EXERCISE 2:
Public Goods

Show all work!

1. Consider an economy with one private good x and one public good q. There are N individuals.

Assuming that each individual has and endowment of private good (only) $w^i = 10$, preferences represented by the utility function

$$u^i(x^i, q) = x^i + \theta^i \ln Q$$

where x_i is his/her consumption of the private good and Q is the amount of public good and the production technology of the public good is

$$f(q) = q$$

where q is the quantity of private good used in production

a) what are the Pareto optimal allocations? How does your answer vary with the choice of N

b) if each individual *simultaneously* contributes part of his endowment to the public good, what is the voluntary contribution Nash equilibrium? How does your answer vary with the choice of N.

c) if the government chooses to tax every agent with a lump-sum tax ε and proved $N\varepsilon$ units of the public good and the agents then have to decide how much, if anything, to contribute in addition to what is provided by the government, what would be the total amount of the public good provided (you may assume that ε is very small)?

d) if the government can only impose an income tax at the rate τ, what is the tax rate that would ensure efficient public good provision (assume all taxes collected are used to provide the public good and individuals do not provide any public good)? If individuals vote for the tax rate (they are aware of the link between the taxes and the quantity of public goods), what would be the Condorcet-winner tax rate? How does it compare with the efficient tax rate if all agents have the same preference parameter θ?

2. Repeat parts (a-c) of exercise 1 assuming the preferences are represented by the utility function

$$u^i(x^i, q) = \ln x^i + \theta^i \ln Q$$

How did your answer change? What stays the same? Explain.

3. Suppose the preferences are represented by the utility function

$$u^i(x^i, q) = x^i + \theta^i \sqrt{Q}$$

Construct a Groves-Clarke mechanism (message space, public good provision rule and transfers) that would implement efficient public good provision in this case.