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Sample midterm

Answer EVERY question. All questions have equal weight. You have 90 minutes to do this. Good luck!

1. Consider a 3-person economy with one private good x and one public good q.

Assuming that each individual has and endowment of private good (only) $w_1 = 3$, $w_1 = 1$, $w_3 = 1$, and preferences represented by the utility function

$$\begin{aligned} u^1(x^1, q) &= x^1 + \sqrt{q} \\ u^2(x^2, q) &= x^2 + 2\sqrt{q} \\ u^3(x^3, q) &= x^3 + 3\sqrt{q} \end{aligned}$$

where x_i is his/her consumption of the private good and q is the amount of public good and the production technology of the public good is

$$q = x$$

where x is the quantity of private good used in production

a) what is the Pareto optimal level of q?

b) if each individual *simultaneously* contributes part of his endowment to the public good, what is the *game* that the agents play. What is the Nash equilibrium of this game? Compare the equilibrium public good level q with the level in part (a). Discuss.

c) if the government taxes **both agents** using a small lump-sum tax τ and applies the tax revenue to the public good, and the agents are allowed to provide more of the public good if they want, as in part (b), will the equilibrium level of the public good change?

2. Continuing with the model in question 1, suppose that the public good provision can only be financed by a proportional tax on the agents' endowments t ("income tax"). That is, if the tax rate is $t \in [0, 1]$ the agents will have $(1 - t) w^i$ units of the private good left for their consumption and the government will provide $q = t\left(w^1 + w^2 + w^3\right)$ units of the public good. Assume that no voluntary contributions to the public good are allowed and suppose that agents will vote to choose the tax rate.

a) In terms of the tax rate t are the agents' preferences single-peaked?

b) Does there exist a Condorcet winner in voting for t? If yes, what is it and what is the associated level of public good provision? How does it compare with the efficient level?

c) Discuss your answer. What is the source of inefficiency?

3. There is a society of a continuum of agents whose political views are uniformly distributed over $[0, 1]$, where 0 stands for extreme left and 1 stands for extreme right. Each individual's type represents his/her ideal policy position. Before the election political candidates must announce there ideal position. Voters vote sincerely for the political candidate advocating position closest to their own.

a) Suppose there are two political candidates, A and B, running in the election. Suppose each candidate cares only about the vote share he/she gets, and will declare a position based only on this. Define the game (agents, their strategy spaces and payoffs) the candidates play. Does there exist a pure strategy Nash Equilibrium? If yes, compute Nash Equilibria of this game. If no, explain why.

b). Suppose there is a third candidate, C, who is an "ultra-rightist ideologue" who will advocate position 1 no matter what. He does not seek to maximize his share of vote. A and B play this game taking position of C as given. Repeat part (a) with the "right-winger" C present. Interpret your answer.

c) How does your answer change if all three candidates are vote-share maximizers.