K+2, ..., K+M$ committee pairs S^k, T^k we may write

$$\sum_{j=1}^{n} w_j a_j^k = 0$$

where $a_j^k = (1_{S^k}(j) - 1_{T^k}(j)) \in \{-1, 0, 1\}$, and the weights $\sum_{j=1}^{n} w_j$ may be interpreted as a "reconstruction" of the individual vote difference consistent with the observed choice structure (note, in particular, that this would imply that $\sum_{j=1}^{n} w_j 1_S(j) = 0$ for every S such that $f(S) = 0$).

Suppose this is impossible. Then by Theorem 4.2 in Fishburn (1970), known as the Theorem of the Alternative, there must exist a collection of numbers $r_{k,}$ $k = 1, 2, ..., M+K$, such that the first K of these are non-negative and not all zero so that for every $j = 1, 2, ..., n$

$$\sum_{k=1}^{K+M} r_k a_j^k = 0$$

In fact, since all a_j^k are rational by construction, all r_k may be chosen to be integers. If for some $k > K$ there is an $r_k < 0$ one may replace a_j^k with $-a_j^k$ to make it positive (this is possible since if $f\left(S^k\right) = f\left(T^k\right)$ one may interchange S^k and T^k). Consider now two committee collections $\mathbf{S}$ and $\mathbf{T}$ such that each committee S^k is repeated r_k times in $\mathbf{S}$ and each committee T^k is repeated r_k times in $\mathbf{T}$. By construction the cardinality of each committee collection is equal to $\sum_{k=1}^{K+M} r_k$ and from the preceding equation it follows that the number of times each individual is included in committees in each collection is

$$n_j(\mathbf{S}) = \sum_{k=1}^{K+M} r_k 1_{S^k}(j) = \sum_{k=1}^{K+M} r_k 1_{T^k}(j) = n_j(\mathbf{T})$$

and, hence $\mathbf{S} \approxeq \mathbf{T}$. But by construction we have $f\left(S^k\right) \geq f\left(T^k\right)$ for all $k = 1, 2, ...K+M$, with the first K inequalities strict. Hence, the strong reinforcement of the committee choice structure is violated. QED

Notably the proof of the theorem provides us not only with a way of falsifying the "weighted majority theory" of the committee decision-making, but also, assuming the strong reinforcement holds, with a way of calculating individual weights consistent with choices: except for the abstaining individuals, these would be the solutions w_j to the inequality system used in the proof.

4 Scoring and multiple alternatives

In the previous section I have assumed that the (not directly observed) weight v_i was an intrinsic characteristic of individual i. Alternatively, one may take a view that the weights arise from the manner in which a decision rule "converts" individual preferences into scores. Thus, for instance, in a simple (unweighted) plurality rule a score of 1 is assigned to the top choice of an individual and scores zero to all other alternatives, while under the Borda count the top alternative gets the maximal score n, the next best alternative gets a score of $n-1$, etc. The class of rules, such as plurality and Borda count, in which individuals are asked to provide each alternative with a numeric score (reflecting their preferences), the individual scores are added up and the alternative with the highest aggregate score is chosen is known as *scoring*. These rules have long been characterized by social choice theorists (see, for instance, Young 1975 or Myerson 1995) and are frequently used in practice. In this section I shall consider the problem of which committee choice structures could be rationalized by *some* scoring rule. This problem has previously been considered this problem in my earlier note (Gomberg 2011), though that note provides only a necessary consequence of decision-making by scoring, rather than a complete characterization of the committee choice structures rationalizable with scoring.

For the case of two alternatives, $X = \{a, b\}$, in fact, the problem is, essentially, identical to the weighted majority problem I considered in the previous section, the only difference being that the weights are now taken to be a property of an (directly unobserved) voting rule rather than of the (directly unobserved) relative power of agents. However, this view provides a natural interpretation for the extension to the case of multiple alternatives, $X = \{x_1, x_2....x_n\}$, which I shall consider here.

The scoring rules require agents to report a ranking of alternatives in X by means of their votes $v_i : X \to \mathbb{R}$. As before, following Myerson (1995), it is not necessary to assume that these actually come from an underlying preference, though, for interpretational reasons we shall still find it convenient to assume that $v_i(x) \geq v_i(y)$ if and only if $x \succcurlyeq^i y$. In the case of multiple alternatives, however, strategic voting incentives may arise even when the group's task is to aggregate preferences, rather than information. In this sense, the interpretation of the characterization that follows as providing a test for *sincere* voting, becomes most natural.

Similarly to the previous section, if a committee choice structure $(\mathcal{E}, C(.))$, where $C : \mathcal{E} \rightrightarrows X$ is a non-empty-valued: correspondence, is such that for any $S \in \mathcal{E}$

$$C(S) = C^{scoring}(S)$$

where the votes are consistent with preferences for some rational preference profile $\succsim$. I shall say that $\succsim$ rationalizes $(\mathcal{E}, C(.))$ via a scoring rule.

If there are three or more alternatives the problem cannot be reduced to that of an existence of a single measure on the committee space. Nevertheless, as long as all the committees are facing the same choice problem (i.e., the budget set B is not varied), the linear structure of the scoring rules utilized in the previous section allows for a very similar formulation.

Our basic objective remains the same: to find vote scores for each individual that would explain the observed committee choices. Notably, once there are at least three alternatives, we now will have to avoid "scoring cycles", as the following example shows.

Example 3 *(Gomberg 2011) Consider the alternative set $X = \{a, b, c\}$ and the four disjoint committees S_1, S_2, S_3 and T. Let $C(S_1) = a$, $C(S_2) = b, C(S_3) = c$, $C(S_1 \cup T) = b, C(S_2 \cup T) = c, C(S_3 \cup T) = a$. It is not hard to see that this implies that in committee T, b would have to gets higher score than c, c higher than a and a higher than b, which, of course, is an impossible cycle.*

As the example above suggests, the scores may be "revealed" through observed committee choices (the revelation idea first introduced in Gomberg 2011). For the rest of this section, I shall assume that there is a finite set of alternatives , $X = \{x_1, x_2, ...x_m\}$. The rest of the model is as before.

- *Direct revelation.* For each $S \in \mathcal{E}$ a pair of nested binary relations $P_S^* \subset R_S^*$ on X is defined by

  (i) let $x \in C(S)$ then xR_S^*y for any $y \in X$

  (ii) let $x \in C(S)$ and $y \notin C(S)$ for some $y \in X$ then xP_S^*y

This constitutes a record of direct preference revelation: if an alternative is chosen, it implies it received at least as high a score as any other feasible alternative and a strictly higher score than any feasible alternative not chosen.

Consider the total set of observations we have. If our theory is correct and this choice is rationalized with scoring, in the actual vote count each observation of xP_S^*y it must have been obtained from $\sum\limits_{i\in S} v_i\left(x\right) > \sum\limits_{i\in S} v_i\left(y\right)$ and each xR_S^*y from $\sum\limits_{i\in S} v_i\left(x\right) \geq \sum\limits_{i\in S} v_i\left(y\right)$. These are, of course, linear inequalities. In fact, the set of all "revealed scoring" statements must have been generated by a system of linear inequalities, which would have to hold simultaneously for the rationalization to be possible.

As the cardinality $\#X = m$, consider a vector $w = (w_1, w_2, ..w_n, w_{n+1}, ...w_{2n}, ...w_{nm}) \in R_+^{nm}$ where w_{kn+j} corresponds to the reconstructed vote that agent k emits for alternative j. As in the previous section, I shall consider each revealed scoring statement (taking care to track the committee by which it has been generated). As the total number of such statements is finite, let K be the number of strict statements xP_S^*y and M be one half of the rest.

Consider a list of all such revealed scoring pairs If the kth pair is $x_pP_S^*x_r$ (for the first K elements of the list) or $x_pR_S^*x_r$ (for the rest) then one can define $a_{jk} = 1$ for all $j = p + ms$, where $s \in S$, $a_{jk} = -1$ for all $j = r + ms$, where $s \in S$, and $a_{jk} = 0$ otherwise. The corresponding matrix $A = (a_{jk})$, which contains the relevant information about the observed choices, shall be called the *scoring matrix.*

As in the case of two alternatives, if for each of the first $k = 1, 2..., K$ revealed preference scoring relations defined above we may write

$$\sum_{j=1}^{n} w_j a_{jk} > 0$$

and for each of the following $k = K + 1, K + 2, ...K + M$ revealed scoring relations we may write

$$\sum_{j=1}^{n} w_j a_{jk} = 0$$

we would rationalize the observed choice structure.

As in the previous section, the Theorem of the Alternative allows one to restate the problem of existence of a solution to this system of inequalities as a problem of existence of a solution to the equation

$$\sum_{k=1}^{K+M} r_k a_{jk} = 0 \qquad (*)$$

where $(r_1, r_2, ...r_{K+M}) \in \mathbb{Z}^{K+M}$ with the first K terms non-negative and not all equal to zero. This, of course, constitutes the proof of the following proposition:

Proposition 2 *A committee choice structure $(\mathcal{E}, C(.))$ may be generated by a scoring rule strictly consistent with rational preferences if and only if the equation $Ar = 0$ (where A is the associated scoring matrix) has a non-zero integer solution r, with the first K coordinates of r all non-negative.*

As in the case of two alternatives, this condition is, in fact, necessary and sufficient for the existence of rationalization by scoring, though it is harder to get its intuitive interpretation. A greater feeling for its implication may be obtained if we reformulate a necessary implication of it in a more familiar "revealed preference" form (as earlier proposed in Gomberg 2011).

Consider, for instance, the "indirect revealed scoring" implied by the reinforcement property of the scoring rules (which, as noted above states that if two disjoint committees make the same choice from a given budget set, so should their union). We can then define the following .

- *Reinforcement*[11]

  The binary relations $P_S \subset R_S$ on X are defined by

  (i) xP^*y implies xPy, xR^*y implies xRy,

  (ii) For any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \cap T = \varnothing$, xR_Sy and xR_Ty imply that $xR_{S\cup T}y$

  (iii) For any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \cap T = \varnothing$, xP_Sy and xR_Ty imply that $xP_{S\cup T}y$

  (iv) For any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \subset T(T \setminus S \neq \varnothing)$, xP_Sy and yR_Tx imply that $yP_{T\setminus S}x$

  (v) For any $S, T \in 2^N \setminus \{\varnothing\}$ such that $S \subset T(T \setminus S \neq \varnothing)$, xR_Sy and yP_Tx imply that $yP_{T\setminus S}x$

[11] Note that the example above shows that a stronger indirect extension could be imposed here. However, reinforcement is more intuitive, so I stick to it as a necessary implication of rationalizability.

With this in mind we may now define a simple acyclicity condition, motivated by the example above:

$S1$ (Committee Axiom of Revealed Preference, CARP)[12]: For any $S \in 2^N \setminus \{\varnothing\}$ and any $x_1, x_2, ...x_n \in X, x_1 R_S x_2, x_2 R_S x_3 ... x_{n-1} R_S x_n$ implies $\neg (x_n P_S x_1)$

It is straightforward to see that CARP is, in fact, implied by scoring (its violation would imply a committee S assigning to an alternative x_1 both a higher and a lower score then to an alternative x_n, which is impossible). Hence, we have the following proposition:

Proposition 3 *A committee choice structure $(\mathcal{E}, C(.))$ may be generated by a scoring rule strictly consistent with rational preferences only if the implied R_S and P_S satisfy CARP for each $S \in 2^N \setminus \{\varnothing\}$.*

5 Related Literature

It would be incorrect to claim, that using group choice data to obtain revealed-preference-like conclusions has never been suggested. Thus, for instance, when Blair *et al.* (1976) characterized such restrictions on choice structures as would derive from maximizing preferences that are merely acyclic, rather than transitive, this could, of course, be interpreted as characterizing choices made by committees of rational members with some of those members exercising veto power. The previously mentioned "non-falsification" result by McGarvey (1953) has been extended by Deb (1976) to certain voting rules other than majority, strengthening the point that without more data theories of internal committee rules may not be testable. However, when the size of the committee is known, restrictions implied by various group choice rules on the minimal cycle length in choices have been studied by Nakamura (1979).[13] In a different context, Peters and Wakker (1991) discussed empirical consequences of bargaining solutions, as did, for instance, Chambers and Echenique in a recent paper (2013).[14] Characterization of the empirical

[12]The naming suggestion for this axiom, originally introduced in Gomberg (2011), belongs to Norman Schofield.

[13]See Chapter 3 of Austen-Smith and Banks (1999) for a textbook treatment.

[14]Notably, in that latter paper the authors demonstrate that, unless the threat point data is available and variable, the variation on the size of the surplus is not sufficient to distinguish among some major bargaining solution concepts.

consequences of such behavior for the household demands by Browning and Chiappori (1998) has been particularly influential.

However, the context in which the revealed preference approach has, perhaps, been most productive recently, is that of individual decision-making, in particular in modeling "boundedly rational" decision-making procedures different from the usual rational preference maximization. Perhaps the most obvious parallel to the present study in the new behavioral literature is presented by the "multi-self" models of decision-making, such as Kalai *et al.* (2002), Green and Hojman (2007) and Ambrus and Rozen (2013).[15] Thus, Green and Hojman's (2007) approach to individual choice behavior as resulting from aggregation of multiple rational rankings arising from distinct *motivations*, has most clear similarities with mine, in as much as they consider the use of scoring rules to do the aggregation. In fact this approach is formally analogous to the preference aggregation in a committee (with each of the "motivation" corresponding to a committee member). Somewhat similarly, Ambrus and Rozen (2013) consider behavior that can be explained by aggregation of multiple utility functions (those of different "selves" in one's mind). As both Green and Hojman (2007) and Ambrus and Rozen (2013) observe, without any restrictions on the number of motivations/utility functions inside one's mind, most aggregation rules would not provide falsifiable restrictions on the resulting choices. However, if the cardinality of the set of "selves" were to be known, such restrictions would, indeed, emerge.[16]

One key distinction between Green and Hojman's (2007), Ambrus and Rozen (2013) and the present paper is, in fact, in the sort of the data that is available for testing the theory. Whereas the "selves" inside one's mind are not directly observable and may only be inferred from individual behavior, in the context of collective decisions the group membership might itself reasonably constitute part of the observed data. Not only that, but it may be natural to assume that one could observe variation not only of the set of available alternatives, as is standard in the revealed preference literature, but also of group composition itself: for instance, we may observe decisions on a given issue of different parliamentary committees and subcommittees, the membership of which is known, or use changes in court or monetary

[15] Another related result is provided by Bogomolnaia and Laslier (2007), who establish results on a number of policy space dimensions that would suffice for representing a given preference order with a Euclidean distance.

[16] This would be the interpretation most similar to the Nakamura (1979) results mentioned above.

committee composition.

As noted in the introduction, the objective of this paper clearly comes close to the study of empirical content of sincere (versus strategic) voting by Degan and Merlo (2009) and to Kalandrakis (2010) work on rationalizing individual voting decisions. Another closely related line of research is that by Apesteguia *et al.* (2014), who characterize observable outcomes of sincere and strategic application of various agenda rules employed in legislatures. To reiterate, where my approach crucially differs from this earlier work, however, is that I do not assume observability of individual votes (nor do I impose anything, in addition to rationality, on individual preferences), but try to infer votes from observing the group choice data.

Finally, it should be noted that the idea that committee membership variation may be used as a source of "revealed voting", as well as Example 3, the CARP, and a version of Proposition 3 in this paper (all of which are included here, mostly, for completeness), were previously presented in my earlier volume note (Gomberg 2011). With those exceptions, all formulations and results of this paper, however, postdate that note.

6 Conclusions and further research

The objective of this paper is to explore how observations of group actions may used to test theories about within-group decision procedures. For this purpose, I introduce the notion of a committee choice structure and establish a necessary and sufficient condition for such a choice structure to be rationalizable via two common voting rules: unanimity with a default and scoring, when the committees decide over the fixed budget set, with a natural interpretation for the case of two alternatives.

A key insight of the paper is that observations of collective decisions by themselves might be revealing, as long as we may observe variations on group membership. The data-theoretic characterizations developed here may be used to develop tests for voting sincerity based purely on the outcomes of group decisions.

As mentioned in the introduction, data sets resembling those suggested in this paper might emerge from a corpus of court decisions, especially those from legal systems in which prior decisions do not constitute legal precedent. Compiling and analyzing such data set for insights on the preferences of court members remains to be done.

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