A laboratory study of Demand Reduction and Collusion in Uniform- and Discriminatory-Price Auctions [*]

by

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Abstract

We report results of an experimental study of multi-object uniform and discriminatory-price auctions in an environment of publicly known common values, concentrating on an environment where theory predicts sharply different results of the two auction formats. We find that the bidding behavior in the uniform case exhibits two clear regularities: agents consistently play weakly dominated strategies by overbidding on the first unit and have moderate difficulty coordinating on the high payoff (low auction revenue) equilibrium predicted by theory. However, subjects with experience in the same environment are better at reducing demand to achieve higher payoff. Bidders in discriminatory auctions, as predicted, tend to submit bids close to value for all units and are not generally successful in attempts at collusion.

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1 Introduction

During the 1990s the U.S. Treasury Department conducted a large-scale "field experiment" by auctioning the 2- and 5-year notes using a uniform-price format. Having concluded that the initial trial was successful in encouraging more aggressive bids, compared with the alternative discriminatory-auction format (Malvey and Archibald 1998), it, eventually, extended the use of the uniform-price auctions to sales of other government bonds as well. Laboratory testing of uniform- and discriminatory-price auctions in the common value no uncertainty environments, which approximate sales of nearly riskless government bonds, has been conducted by Goswami *et al.* (1996) and Sade *et al.* (2006). Notably, the results of these studies seem to be inconsistent. In fact, contradicting the Goswami *et al.* (1996) findings, as well as some theoretical predictions, but in accordance with the U.S. Treasury experience, Sade *et al.* (2006) find that uniform auction format results in higher revenue. The interpretation of the outcomes of these studies is somewhat complicated by equilibrium multiplicity (inherent especially in the uniform-price auctions), which require multiple (and, perhaps, somewhat *ad hoc*) refinements to obtain clear theoretical comparisons. In particular, while theory predicts that low-revenue equilibria exist in uniform-price settings, the analysis of relatively complicated environments studied until now involves choosing among a large number of equilibria, only some of which imply low revenues for the auctioneer. It, thus, remains unclear under which conditions such low-revenue outcomes are likely to be observed. We believe our experimental results, in a simple environment structured to give sharp theoretical predictions, may help answering this question.

The basic uniform-price set-up is to auction a number of identical objects and to allow bidders to submit a schedule specifying how many of these they'd be willing to purchase at each given price. It can be implemented by asking each agent to submit as many separate bids as there are objects to be auctioned. The highest bids get allocated the objects (ties resolved randomly) at the uniform market-clearing price. In contrast, in the alternative discriminatory-price format each winning bid has to be paid in full.

At a first glance, the uniform-price auction seems an easy generalization of the standard Vickrey second-price auction However, already Vickrey (1961) observed that when individual agents may bid for multiple objects, the uniform-price auction is not the appropriate extension. Since then, Wilson (1979), Back and Zender (1993), Ausubel and Cramton (1996), Engelbrecht-Wiggans and Kahn (2005), among others, have shown that in a wide variety of environments uniform-price (and the related simultaneous-bid ascending) auctions may (in some equilibria) allow for equilibrium bidder collusion leading to extremely low seller revenue (this is impossible in the discriminatory auction).

Our approach in this paper is to design an environment, in which low-revenue equilibria of the uniform-price auctions would be naturally selected. We concentrate on a setting (selling a fixed number of cash bundles to an identical number of subjects, while allowing arbitrary non-negative bids) in which a uniform-price auction implies that the unique equilibrium, surviving one round of elimination of weakly dominated strategies, is highly collusive, resulting (theoretically) in full surplus extraction by the buyers. In contrast, the seller should receive (nearly) the entire surplus if a discriminatory-price auction is used in an otherwise identical situation. In a sense, what we attempt to do is to design perfect conditions for a low-revenue "disaster" (something that we can deliberately do in a lab) and see if it materializes.

In the process of testing both auction environments in the lab we discovered that successful cooperation on low-revenue equilibria is not straightforwardly achieved even in this case. However, experienced subjects, who have an opportunity to communicate with each other out of the lab, substantially improve their performance (this result recalls the earlier finding of Goswami *et al.* 1996). Notably, out-of-equilibrium collusion in discriminatory-price auctions, even when attempted, seems difficult for the subject s to sustain.

Within the larger literature on experimental uniform-price auctions in various environments (with exogenous uncertainty), our experimental design is related to Kagel and Levin (2001).

The rest of this paper is organized as follows. Section 2 presents the description of the game and of its equilibria. Section 3 explains the experimental design. Section 4 presents experimental findings and Section 5 concludes.

2 The Game

Unlike Goswami *et al.* (1996) and Sade *et al.* (2006), who sell a large number (respectively, 100 and 26) of objects to relatively numerous (respectively, 11 and 5) bidders, we choose to concentrate on smaller groups and fewer objects. Throughout the paper, we shall restrict our attention to a fixed number of three bidders $i = 1, 2, 3$. In every auction we consider selling n identical and perfectly substitutable objects valued at $v \in \left[0, \bar{v}\right]$. This value is common knowledge among the bidders who can submit as many bids as there are objects on sale. We shall denote the individual vector of bids as $b_i = (b_{i1}, b_{i2}, ..., b_{in}) \in \left[0, \bar{b}\right]^n$ $(i \in I)$; it should be noted that, unlike in the two above-cited studies, arbitrary bids (including those above value) were allowed.[1] For the uniform-price auctions the winners have to pay the same market-clearing price p (we shall assume that all ties are resolved randomly). We choose this price to be equal to the highest loosing ($(n+1)$st-highest) bid. In every auction, therefore, an agent's payoff will be equal to $(v - p)$ times the number

[1] In practice, this was implemented by using graphical software to allow subjects to submit fractional bids on a very fine grid (without loss of generality we shall assume that the bids are ordered: $k > j$ implies $b_{ij} \geq b_{ik}$).

of objects s/he wins and the revenue of the seller is equal to np. In contrast, in the discriminatory-price auctions each winning bid gets paid in full.

As usual, in the uniform-price auctions there is a continuum of Nash equilibria. Going back to Vickrey (1961) weak dominance has been considered a suitable refinement for analyzing second-price and uniform auctions. As in second-price auctions, every weakly undominated strategy involves never bidding above value, but submitting the full-value bid for, at least, one object (in fact, the argument is, essentially, the same as the one originally employed by Vickrey 1961). If one assumes that others, playing undominated strategies, have, in total, submitted, at least, 2 "honest" bids, then, for a two-object auction, there is no strategy that would result in a positive payoff. However, if the same assumption is made by a participant in a three-object auction, there is, among his/her own undominated strategies, one that is always a best response, irrespective of which undominated strategies his/her competitors choose. Indeed, if s/he submits more than one bid equal to value, s/he guarantees the zero payoff for him/herself (as well as for the others). Lowering the second-largest bid ensures that it would not be winning, but it still might determine the clearing price. Hence, no matter what the others do (other than play an undominated strategy) setting the second bid as low as possible is optimal! The same argument allows one to reduce the number of equilibria in 4-unit auctions as well, but the multiplicity remains due to the asymmetry of this setting: unless the bidders can resolve who gets the fourth object, competition will drive up their second-largest bids, and dominance alone does not allow those to be pinned down. Finally (this follows, for instance, from Theorem 3 in Beck and Zender 1993), in discriminatory auctions the seller is guaranteed to be receiving the entire surplus, as all bids have to be equal to value (this is implied by the continuity of the bid space: if bids were restricted on a discrete grid, equilibrium underbidding would be possible; thus, choosing, in practice, a very fine grid allows us to further differentiate between the uniform- and discriminatory-price predictions). The following summarizes theoretical predictions for the auction environments in this paper:

Proposition 1 *(i) In the uniform-price auctions, given any number of objects n, for each agent $i \in I = \{1, 2, 3\}$ any strategy $b_i = (b_{i1}, b_{i2}, ...b_{in})$ such that $b_{i1} \neq v$ is weakly dominated by $b'_i = (v, b_{i2}, ...b_{in})$.*

(ii) In the uniform-price auctions if $n = 2$ then in every Nash equilibrium in weakly undominated strategies the clearing price $p = v$.

(iii) In the uniform-price auctions if $n = 3$ then if agents eliminate all weakly dominated strategies of other bidders, the strategy $b_i = (v, 0, 0)$ weakly dominates (v, x, y) for any $v > x, y > 0$. This implies the clearing price $p = 0$.

(iv) In the uniform-price auctions if $n = 4$ then for every agent bidding $b_i = (b_1, b_2, b_3, b_4)$ such that $b_3 b_4 \neq 0$ is weakly dominated by bidding $(b_1, b_2, 0, 0)$ in every pure strategy Nash equilibrium there must be at least one agent bidding $b_{i2} \geq \frac{v}{2}$.

(v) In the discriminatory-price auctions Nash equilibrium implies that at least n highest bids b_{ij} are equal to value v

Proof. *See appendix.* ■

3 Experimental Design

All experimental sessions were conducted at Instituto Tecnológico Autónomo de México (ITAM) in Mexico City and the subjects were undergraduates recruited in introductory economics courses. The experiments were computer-administered. Each experimental session had 15 to 30 participants per session.

During each session a constant number n of identical objects were repeatedly auctioned to each three-person group using the above-described uniform-price or discriminatory-price format. For each period agents were randomly matched into groups of three to participate in an auction (groups were randomly formed anew after each period and agents were not aware with whom they were playing in each round). The total value of all objects on sale to each group randomly varied each period between MN$20 and MN$100 Mexican pesos (slightly less than US$2 to US$10). This value was announced to agents at the beginning of each session before they made their bids and they were explicitly told that other members of the group received the same announcement. The agents had to make n non-negative bids (not exceeding $\frac{100}{n}$). After each round, agents learned the size of the top $(n+1)$ bids in their auction.

We conducted a total of 15 uniform-price sessions with 2, 3, or 4 objects sold and 6 discriminatory-price sessions with 3 objects sold. In addition, during the same time frame, we conducted 8 sessions of single-object second-price auctions and 2 pilot sessions of uniform-price auctions with, respectively, 6 and 15 objects sold (we report the results of these sessions, conducted for a separate study, in Elbittar and Gomberg 2007). Each session consisted of 5 practice periods followed by 20 periods of play for money. The total duration of a session (including detailed discussion of instructions and answering subjects' questions) was somewhat under 2 hours.

At the beginning of each session agents received a balance of MN$60 pesos. All earnings/losses were added each period to this balance. If a subject's balance fell below MN$20 pesos s/he was not allowed to bid further and was paid that remaining balance (in a couple of cases, where a subject's balance fell below zero - in no case this amount was worth more than a few U.S. cents - they were paid nothing). Since in this case the number of subjects in the room was no longer divisible by 3, some subjects would be randomly chosen each period not to participate (consequently, up to the period 19, the termination time remained random for individual subjects; our results seem to be robust to eliminating the data from period 20). After session 20 the accumulated balance was paid out to subjects in cash.

Treatment	No Experience	Relevant Experience	1-object experience
2-object uniform	2 sessions	2 sessions	
3-object uniform	4 sessions	2 sessions	1 session
3-object discrim.	4 sessions	2 sessions	
4-object uniform	2 sessions	2 sessions	

Table 1: Treatments Run

In order to study the impact of experience and communication on outcomes, we deliberately recruited subjects who had participated in earlier sessions. To encourage them returning, those who had participated in at least one prior session were offered an additional MN$60 participation fee for each new session they took part in. In order to facilitate communication among experienced subjects we made a particular effort to recruit participants in a given prior session to return together for another session. In these repeat sessions, experienced subjects were mixed with new inexperienced subjects. Subjects were not told that they would come back for the same experiment, and, in fact, for one three-object session we invited subjects "trained" in a single-object second-price auction. No attempt was made to prevent pre-session discussion of the experiment (in fact, such communication was repeatedly observed). Overall, there does not exist a way for us, without further experiments, to separate the effect of experience *per se*, and that of communication between subjects during the period between sessions. Even though, for brevity, in what follows we shall consistently refer to “experience”, we are cognizant of the possibility that what matters is the ability of subjects to communicate outside the lab. Table 1 summarizes the treatments we ran.

4 Results

We concentrate on, firstly, trying to determine whether the theoretical predictions of Proposition 1 are observable in the lab and, secondly, on the role experience (both within a session and in prior sessions) plays in determining the subject behavior. Our results, broadly, show two phenomena in the uniform-price auctions: agents tend to overbid on their higher bid, and, though reducing the size of their further bids (we shall call this demand reduction in what follows), do not, most of the time, do this sufficiently to reap high predicted payoffs. Neither overbidding on the highest bid, nor the demand-reduction on the second-highest bids is normally observed in the discriminatory-price auction (except in a small group of overbidding subjects that quickly exit due to bankruptcy).

As the subjects received a MN$60 peso show-up fee which they could have safely preserved by never overbidding (bidding above value) in any auction. Thus, playing undominated strategies would imply that no agent ever receives a total payoff of less then

Treatment	No Experience	Relevant Experience	1-object experience
2-object uniform	51/60	61/51	
3-object uniform	45/42/54/49	90/66	47
3-object discrim.	53/53/59/54	62/96	
4-object uniform	65/46	210/71	

Table 2: Average Payoffs per subject in each session (pesos)

MN$60 pesos. In fact, for the two-object auctions, in which the object price should be equal to value, this payoff is indeed what proposition 1 suggests for the subject payoffs after any number of experimental sessions. In contrast, in 3-object auctions the proposition suggests that in each auction one object should be assigned to each subject at no cost. Since the total value of objects on sale in each period averaged $60 pesos, this surplus (shared equally among the three bidders in each auction) should have accumulated, on average, MN$400 pesos after 20 rounds. The total predicted payoff for these sessions (including the show-up fee) was, therefore, equal to MN$460 (over US$40) per agent. For the four-object auction proposition 1 provides no precise prediction for payoffs, though the low revenue equilibria, as in the three-object case, are still possible. In the discriminatory-price auctions the predicted payoff is the initial $60 pesos. Table 2 presents the empirically observed average payoffs (in pesos, excluding the additional MN$60-peso show-up fee paid to experienced subjects for a repeat participation) per subject for each session.

Of the 13 sessions where subjects were not recruited to have experience in the same auction type, in 12 sessions agents on average lost between a few centavos (one of the two-object sessions) and 18 pesos, where inaction would have guaranteed them no losses! The improvement in sessions for which some of the subjects were recruited from the pool of those with experience in the same auction format is noticeable: of the 8 such sessions in only 1 there are any losses and in all four 3- and 4-object sessions there are substantial over the course of the experiment. In one of these sessions (a 4-object session) the gains are, in fact, quite striking: each agent went home with, on average, $MN210 pesos (almost US$20, nearly half of the maximum predicted). Still, in the rest of these sessions payoffs of the magnitude comparable to the prediction did not materialize. Notably, sizeable gains were recorded in one of the discriminatory-price sessions, where no such gains should have been possible in equilibrium.

Table 3 presents the summary statistics for the bids (as a proportion of value). The highest bids in uniform-price auctions, on average, show marked overbidding. There is also noticeable bid reduction for lower bids (especially pronounced in 4-object auctions), with extremely high standard deviations reflecting bimodal bid distributions. In contrast, in discriminatory auctions we obtain flat bids at close to value. The picture becomes even more suggestive if we only consider subjects with prior experience, who seem to be

Treatment	Highest bid	2nd bid	3rd bid	4th bid
2 unif. all (1854 obs.)	1.110 (0.356)	0.760 (0.446)		
2 unif., same exp. (476 obs.)	1.073 (0.229)	0.560 (0.435)		
3 unif. all (3045 obs.)	1.159 (0.423)	0.856 (0.472)	0.726 (0.445)	
3 unif, 1-obj. exp.(287 obs.)	1.092 (0.287)	0.920 (0.288)	0.827 (0.308)	
3 unif, same exp. (753 obs.)	1.147 (0.381)	0.599 (0.451)	0.510 (0.431)	
3 disc. (2130 obs.)	0.987 (0.184)	0.967 (0.200)	0.935 (0.236)	
3 disc., same exp.(660 obs.)	0.873 (0.265)	0.853 (0.281)	0.821 (0.303)	
4 unif. all (1122 obs.)	1.211 (0.489)	0.762 (0.552)	0.591 (0.536)	0.527 (0.492)
4 unif., same exp. (315 obs.)	1.187 (0.410)	0.540 (0.469)	0.335 (0.433)	0.303 (0.408)

Table 3: All Bidders. Average bids as a proportion of value, general and by experience type (standard deviation in brackets)

Treatment/Experience (obs.)	Highest	2nd bid	3rd bid	4th bid
2 unif./ no exp. (1378 obs.)	4 (0.3%)	120 (8.7%)		
2 unif./ same exp. (476 obs.)	0 (0.0%)	130 (27.3%)		
3 unif./ no same exp. (2292 obs.)	7 (0.3%)	161 (7.0%)	271 (11.5%)	
3 unif./ same exp. (753 obs.)	2 (0.3%)	220 (29.2%)	256 (34.0%)	
3 disc./ no exp. (1770 obs.)	13 (0.7%)	21 (1.2%)	41 (2.3%)	
3 disc./ same exp. (660 obs.)	31 (4.7%)	32 (4.8%)	42 (6.4%)	
4 unif./ no exp. (807 obs.)	1 (0.1%)	151 (18.7%)	216 (26.8%)	225 (27.9%)
4 unif./ same exp. (315 obs.)	0 (0.0%)	127 (40.3%)	195 (61.9%)	203 (64.4%)

Table 4: Number of bids equal to or lower than 1 peso (share of total number of bids)

particularly good at demand reduction on lower bids (high standard deviations still being observed) in all treatments (even in the two-unit auctions where this does not normally result in higher payoffs). Interestingly, subjects with experience in single-unit auctions do not seem to reduce demand when given a chance to play in a three-unit auction. A noticeable reduction of all bids in the discriminatory auctions comes almost entirely out of a single session.

Table 4 reports how often bidders came close to full demand reduction (predicted by theory for at least some uniform-price auctions). Few subjects ever submit all their bids this low in the uniform-price auctions, though some do in the discriminatory-price treatment. However, the full demand reduction becomes fairly common on the second bid, especially with experience: 27% of bids by bidders with relevant experience in the 2-object treatment, 29% in the 3-object uniform-price treatment, and 40% of bids in the 4-object treatment belong to this category. Interestingly, only in the 4-object treatment there is a further substantial increase of such low bids: nearly 62% of third-highest bids in the 4-object auctions do not exceed MN$1 peso. This further reduction suggests that second-highest bids might have been elevated by competition for the 4th unit. Second- and third-highest zero bids in discriminatory-price auctions are comparatively rare.

To study demand-reduction (or lack thereof) in multi-unit auctions we separately estimated the demand functions for the multi-unit auctions, using the GLS random effects models. The independent variable in all regressions is the bid as a proportion of object value. The *saexperience* variable is a dummy, taking value 1 for bids by subjects with experience in the same treatment. Similarly, $1uexperience$ dummy stands for the subjects with experience in single-unit auctions. The continuous *period* variable is there to capture bid dynamics. Finally, the *bankrupt* dummy captures the behavior of those subjects who were forced to stop bidding before the 20th round by early bankruptcy. The $2ndbid$ variable is the dummy for the second bid, the coefficient on which may be interpreted as the difference between the highest and the second bids (essentially, the demand reduction we are interested in) and the $2ndbid/saexperience$ variable is a dummy for the second bids of experienced subjects, designed to capture the difference between the second bids of inexperienced subjects and the same for experienced subjects. The rest of the second bid variables are constructed analogously. The dummy variables for the lower bids are nested, so that the coefficients can be directly interpreted as demand reduction from the higher bids. This is done by setting the $2ndbid$ dummy equal to 1 both for second and third bids, and setting the $3rdbid$ dummy to 1 for the third bid only; the interaction dummies, such as $3rdbid/bankrupt$ the $4thdbid/period$ variables are constructed similarly. Thus, the coefficient on the $3rdbid$ can be interpreted as the average amount by which subjects' third bids are smaller than their second bids and $4thdbid/saexperience$ shows how the experienced subjects differ from inexperienced subjects in terms of the reduction from the 3rd to the 4th bid. These regressions are presented in Table 5 and the estimated first period demands are plotted in Figure 1.

What we observe is that, whereas the highest bids in uniform-price auctions do reflect overbidding, the second bids in all uniform-price auction environments are substantially (15% to 36% of value lower), and prior experience induces a further (22% to 32% of value) reduction. In the four-object treatment there was further demand reduction by some 12% of value from the second to the third bid (in the three-object treatment, experienced subjects reduced their demand by roughly the same proportion as inexperienced), possibly signifying elevated second bids due to the competition for the fourth object sold (a further minor reduction in demand on the fourth bid occurred only among inexperienced subjects). In contrast, in the discriminatory-price auctions there is no overbidding, but rather slight, though statistically significant (about 4% of value) underbidding even on the highest bid, which substantially increases (by about 13%) as a result of prior experience (it should be noted that this last decrease is almost entirely due to the outcomes of a single experimental session). The demands in the discriminatory auctions are remarkably flat, compared to those in the uniform auctions, with only a slight reduction beyond the highest bid.

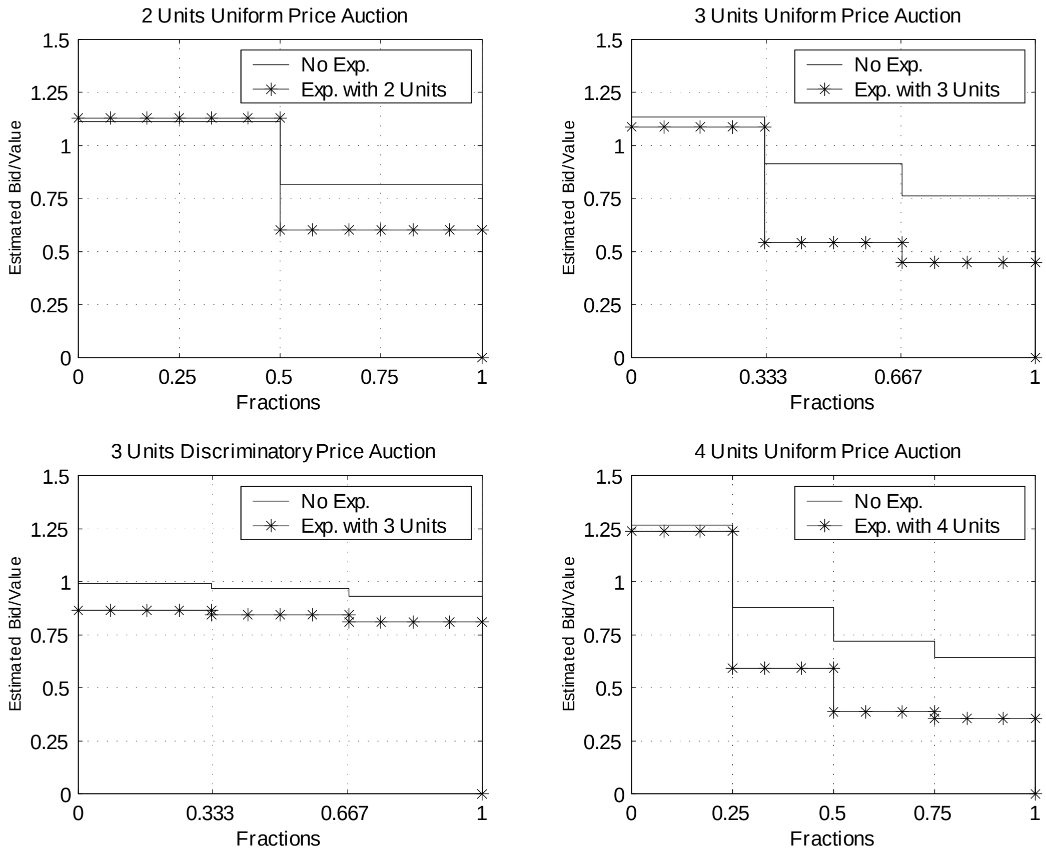
Only in two of the 4 environments were we able to detect substantial intra-session dynamics. In three-unit uniform-price auctions there was a slight increase of the highest bids during the course of a given session, but it was more than overwhelmed by a simultaneous substantial decrease in second bids (over the 20 periods the second bids in comparison to the highest bids decreased by about 14% of value beyond the initial 15% difference). Though non-negligible, this effect was small compared to decrease in second bids exhibited by subjects that return for a follow-up session. In contrast, in the three-object discriminatory-price auctions we observed significant intra-session increase in both the highest and second bids (the 6% by which the highest bids grew over 20 periods is more than the size of the initial underbid by inexperienced subjects and would remove, roughly, half the initial underbid by experienced subjects). No significant intra-session dynamics was observed in the 2- and 4- object auctions.

	Variable	2-obj. uniform	3-obj. uniform	3-obj. disc.	4-obj. uniform
Highest bid	constant	1.110**# (0.027)	1.101**# (0.025)	0.962*# (0.017)	1.233**# (0.051)
	saexperience	0.016 (0.029)	0.006 (0.040)	-0.127** (0.010)	-0.030 (0.077)
	1uexperience	-	-0.048 (0.048)	-	-
	period	0.000 (0.001)	0.0034* (0.0017)	0.003* (0.001)	0.0035 (0.0023)
	bankrupt	0.394** (0.094)	0.591** (0.121)	0.265** (0.037)	0.190 (0.107)
2nd bid (diff. w/highest)	2nd bid	-0.274** (0.040)	-0.153** (0.030)	-0.042** (0.013)	-0.357** (0.068)
	2ndbid/saexperience	-0.215** (0.079)	-0.323** (0.082)	0.003 (0.013)	-0.257 (0.141)
	2ndbid/1uexerience	-	0.054 (0.073)	-	-
	2ndbid/period	-0.002 (0.002)	-0.007** (0.002)	0.002* (0.001)	0.001 (0.002)
	2ndbid/bankrupt	0.086 (0.080)	-0.082 (0.199)	-0.006 (0.035)	0.237* (0.100)
3rd bid (diff. w/ second)	3rd bid	-	-0.155** (0.019)	-0.048** (0.013)	-0.121** (0.032)
	3rdbid/saexperience	-	0.061** (0.023)	0.004 (0.013)	-0.046 (0.057)
	3ndbid/1uexerience	-	0.057 (0.045)	-	-
	3rdbid/period	-	0.001 (0.001)	0.001 (0.001)	-0.037 (0.002)
	3rdbid/bankrupt	-	0.006 (0.050)	0.010 (0.035)	-0.074 (0.018)
4th bid (diff. w/ third)	4th bid	-	-	-	-0.073** (0.002)
	4thbid/saexperience	-	-	-	0.045 (0.024)
	4thbid/round	-	-	-	-0.000 (0.001)
	4thbid/bankrupt	-	-	-	0.003 (0.029)

**- signif. at 1% and *-signif. at 5%

- diff. from 1

Table 5: Demand in multi-unit auctions (std. errors)



Figute 1: Demand Functions

5 Conclusions

In an experimental no-uncertainty setting with common values we find that subject behavior substantially differs between uniform-price and discriminatory-price auctions. Though the former do not readily lead to low-revenue equilibria even when these are the only ones surviving two rounds of iterated elimination of weakly dominated strategies, some agents do make an effort to reduce demand. If experienced agents are recalled into the lab, having been able to think through and/or communicate the right equilibrium to each other, the demand-reduction efforts become more pronounced, and, frequently, successful. In addition, at least in the three-object environment, intra-session dynamics reinforced the observed demand-reduction. In contrast, the discriminatory-price auctions result in agents submitting flat demands, with only small decrease from the highest to the second bid. While we do observe attempts at out-of-equilibrium collusion by experienced subjects in the discriminatory auctions, these are not entirely successful, substantially unraveling during the course of the experimental session. Low revenue outcomes were obtained in both auction types in sessions with experienced subjects.

It should be noted, that the uniform-price revenue is somewhat increased compared to the discriminatory-price revenue due to a consistent pattern of overbidding (i.e., playing weakly dominated strategies) in uniform uniform-price auctions. This, of course, recalls the well-established experimental finding of Kagel *et al.* (1987) about the consistent overbidding in the second-price auctions, which has itself spawned an important literature (see, for instance, Kagel and Levin 1993, or Harstad 2000). No such overbidding normally occurs in discriminatory-price auctions. Still, it seems that equilibrium collusion in uniform-price auctions was substantially more successful than out-of-equilibrium collusion in discriminatory-price auctions.

6 Appendix A: Proof of Proposition 1

Proof. (i) This follows the standard dominance argument for second-price auctions. Consider the choice vector of bids b_1 by agent 1 (as the game is symmetric this is without loss of generality), given any profile of bids b_2,b_3 If the total number of bids $b_{jk} > v$ $(j \neq 1)$ is smaller than n then setting $b_{11} \geq v$ gains an object at some price $p \leq v$ that is independent of b_{11}. Otherwise, if the number of bids $b_{jk} > v$ $(j \neq 1)$ is bigger than or equal to n, any $b_{11} > v$ guarantees that $p \geq v$, in which case the agent weakly prefers to have chosen any $b_{11} \leq v$. Clearly, no matter what the bids by others, $b_1 = (v, x, y)$ dominates (b, x, y) for any $v \geq x, y \geq 0$ and any $b \neq v$.

(ii) If $n = 2$ and all agents play a weakly undominated strategy, there are no bids above v, and, at least, three bids $b_{jk} = v$, which guarantees that the price $p = v$ no matter what else the bidders do.

(iii) If $n = 3$ and all other agents play a weakly undominated strategy then there are no bids above v and, at least, two bids by other agents such that $b_{jk} = v$. Then the only way for an agent 1 (once again, without loss of generality) to win more than one object is by setting $b_{11} \geq b_{12} \geq v$. But that guarantees that the price $p \geq v$ in which case he can't have positive payoffs . Furthermore, any bid b_{12} is either the fourth-highest (in which it determines the price), or not: in the former case agent 1 prefers to set it as low as possible, and in the latter s/he does not care what it is. Hence, setting $b_1 = (x, 0, 0)$ weakly dominates any $b'_1 = (x, y, z), x \geq y, z > 0$, assuming the agent expects others to play weakly dominant strategies.

(iv) If other agents play a weakly undominated strategy then there are, at least, 2 bids $b_{ik} = v$. Therefore, the only way for a bidder j to make his third bid winning is by setting $b_{j1} \geq b_{j2} \geq b_{j3} \geq v$, so that he can't have positive payoffs from winning (the rest of the argument is as in (iii). Furthermore, suppose for all agents $i \neq j$ for all $k = 2, 3, 4$, $b_{ik} < \frac{v}{2}$. Clearly, setting any $b_{j1} \geq b_{j2} = v$ gains two objects at the price $p < \frac{v}{2}$, ensuring a payoff of strictly more than v, which is strictly bigger than the payoff from winning a single object at any non-negative price. Therefore, if all three agents propose setting $b_{j2} < \frac{v}{2}$ there will always exist a profitable deviation by an agent not winning more than one object. It remains to consider the strategy profiles with tied second bids, where all agents have at least a 1/3 probability of gaining the second object. Clearly, an arbitrarily small increase of the second bid by any agent would result in him/her getting the second object for sure, strictly increasing the payoff, so such a tie at less than half of value would still not be an equilibrium.

(v) Given a continuos price space, if the n'th-largest bid is less than v, submitting a bid between it and value gains an object at less than value, without affecting the prices of other objects an agent might obtain. Overbidding always incurs a loss. ■

7 Appendix B: Experimental Instructions and Proceedings for a 3-unit uniform-price auction

The following is the verbatim translation (from Spanish into English) of experimental instructions administered to subjects at ITAM (the Spanish original is available from the authors upon request).

Instructions

This is an experiment about decision-making in auctions. The CONACYT has provided money for this experiment. The instructions are simple and, if you follow them carefully and take good decisions, you may win a CONSIDERABLE AMOUNT OF MONEY, which shall be PAID OUT IN CASH at the end of the experiment.

General Proceedings.

In this experiment you shall participate in an auction as a buyer of a fictitious good. The experiment shall consist of 25 periods of buying: 5 practice periods and 20 periods to be played for money. The monetary value of the good shall be randomly chosen for each period between $20.00 and $100.00 pesos. Any value within this interval shall have the same probability of being chosen. The value of the good in each period shall be chosen independently from the values in the previous periods.

Once chose, the value of the good shall be divided into three[2] fractions, which shall be sold simultaneously but separately. Your job shall be to offer the money for the distinct fractions, while competing with other buyers. The value for each fraction (VF) of the good is the value of the entire object, divided into 3. For instance, if the value of the good is $100.00 pesos, the value for each fraction shall be $33.30. If the value of the good is $20.00, the value for each fraction shall be $6.60.

In each period groups of three buyers shall be formed. In each group the buyers shall compete for the fractions of the same good. The membership of each group shall change randomly, so that the same group shall be formed by different buyers in each period. You shall never know with whom you are participating. The value of the good shall be the same for all members of a group, and may be distinct for each group.

Specific Proceedings.

At the beginning of each period each buyer shall write and send a bid for each fraction of the good. The three highest bids shall obtain the value equal to that of a fraction of the good. In case of a tie among the bids, the computer shall randomly choose the three winning bids. Nobody can offer for a fraction of the good less than $0 pesos or more than $33.30 pesos. Neither it is possible to offer more for a second fraction than for the first one, nor for the third fraction than for the second one.

The price to be paid for each fraction shall be the amount equal to the fourth largest bid[3]. Thus, the monetary gain (or loss) for each fraction received by a winner shall be equal to the value of the fraction less the price to be paid for a fraction. The other buyers shall get the payoff of zero. The total gain of a buyer shall be equal to the sum of gains (or losses) for all the objects obtained.

At the end of each round, the participants of each group shall learn the winning offers, the price paid and their individual gains (or losses). After this, they shall proceed to the next round.

Example:

[2] In this particular session, the good was divided into three. The number of divisions varied across sessions.

[3] The italicized part changes in different treatments. In all $n-$fraction uniform treatments it is the $(n+1)$ *st*-largest bid, instead of the "fourth largest bid". In the discriminatory auctions the words are replaced with "to each winning bid".

For an example we shall now look at a table with offers submitted buy a group of buyers: buyers C1, C2 and C3 have given an offer for each of the fractions of the good. The three highest offers in the group are marked by a double asterisk: 20.55 - C1, 16.50 - C2, 15.50 - C1. The winners of the three fractions of the good shall pay the fourth highest bid: 13.40, which is masked by a single asterisk. The gain (or loss) for each fraction is shown in the last column, where the fraction to be paid is subtracted from the value per fraction.

Buyer	Fractions	Bids	Winners	Price	Benefit
C1	F1	20.55	**		VF-13.40
	F2	15.50	**		VF-13.40
	F3	10.00			0.00
C2	F1	16.50	**		VF-13.40
	F2	12.40			0.00
	F3	12.40			0.00
C3	F1	13.40		*	0.00
	F2	12.50			0.00
	F3	10.00			0.00

Do you have any questions about this example?

Exercise:

For an exercises and to clarify doubts, indicate with double asterisks the three winning offers and with a single asterisk the price that would be paid for each of the fractions. Assuming the value or a fraction is VF, indicate in the last column what shall be the gain of each buyer for each unit.

Buyer	Fractions	Bids	Winners	Price	Benefit
C1	F1	20.65			
	F2	18.50			
	F3	15.00			
C2	F1	26.16			
	F2	22.10			
	F3	22.00			
C3	F1	30.40			
	F2	20.50			
	F3	11.00			

Do you have any questions about the exercise?

Initial balance, accumulated balance and minimal balance.

Each buyer shall start the experiment with the initial balance of $60 pesos. The total gains (or losses) of each round shall be added (or subtracted) from the balance accumulated in the previous period.

If the balance of a buyer for any period is less than $20 pesos, he shall not be allowed to continue participating. He shall be paid out his final accumulated balance and shall have to leave. In case the balance is negative, he shall receive no payment.

In case all groups cannot be formed due to the exit of some of the participants, the computer shall randomly form as many groups as possible. The participants that cannot be included in a group for certain period shall have to wait for the following rounds to be able to participate again as buyers.

Payment procedures.

Your balance accumulated at the end of the last round shall be paid out in cash at the end of the experiment.

Do you have any questions about the instructions?

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