Bidding and Drilling on Offshore Wildcat Tracts*

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Abstract

I study a two-stage bidding drilling game. If a firm drills, her neighbor learns the state of the world. Depending on parameter values there either exists a separating, or there exist two types of semi-separating equilibria. In the first one, a low-type player sometimes submits a "high" bid to influence her neighbor's drilling decision. In the second one, a high-type player sometimes submits a "low" bid. In this equilibrium bids are used as a coordination device: If player i bid "low" while player $-i$ bid "high", player i waits while player $-i$ drills. Normative and positive implications are discussed.

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1 Introduction

In recent years, some countries decided to put part of their offshore oil (and gas) reserves under the hammer. Brazil, Cuba, Libya, Nigeria, Russia, and the U.S., for example, organized offshore oil (and gas) auctions in the past decade. Those auctions often generate huge revenues and secure the supply of crucial energy resources. In a seminal paper, Hendricks and Porter (HP, 1996) argued that offshore drilling suffers from a public good problem: If a firm drills and finds oil, this is costlessly observed by her neighbor(s). They next showed that the drilling data is consistent with the idea that — on average — firms do *not* coordinate their drilling decisions, i.e. firms typically play a war-of-attrition to determine who will drill first. This finding raises several interesting questions: How do bids influence drilling decisions (and vice versa)? Does equilibrium behavior in a bidding-drilling game shed additional light on their empirical findings? Can bids sometimes be used as a coordination device? If so, how? Does this increase or hurt revenues? How should offshore oil-and-gas auctions be designed?

To tackle those questions, I analyze a two-unit, two-player bidding-drilling game. I assume that if both players end up owning adjacent tracts, and if both players possess the same posterior at the start of the drilling game, they play a war-of-attrition (i.e. they do not coordinate their drilling decisions). I first show that there exist equilibria in which a high-type player (i.e. a player who possesses favorable private information) bids "high" (with probability one) as she wants to secure the purchase of the tract. A low-type player (i.e. a player who possesses unfavorable private information) faces the following trade-off: If she bids "low" she might buy her tract "cheaply" but her low bid also reduces her neighbor's posterior probability of finding oil. This, in turn, reduces the probability that her neighbor will drill (and thus hampers her free-riding opportunities). Depending on the values of the parameters, she either prefers to bid "high" (in which case there exists a pooling or a semi-separating equilibrium) or she prefers to bid "low" (in which case there exists a separating equilibrium). Next, I also show that there exists another semi-separating equilibrium in which low-type players bid low (with probability one), while high-type ones (with some probability) bid as if they have bad private information. To understand the intuition behind this equilibrium, suppose player 1 bid "high" while player 2 bid "low". Suppose both players won their tracts. As a low bid may have been submitted both by a high-type and a low-type player, both players do *not* possess the same posterior at time one. Hence, they do not have to drill with the same probability. I then assume that both players focus on a continuation equilibrium in which player 1 drills while player 2 waits. Hence, in this equilibrium a high-type player faces the following trade-off: If she bids "low", she reduces her probability of winning the tract. Conditional upon winning, however, she increases the probability that she will free-ride on her neighbor's drilling cost. In equilibrium the probability with which a high-type player bids low is chosen to balance its advantage with its disadvantage. In section 4, I argue that the latter semi-separating equilibrium is consistent with HP's empirical findings and need not result in lower revenues (despite the strategic "low" bidding behavior of the high-type player). I also argue that the linkage principle (Milgrom and Weber (1982)) need not hold in my setting.

HP developed a bidding-drilling model in which different types are (exogenously) assumed to bid differently. I provide sufficient conditions for such a bidding behavior to arise endogenously. My model also provides a competing explanation behind some of their findings, and provides an answer to the many questions I listed in my first paragraph.

This is not the first paper to analyze an auction as part of a larger market interaction. Haile (2000), considers a game in which players can resell after the auction took place. Jehiel and Moldovanu (2000) and Goeree (2003) analyze an auction followed by some downstream interaction among all players. In contrast to this paper, downstream interaction is not modelled explicitly. Instead they take a reduced-form approach in which player i's payoff depends on the outcome of the auction.[1] Das varma (2003) models post-auction (Betrand and Cournot) competition explicitly and obtains essentially the same results as Goeree (2003). Arozamena and Cantillon (2004) analyze incentives to invest in a cost reducing technology prior to a procurement auction. Burguet and McAfee (2005) analyze a model with budget constrained bidders and in which the auction stage is also followed by Cournot competition. Haile, Goeree and Das Varma find that — in the presence of post-auction interaction — it becomes harder to obtain a separating equilibrium because of signaling considerations at the bidding stage. In contrast to my paper, all three papers restrict attention to separating equilibria. Furthermore, Jehiel and Moldovanu and Goeree assume that the payoff function is differentiable in players' types (or in a player's perceived type). This assumption is reasonable if one thinks of either Cournot or Bertrand competition as the (implicit) post-auction interaction. In this paper, however, after the auction players engage in a battle-of-the-sexes game which typically possesses three different equilibria (player i drills while her neighbor waits, player $-i$ drills while i waits, and the mixed-strategy Nash equilibrium). I then show that, even if one restricts attention to the class of the strongly symmetric strategies, bids may select a continuation equilibrium in which one player waits while the other one drills. Hence, depending on the selected equilibrium, my payoff function is not continuous in bids: If player i bids below a certain threshold (and wins her tract), her

[1] In Goeree a player's payoff depends (i) on whether she won the object or not, (ii) on her true type and (iii) on her perceived type in the post-auction game. Jehiel and Moldovanu consider a two-player set-up in which the payoff from not winning the object depends on both players' types (the winning bidder's payoff only depends on her type). They also assume that a player's type becomes common knowledge after the auction.

payoff jumps upwards. Finally, Avery (1998) studies an English auction in which players "jump bid" to signal that their valuations lie above some threshold level and to select an asymmetric continuation equilibrium. I show that bidders in OCS auctions behave similarly: In one semi-separating equilibrium a low bid (partly) signals a low valuation and selects an equilibrium in which the high bidder drills while the low bidder waits.

This paper is organized as follows: In section 2, I explain some institutional features which are most relevant for understanding the game I will study and which also motivate some of my modeling assumptions. In section 3, I build my two-stage game and prove existence of all the equilibria discussed above. In section 4, I argue that the existing empirical evidence does not allow me to rule out any equilibrium, and that the linkage principle need not hold in my set-up. A final comment is summarized in section 5.

2 Some institutional features

In this paper I focus on wildcat tracts. Such a tract is situated in an offshore geographical area where no exploratory drilling has occurred in the past. Tracts that are situated next to already developed ones are called drainage tracts. Hendricks and Porter (1988) showed that firms possess an informational advantage over the value of a neighboring tract. In contrast, no firm should possess superior information about the value of a wildcat tract.

At the start of the auction process, firms express their desire to drill in some geographical area of the outer continental shelf. The U.S. government then organizes an auction in which a huge number of tracts (situated in the desired area) are simultaneously offered for sale. A tract covers an area not exceeding 5,760 acres ($\approx 23.3km^2$). Firms then bid on a small subset of the tracts offered for sale. For example, between 1998 and 2005 (inclusive) the U.S. government organized 22 such auctions. On average 3,145 tracts were offered in each one of them.[2] On average only 305 of them received at least one bid.[3] Hence, in those auctions the number of tracts offered for sale by far exceeds total demand. As a result of this, few of the tracts offered for sale receive more than one bid. To illustrate this point consider Figure 1. The Figure reveals that between 1998 and 2005 (inclusive), Pr(tract i receives only one bid|tract i receives at least one bid) always exceeded 74%.[4]

[2]Observe, however, that not all those 3,145 tracts were wildcat ones. Some tracts were drainage tracts. Some tracts may have been re-offered for sale as the past owner of the tract let her lease expire without drilling any well (those ones are called development tracts).

[3]Source: own computations based on data taken from http://www.mms.gov/econ/EconDiv.htm.

[4]Solo bidding, however, has not always been the norm in OCS auctions. In particular, Hendricks, Porter

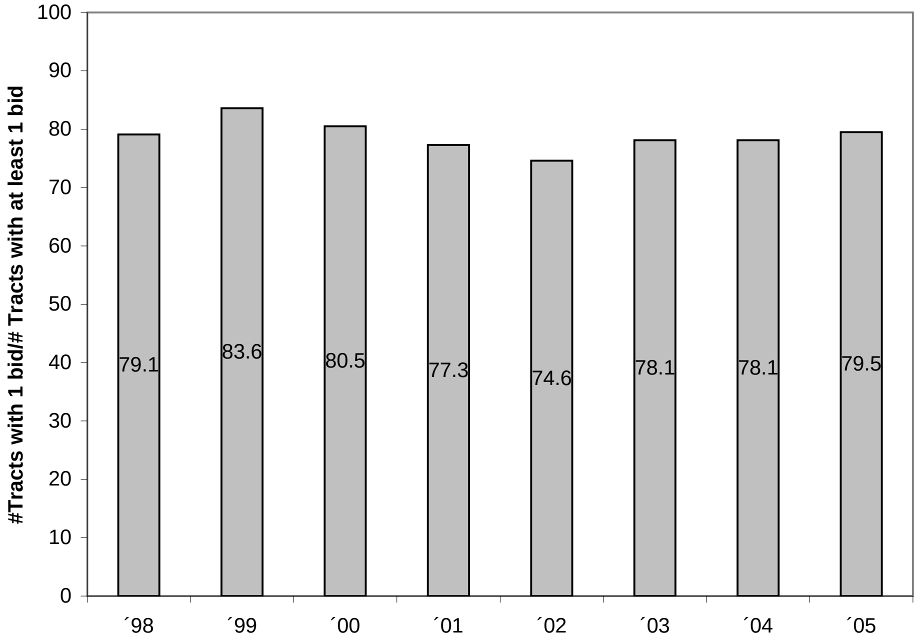

Figure 1: Solo Bidding in OCS Auctions

A bid is a dollar figure that the firm has to pay if it wins the tract. Apart from the bid, firms also must pay a royalty fee to the government which typically equals one sixth of the value of the extracted oil.[5] Firms submit their bids simultaneously. If a tract happens to possess only one bid, then the U.S. government decides whether or not to reject the bid. To do so, it estimates the "fair market value" of the tract. Henceforth, this fair-market-value estimate will be called the (government's) reservation price. A tract which received only one bid is sold if the bid exceeds the reservation price. The reservation price is computed after all bids were submitted. Hence, ex-ante bidders don't know what the realization of the reservation price will be. This insight, combined with my earlier finding that few tracts receive more than one bid, indicate that a player's bidding strategy is primarily determined by her desire to "beat" the reservation price rather than to "beat" a hypothetical competing bid. So far, only Hendricks, Porter and Spady (HPS, 1989) analyzed the government's rejection decision on offshore tracts. They focussed on drainage and development tracts that were sold during the period 1959 - 1979. Unfortunately, wildcat tracts were not included in their sample. They found that the rejection decision on drainage tracts was positively correlated with a tract's size, with the average wellhead price of offshore oil and with the identity of the highest bidder (i.e. the government was more

and Boudreau (1987) documented that Pr(tract i receives only one bid|tract i receives at least one bid) was approximately 32% for wildcat auctions held during the period 1954-1969.

[5]Firms can obtain a reduction of this royalty fee if they drill in waters deeper than 200 meters (source: www.mms.gov/ld/PDFs/GreenBook-LeasingDocument.pdf).

likely to reject a given high bid submitted by a neighbor firm than by a non-neighbor one). The rejection decision was also negatively correlated with the value of the highest bid. The decision, however, was *not* significantly correlated with the amount of oil extracted nor with the bidding history of the neighboring tract. As the reservation price on drainage tracts did not depend on the expected quantity of oil nor on the neighbors' bids, there is no reason to assume that the contrary situation would prevail on wildcat tracts. After firms submitted their bids, but before the first drilling date, the government releases the identity of all bidders along with their bids.

After winning her tract, a firm is given five years to initiate an exploratory drilling program. If after five years it has not drilled its tract, its lease expires and the tract is returned to the government which may decide to resell it in some future auction. The tracts are usually smaller than the size of the deposits. For example, Lin (2007) documents that the largest petroleum field in the Gulf of Mexico spans 23 tracts. Depending on water depth, 57% to 67% of all productive tracts had to share their deposits with at least one neighboring firm. Furthermore, the costs of drilling an exploratory well are not trivial. According to Zampelli (2000) in 1996 the average exploratory well had a depth of 11,203 feet (3,414 meters) and cost 3.3 million USD. This cost dramatically increases with well depth: A 15,000 feet (4,572 meters) exploratory well cost 10 million USD.

Given those facts, one would expect firms to play a waiting game, i.e. a firm has an incentive to postpone its exploratory drilling in the hope that its neighbor drills first. This plausible strategic behavior is not inconsistent with the available empirical evidence. HP documented that the hazard rate of drilling (i.e. the probability to drill at time t given that the tract has not been drilled before) features a U-shaped pattern. A tract is most likely to be drilled at the start or at the end of her lease term. In years 2, 3 and 4, however, the hazard rate is significantly lower. If a firm drills its tract during the final year of her lease, this indicates that it must hold sufficiently optimistic beliefs about her prospects of finding oil. The fact that it postponed its drilling decision indicates that there was a positive option value of waiting. A plausible explanation behind this option value of waiting is that the firm hoped to learn from its neighbor's drilling outcomes. Furthermore, HP also found that the probability to drill during the second and the third year of the lease term is positively influenced by the number of past successful drilling outcomes.

3 The Model

3.1 The general set-up

Two risk-neutral players are interested in acquiring one of two adjacent offshore tracts. The seller offers them in two simultaneous first price auctions. Each of the players bid in one of the two auctions.[6] The value of both tracts depends on the realized state of the world. In particular, I assume that the state of the world is either high (H) or low (L). If the state of the world is high (low), then the value of the oil (underneath both tracts) is equal to one (zero). The probability that the state of the world is high is equal to $\frac{1}{2}$.

Both players possesses an informative, but imperfect signal concerning the realized state of the world. Formally, if the state of the world is H, a player receives signal h with probability $p \in (\frac{1}{2}, 1)$, and signal l with probability $(1 - p)$. Similarly, if the state of the world is L, a player receives signal h with probability $(1 - p)$, and signal l with probability p. Signals are (conditionally) independent. Henceforth, a player with signal l (h)is called a low-type (high-type) player. I denote the common drilling cost by c. A tract is called marginal if $\frac{1}{2} < c < p$. A tracts is called non-marginal if $1 - p < c < \frac{1}{2}$.

The (nominal) value of the oil is equal to PQ, where P and Q respectively denote the price and quantity of oil. As $Q \in \{0, 1\}$, the real value of the oil is either equal to zero or equal to one. Furthermore, suppose the government's (nominal) reservation price on tract i, is given by: $R_i = f(P) + \epsilon$, where $\epsilon \sim U[\underline{\epsilon}, \bar{\epsilon}]$ and where f denotes an arbitrary function. This is consistent with the empirical findings of HPS which showed that the government's rejection decision was only correlated with (i) the tract size, (ii) the winning bid, (iii) the identity of the winning bidder and (iv) the price of oil. Recall that the quantity of oil and a neighbor's bid were *not* significant in their regression equation. Hence, there is no reason to assume that R_i is contingent on the bid on tract $-i$ or on Q. In my model both tracts have the same size and both bidders do not own a

[6]Implicitly, I am making two assumptions here. First, I assume that bidders have unit demand. Second, I assume that there is only one bidder per tract. The first assumption can be defended on the grounds that firms may not want to bid on all the tracts offered for sale (recall that in the period 1998-2005 on average 3,145 tracts were simultaneously offered for sale!) either because of bidding constraints, or because of a bottleneck in the supply of drilling rigs or because of risk-aversion. None of those reasons, however, are explicitly modeled here. Next, because of the information externality, a firm's valuation of a particular tract is nondecreasing in the number of neighboring tracts it wins in the auction. Recall, however, that I study how the information externality at the drilling stage affects bidding behavior (and vice versa). Introducing supermodular utility functions in the analysis would therefore unnecessarily complicate matters. The second assumption considerably simplifies computations and is consistent with the recent U.S. experience as shown in Figure 1. Moreover, I do not believe my main results to hinge on that assumption.

neighboring tract. Perform the following normalizations: $r_i \equiv \frac{R}{P}$, $\underline{\epsilon} \equiv -f(P)$ and $\bar{\epsilon} \equiv P - f(P)$. Then $r_i \sim U[0,1]$. Finally, I assume that both tracts possess the same reservation price (i.e. $r_i = r_{-i}$). While this assumption is not supported by HPS's findings, it should not affect any of the results presented in this paper.

After players submitted their bids, but before players decide whether or not to drill, the seller (government) discloses both player's bids. Players discount the future at a rate $\delta < 1$. I assume that

Assumption 1 *$1 - p < c < p$.*

The assumption implies that a player who received signal h is - a priori - willing to drill ($\Pr(H|h) = p > c$), and that a player who received a signal l is a priori not willing to drill ($\Pr(H|l) = 1 - p < c$). I consider the following sequencing of events:

- -1 Nature draws the state of the world, the reservation price and players receive their signals.
- 0 Player one bids on tract one, player two bids on tracts two.
- $\frac{1}{2}$ The auctioneer publicly announces all bids and whether they were higher or lower than the reservation price.[7]
- 1 If player i won her tract, she decides whether to drill or wait.
- 2 In case player $-i$ drilled, player i observes the state of the world. If player i waited, she decides whether to drill or not to drill.
- 3 Players receive their payoffs and the game ends.

As should be clear from above, I assume that both tracts possess the same value and that they are adjacent. This is a simplifying rather than a crucial assumption. At the end of section 3, I will argue that my main results should go through if both tracts are "not-too-distant" neighbors with (imperfectly) correlated values. I also assume that players never cooperate. Prior to bidding, firms must invest in geophysical surveys and in a team of experts to interpret the seismic data. Understandably, firms are reluctant to share this private information with firms which are uninterested in drilling (and bidding) in their geographical area.[8] If firms were to know the identity of their (not-too-distant) neighbor, they might decide to create a joint venture

[7]In reality, the reservation price for tracts with a rejected high bid can be downloaded from http://www.mms.gov/econ/EconDiv.htm. The reservation price for tracts with an accepted high bid, however, is not made public.

[8]Hendricks and Porter (1992) provide empirical evidence which is consistent with this explanation.

prior to bidding. Unfortunately, the identity of a firm's neighbor only becomes available after the auction. Post-auction (but pre-drilling) cooperation is also problematic. First, firms need to agree on whether or not to drill. Reaching an agreement on this issue need not be simple as both firms may still possess a substantial amount of private information after the bidding-stage.[9] Second, as firms possess different private information about future oil and gas prices, they need to agree on when to drill. Third, firms need to agree on where to drill. As (most likely) firms possess different private information about the value of the informational externality, it might be hard to come to an agreement on this issue. This is especially true if both tracts are not adjacent (but possess correlated values).[10] Fourth, firms need to agree on a payment (one firm might not be willing to pay more than 40% of the joint drilling costs, while the other one might insist on splitting the costs evenly). The three sources of private information mentioned above influence a firm's willingness to participate in a joint-drilling effort and agreement on this fourth issue may also be pretty hard.

3.2 Equilibrium

Let $\mathtt{h}_t (t = 0, 1, 2)$ denote the history of the game at time t. Thus, $\mathtt{h}_0 = \{\emptyset\}$, $\mathtt{h}_1 = (b_i, b_{-i})$ and $\mathtt{h}_2 = (h_1, a_{i,1}, a_{-i,1}, \xi)$ where $a_{i,1} \in \{drill, wait\}$ represents player i's time-one action and $\xi = \{\emptyset\}$ if $a_{i,1} = a_{-i,1} = wait$ and is equal to the state of the world if at least one of the two players drilled at time one. H_t denotes the set of all possible histories at time t. Let $H \equiv \bigcup_{t=1}^{2} H_t$. A symmetric behavioral strategy is a (β, λ) where $\beta : \{h, l\} \to \Delta[0, 1]$ and $\lambda : \{h, l\} \times H \to [0, 1]$. $\beta(s_i)$ represents a distribution function over player i's possible bids. $\lambda(s_i, \mathtt{h}_1)$ and $\lambda(s_i, \mathtt{h}_2)$ represent the probabilities with which player i will respectively drill at times one and two. If $r > b_i$ (i.e. if player i does not own tract i), then player i can never drill and, thus, $\lambda(s_i, \mathtt{h}_1) = \lambda(s_i, \mathtt{h}_2) = 0$. A player can only drill once. Therefore, $\lambda(s_i, \mathtt{h}_2) = 0$ if $a_1^i = drill$.

When solving my game, I rely on two equilibrium selection criteria. First, I require a candidate equilibrium to belong to the class of the perfect Bayesian equilibria. In a perfect Bayesian

[9] I will shortly prove that bids need not perfectly reveal a player's private information. A low bid, for example, may have submitted from a firm with a low posterior about the probability of finding oil. However, it could also have been submitted by a firm with a better posterior in the hope that this would induce its neighbor to drill (or to pay more in a joint drilling effort).

[10] Firms could exchange their seismic information after the auction and then decide where to drill. This "solution" faces two potential problems. First, firm i needs to reveal how it interpreted the seismic data. This may increase firm $-i$'s expertise knowledge in interpreting seismic data and allow her to bid more aggressively in future auctions. Second, a firm may also be reluctant to divulge her private information fearing that it might "destroy" her neighbor's incentives to drill.

equilibrium (PBE) strategies and beliefs (concerning the other player's type) must be such that (i) player i cannot gain by choosing a $\beta \neq \beta^*$ and a $\lambda \neq \lambda^*$ given her beliefs and (ii) beliefs must be computed using Bayes's rule whenever possible. I define a separating equilibrium as a PBE in which a low-type player bids b_l with probability one while a high-type player bids b_h ($\neq b_l$) with probability one. In such an equilibrium player i can infer player $-i$'s signal out of her bid. A pooling equilibrium is a PBE in which both types bid the same amount with probability one. In such an equilibrium bids have no informational content and do not affect posteriors. A semi-separating equilibrium is a PBE in which one type bids y with probability one, while the other type randomizes her bid between z ($\neq y$) and y.

Second, I restrict attention to the class of the *strongly symmetric* strategies. A strategy is said to be strongly symmetric if it is symmetric and if a player who believes that her rival possesses the same time-one posterior as herself, computes her time-one drilling probability under the assumption that her rival will drill with the same probability as herself. To illustrate this restriction, suppose beliefs are updated under the assumption that high-type players always bid y while low-type players always bid z ($\neq y$). Suppose player one is an optimist while player two is a low-type player. Suppose player one bids y while player two bids z. At time one, player one's posterior ($= \Pr(H|h, b_2 = z)$) is then equal to the one of player two ($= \Pr(H|l, b_1 = y)$). As both players possess different private information, a symmetric strategy does not put any restriction on their time-one drilling behavior.[11] However, as both players possess the same time-one posterior, a strongly symmetric strategy prescribes them to drill at time one with the same probability.

Observe that some $\lambda^*(\cdot)$'s are easy to compute. For example, suppose $b_{-i} < r < b_i$. Then, player i knows that player $-i$ will never drill. In that case $\lambda^*(s_i, \mathtt{h}_1) = 1$ if and only if $\Pr(H|s_i, b_{-i}) \geq c$. Similarly, suppose player i owns her tract and that she did not drill at time one. Her time-two equilibrium drilling probabilities are then also easy to compute. For, at time two ξ is either equal to the state of the world or it is equal to the empty set. In the former case $\lambda^*(s_i, \mathtt{h}_2) = 1$ if and only if the state of the world is high. In the latter case, $\lambda^*(s_i, \mathtt{h}_2) = 1$ if and only if $\Pr(H|s_i, \mathtt{h}_2) \geq c$. Therefore, from now on I restrict attention to computing optimal bidding and time-one drilling decisions when both players own their tracts. With a slight abuse of notation, let $\lambda(s_i, b_i, b_{-i})$ denote player i's time-one drilling probability given her signal, her bid, her neighbor's bid, and *given that both players own their tracts.*

[11] Remember that a strategy is symmetric if players with identical private information who face identical histories behave in an identical way.

3.3 Equilibrium behavior in the waiting game

In this subsection, I compute equilibrium behavior in the waiting game for a variety of exogenously given bidding strategies. In the next subsection, I endogenize bidding behavior. The analysis in this subsection is not original. In particular, Hendricks and Kovenock (1989) already analyzed a waiting game in the context of oil exploration. Similarly, Chamley and Gale (1994) analyzed a waiting game when the state of the world is known after all players chose their actions. I therefore decided not to include all the proofs of this subsection in this paper. They are, however, available upon request.

Let $W(s_i, b_i, b_{-i})$ denote player i's undiscounted gain of waiting, given her signal and both players' bids. Formally,

$$\begin{aligned} W(s_i, b_i, b_{-i}) \quad = \quad & \Pr(H, a_{-i,1} = drill | s_i, b_i, b_{-i})(1-c) \\ & + \Pr(a_{-i,1} = wait | s_i, b_i, b_{-i}) \max\{0, \Pr(H | s_i, b_{-i}, a_{-i,1} = wait) - c\}. \end{aligned} \tag{1}$$

Lemma 1 *Suppose* $(\lambda'(h, b_i, b_{-i}), \lambda'(l, b_i, b_{-i})) << (\lambda''(h, b_i, b_{-i}), \lambda''(l, b_i, b_{-i}))$. *Then*

$$W(s_i, b_i, b_{-i}; (\lambda(h,\cdot), \lambda(l,\cdot)) = (\lambda'(h,\cdot), \lambda'(l,\cdot))) < W(s_i, b_i, b_{-i}; (\lambda(h,\cdot), \lambda(l,\cdot)) = (\lambda''(h,\cdot), \lambda''(l,\cdot)))$$

Lemma 1 is intuitive: the higher $(\lambda(h,\cdot), \lambda(l,\cdot))$, the greater the probability that player $-i$ will drill and, thus, the greater the probability that player i will free-ride on her neighbor's drilling cost.

PROPOSITION 1 *Suppose high-type players bid b_h with probability 1 while low-type players bid b_l with probability 1 ($b_l < b_h$). Suppose both players won their tracts. Then, there exists a unique continuation equilibrium in which player i drills with probability*

$$\lambda^*(s_i, b_i, b_{-i}) = \min\left\{1, \max\left\{0, \frac{(1-\delta)(\Pr(H|s_i, b_{-i}) - c)}{\delta \Pr(L|s_i, b_{-i})c}\right\}\right\}. \tag{2}$$

Proof: As bids perfectly reveal a player's type, both players possess the same posterior at time one. Suppose time-one posteriors are such that

$$0 \leq \Pr(H|s_i, b_{-i}) - c < \delta W(s_i, b_i, b_{-i}; \lambda(\cdot) = 1). \tag{3}$$

The last inequality implies that if player i expects player $-i$ to drill with probability 1, it is a best reply for her to wait. My game then possesses three different continuation equilibria. In the first one, player one drills, while player two waits. In the second one, player two drills while player one waits. In the third one, player i drills with probability $\lambda^*(\cdot)$ in order to make player $-i$ indifferent between drilling and waiting. As I focus on the class of the strongly symmetric strategies, I assume that the mixed-strategy Nash equilibrium is played in the continuation game. As bids perfectly reveal a player's type, player i does not learn anything (about s_{-i}) upon observing player $-i$'s time-one action. Hence, in this case equation 1 boils down to

$$\begin{aligned} W(s_i, b_i, b_{-i}) &= \Pr(H, a_{-i,1} = drill|s_i, b_{-i})(1-c) \\ &\quad + \Pr(a_{-i,1} = wait|s_i, b_{-i})(\Pr(H|s_i, b_{-i}) - c) \\ &= \Pr(H|s_i, b_{-i}) - c + \Pr(L, a_{-i,1} = drill|s_i, b_{-i})c \end{aligned} \tag{4}$$

Player i is indifferent between drilling and waiting if $\Pr(H|s_i, b_{-i}) - c = \delta W(s_i, b_i, b_{-i})$. Replacing $W(\cdot)$ by the right-hand side of 4, this indifference equation can be rewritten as

$$(1-\delta)(\Pr(H|s_i, b_{-i}) - c) = \delta \Pr(L, a_{-i,1} = drill|s_i, b_{-i})c. \tag{5}$$

The left-hand side of this last equality represents player i's expected discounting cost of waiting. The right-hand side represents her expected benefit of waiting: if she waits, with probability $\Pr(L, a_{-i,1} = drill|s_i, b_{-i})$ she will realize that there is no oil. She will then not drill at time two and, from a time-one perspective, save δc. Replacing $\Pr(L, a_{-i,1} = drill|s_i, b_{-i})$ by $\Pr(L|s_i, b_{-i})\lambda^*(\cdot)$, equation 5 boils down to $\lambda^*(\cdot) = \frac{(1-\delta)(\Pr(H|s_i,b_{-i})-c)}{\delta \Pr(L|s_i,b_{-i})c}$.[12]

If $\Pr(H|s_i, b_{-i}) < c$, drilling yields a negative expected payoff and $\lambda^*(s_i, b_i, b_{-i}) = 0$ (as stated in the Proposition). It follows from 5 that

$$\begin{aligned} \delta W(s_i, b_i, b_{-i}; \lambda(\cdot) = 1) &\leq \Pr(H|s_i, b_{-i}) - c \\ \Leftrightarrow (1-\delta)(\Pr(H|s_i, b_{-i}) - c) &> \delta \Pr(L|s_i, b_{-i})c \end{aligned}$$

This case prevails when the discount factor is very low. Player i then prefers to drill even if her neighbor were to drill with probability one. Therefore in this case $\lambda^*(s_i, b_i, b_{-i}) = 1$ (as stated in the proposition). ■

PROPOSITION 2 *Suppose both tracts are marginal ones. Suppose high-type players bid b_h with probability 1 while low-type players bid b_h with probability $x \in (0,1]$ and b_l ($< b_h$) with probability $1-x$. There exists then a unique continuation equilibrium in which $\lambda^*(l, b_i, b_{-i}) = 0$, $\lambda^*(h, b_i, b_l) = 0$ and $\lambda^*(h, b_h, b_h) \in (0,1]$.*

The proposition states a.o. that a low-type player does not drill at time one. This is intuitive: as both tracts are marginal ones, even if a low-type player learns that her neighbor possesses a favorable signal, drilling at time one would still result in a negative expected payoff. Hence, if both tracts are marginal ones a low-type player only drills at time two in case her neighbor found oil at time one. The proposition also states that if a low-type player bids low, no one ever

[12] It follows from the inequalities stated in 3 that this probability $\in [0, 1)$.

drills. This is also intuitive: Player i's knowledge that her neighbor possesses signal l, leads to a downward revision of her posterior probability of finding oil. As both tracts are marginal ones, the cost of drilling now exceeds its expected gain and no one drills. As no new information is produced at time one, no drilling takes place at time two either.

More interestingly, suppose player i is a high-type player and that her neighbor submitted a high bid. Player i's gain of drilling then becomes $\Pr(H|s_i = h, b_{-i} = b_h) - c$, which is positive as $\Pr(H|s_i = h, b_{-i} = b_h) \geq \Pr(H|h) > c$. Observe that, if player $-i$ is a high-type player she possesses the same time-one posterior as player i. In a symmetric equilibrium, both players must drill with the same probability. On the basis of the intermediate value theorem, one can show that there exists a unique $\lambda^*(h, b_h, b_h)$. The intuition is similar to the one I explained above: If δ is "low", $\lambda^*(h, b_h, b_h) = 1$ as the cost of waiting outweighs any gain of waiting. If δ is not "low", player i chooses $\lambda^*(h, b_h, b_h)$ such as to make player $-i$ indifferent between drilling and waiting provided that $s_{-i} = h$.

Proposition 3 *Suppose both tracts are not marginal ones. Suppose low-type players bid b_l with probability one, while high-type players bid b_h ($> b_l$) with probability $x \in (0, 1)$ and b_l with probability $1 - x$. Suppose also that $\delta \Pr(L|h, h)c > (1 - \delta)(\Pr(H|h, h) - c)$. Then there exists a continuation equilibrium in which low-type players always wait. If player i is a high-type player and if $(b_i, b_{-i}) = (b_l, b_h)$, then player i waits while player $-i$ drills at time one. If $(b_i, b_{-i}) = (b_h, b_h)$, both players drill at time one with probability $\lambda^*(h, b_h, b_h) = \frac{(1-\delta)(\Pr(H|h,h)-c)}{\delta \Pr(L|h,h)c}$. If $(b_i, b_{-i}) = (b_l, b_l)$, player i drills at time one with probability*

$$\lambda^*(h, b_l, b_l) = \min\left\{1, \frac{(1-\delta)(\Pr(H|h, b_l) - c)}{\delta \Pr(s_{-i} = h|h, b_l) \Pr(L|h, h)c}\right\}.$$

I first explain the case in which both players submitted a high bid. As a high bid is only submitted by a high-type player, this implies that both players possess signal h (and thus face a positive gain of drilling). As before, there exists a $\lambda^*(h, b_h, b_h) = \frac{(1-\delta)(\Pr(H|h,h)-c)}{\delta \Pr(L|h,h)c}$ which makes both players indifferent between drilling and waiting.[13]

Suppose now that both players submitted a low bid. If player i is a low-type player, she computes $\Pr(H|l, b_l) \leq \Pr(H|l) < c$. As drilling yields a negative expected payoff, she waits. If player i is a high-type player, she computes $\Pr(H|h, b_l)$. Observe that

$$\Pr(H|h, b_{-i} = b_l) \geq \Pr(H|s_i, b_{-i} = b_l, a_{-i,1} = wait) \geq \Pr(H|h, l) = \frac{1}{2} > c.$$

The inequalities above are intuitive: as both tracts are not marginal ones, player i faces a positive gain of drilling at time two when her neighbor submitted a low bid and did not drill at time one.

[13]This probability is $\in (0, 1)$ as $\delta \Pr(L|h, h)c$ is assumed to be greater than $(1 - \delta)(\Pr(H|h, h) - c)$.

This also implies that she faces a positive gain of drilling at time one. As a low bid can come both from a high-type player as from a low-type player, at time one player i is still unsure about player $-i$'s type. Suppose player $-i$ drills with probability one (provided she is a high-type player). Player i then prefers to wait (if and only if)

$$\begin{aligned}\Pr(H|h,b_l)-c &< \delta\Pr(s_{-i}=h|h,b_{-i}=b_l)\Pr(H|h,h)(1-c)\\ &\quad+\delta\Pr(s_{-i}=l|h,b_{-i}=b_l)\left(\Pr(H|h,l)-c\right)\\ \Leftrightarrow (1-\delta)(\Pr(H|h,b_l)-c) &< \delta\Pr(s_{-i}=h|h,b_l)\Pr(L|h,h)c. \qquad (6)\end{aligned}$$

Despite my restriction on δ (stated in the proposition) the above inequality need not be satisfied. To see this, suppose x is very high. In that case it is very unlikely that a low bid was submitted by a high-type player. Hence, even if player i anticipates that player $-i$ will drill with probability one (provided $s_{-i}=h$), the above inequality may be violated due to the fact that $\Pr(s_{-i}=h|h,b_{-i}=b_l)$ is very low. In that case, there exists a unique continuation equilibrium in which player i drills with probability one (as stated in the Proposition). In case inequality 6 is satisfied, there exists a unique symmetric continuation equilibrium in which player i drills with probability $\frac{(1-\delta)(\Pr(H|h,b_l)-c)}{\delta\Pr(s_{-i}=h|h,b_l)\Pr(L|h,h)c}$ (as stated in the proposition).

Suppose now that player i submitted a high bid, while player $-i$ submitted a low one. As the tract is not a marginal one, player i, despite observing that $b_{-i}=b_l$, still faces a positive gain of drilling. Player $-i$ knows this. As $x\in(0,1)$, both players do *not* possess the same time-one posterior, and thus are not required to drill at time one with the same probability.[14] This insight, combined with the assumption that $\delta\Pr(L|h,h)c>(1-\delta)(\Pr(H|h,h)-c)$ implies that, within the class of the strongly symmetric strategies, there exists a continuation equilibrium in which player i drills at time one and in which player $-i$, independently of her signal, waits. In essence, in this continuation equilibrium the right to free-ride is allocated to the low bidder.

3.4 Equilibrium bidding behavior

In this section players choose their bids optimally, correctly anticipating how they will affect equilibrium play in the waiting game.

Proposition 4 *If signals are sufficiently precise or if δ is sufficiently high or if δ is sufficiently low, there exists an equilibrium in which player i bids*

$$b_i^*=\frac{1}{2}\sum_{s_{-i}}\Pr(s_{-i}|s_i)\max\{\Pr(H|s_i,s_{-i})-c,0\}. \qquad (7)$$

[14] As a matter of fact, if player $-i$ is a high-type player who bid low, her posterior (=$\Pr(H|h,b_i=b_h)$) is greater than the one of player i.

Moreover, if either signals are sufficiently precise or if both tracts are marginal and δ sufficiently small, this equilibrium is unique.

Observe that equation 7 implies that a low-type player submits a lower bid than a high-type player. The proposition therefore provides sufficient conditions for the existence and uniqueness of a separating equilibrium. I now explain the intuition behind the three conditions which guarantee the existence of a separating equilibrium. Call $b^*_{l,s}$ the bid of a low-type player in a separating equilibrium. Call $b^*_{h,s}$ the bid of a high-type player in a separating equilibrium. Let $E_1(U|s_i, b_i, b_{-i})$ denote player i's expected utility conditional on her signal, her bid, her neighbor's bid, conditional on owning the tract and net of bidding costs.[15] Let

$$E_{\frac{1}{2}}(U|s_i, b_i) \equiv \Pr(b_{-i} = b^*_{l,s}|s_i)E_1(U|s_i, b_i, b^*_{l,s}) + \Pr(b_{-i} = b^*_{h,s}|s_i)E_1(U|s_i, b_i, b^*_{h,s}).$$

Finally, let $E_0(U|s_i, b_i) \equiv \Pr(r < b_i)[E_{\frac{1}{2}}(U|s_i, b_i) - b_i]$ denote player i's time-zero expected utility. Observe that, as $r \sim U[0, 1]$, $\Pr(r < b_i) = b_i$.

To understand the existence (and non-existence) of a separating equilibrium, it is useful to consider first the hypothetical case in which signals instead of bids are revealed at time $\frac{1}{2}$. Suppose player i submits bid b_i and that she wins her tract. Either player $-i$ also won her tract or player $-i$ submitted a bid lower than the government's reservation price. In the latter case, $E_0(U|s_i, b_i) = \max\{\Pr(H|s_i, s_{-i}) - c, 0\}$. Suppose the former case prevails. As signals are revealed at time $\frac{1}{2}$, both players possess the same time-one posterior. As explained in my previous section, in a strongly symmetric equilibrium this implies that $E_0(U|s_i, b_i)$ is also equal to $\max\{\Pr(H|s_i, s_{-i}) - c, 0\}$. At the start of time $\frac{1}{2}$, player i does not know player $-i$'s signal. Therefore,

$$E_{\frac{1}{2}}(U|s_i, b_i) = \sum_{s_{-i}} \Pr(s_{-i}|s_i) \max\{\Pr(H|s_i, s_{-i}) - c, 0\}.$$

Hence, at time zero player i chooses b_i to maximize $b_i(E_{\frac{1}{2}}(U|s_i, b_i) - b_i)$. This is a very simple strictly concave problem: if player i increases her bid, she increases her chances of winning her tract. This benefit, however, comes at a cost of having to put more money on the table. The solution to this maximization problem is given in 7.

Suppose now that bids instead of signals are disclosed and that both players focus on the candidate equilibrium in which players bid according to equation 7. I assume that an out-of-equilibrium bid is supposed to have been submitted by a low-type player (i.e. $\Pr(s_i = l|b_i \notin \{b^*_{l,s}, b^*_{h,s}\}) = 1$). What are both types' incentives to deviate from this candidate equilibrium strategy?

[15] Thus, $E_1(U|s_i, b_i, b_{-i})$ is solely a function of (b_i, b_{-i}) as it influences both players' incentives to drill. Observe, however, that $E_1(U|\cdot)$ a priori also depends on whether player $-i$ won her tract or not.

Suppose $s_1 = h$ and that she bids $b_1 \neq b^*_{h,s}$. Without loss of generality suppose both firms won their tracts. Player two then computes $\Pr(H|s_2, b_1 \neq b^*_{h,s}) = \Pr(H|s_2, l)$. If player two is a low-type player, she computes $\Pr(H|l, b_1 \neq b^*_{h,s}) = \Pr(H|l, l) < 1 - p < c$, and refrains from drilling at time one. Rationally anticipating this, player one gets $\max\{\Pr(H|h, l) - c, 0\}$. More interestingly, suppose player two is a high-type player. Player two then computes $\Pr(H|h, b_1 \neq b^*_{h,s}) = \Pr(H|h, l)$. Player one, however, is now more "optimistic" than player two in the sense that her posterior ($= \Pr(H|h, h)$) is greater than player two's. Furthermore, player two believes that player one possesses the same posterior as herself (even though this is not true). As I restrict attention to the class of the strongly symmetric strategies, she computes her drilling probability under the assumption that player one and herself play a mixed-strategy Nash equilibrium in the waiting game. It then follows from Proposition 1 that player two drills with probability $\max\{0, \min\{1, \frac{(1-\delta)(\Pr(H|h,l)-c)}{\delta \Pr(L|h,l)c}\}\}$. More importantly, in the Appendix I show that player one's gain of waiting then does not exceed her gain of drilling. The intuition is simple: player one, having observed a high bid from player two, became "very optimistic" about the prospect of finding oil. For her "time is money" and she is only willing to postpone her drilling decision if player two drills with a "very high" probability. Player two, however, having observed that player one did not bid high, became much less confident about the prospect of finding oil. This dented her incentives to drill at time one. Hence, if player one deviates, at time one she gets $\max\{\Pr(H|h, s_2) - c, 0\}$, which is the same (time-one) payoff as the one she would have gotten had she not deviated. As the time-zero payoff function is strictly concave, player one strictly loses by submitting any bid different from $b^*_{h,s}$.

I now consider a low-type player's incentives to deviate. Given my hypothesized out-of-equilibrium beliefs, she cannot gain by submitting a bid $\neq b^*_{h,s}$. Suppose she submits $b_1 = b^*_{h,s}$, that she wins her tract and that she waits.[16] Then, at time $\frac{1}{2}$ her gain of waiting equals

$$\Pr(h|l) \left\{ \Pr(H|l, h)\lambda^*(h, b^*_{h,s}, b^*_{h,s})\delta(1-c) + (1 - \lambda^*(\cdot))\delta \max\{\Pr(H|l, h) - c, 0\} \right\} + \Pr(l|l) \left[\Pr(H, r < b^*_{l,s} | s_1 = s_2 = l, r < b^*_{h,s})\lambda^*(l, b^*_{l,s}, b^*_{h,s})\delta(1-c) \right]. \quad (8)$$

The two terms between curly brackets represent player one's expected gain of waiting if player two is a high-type player: With probability $\Pr(H|l, h)\lambda^*(h, b^*_{h,s}, b^*_{h,s})$ player two drills and finds oil in which case player one gets $\delta(1 - c)$. With probability $(1 - \lambda^*(h, b^*_{h,s}, b^*_{h,s}))$ player two waits in which case player one gets $\delta \max\{\Pr(H|l, h) - c, 0\}$. The term between square brackets represents her expected gain of waiting if player two is a low-type player: With probability

[16] Of course, it remains to be seen whether she would prefer to wait. It should, however, be obvious that player one's gain of drilling is unaffected by her bid. To understand player one's incentives to deviate, I can therefore restrict attention to how her bid affects her gain of waiting.

$\Pr(r < b^*_{l,s} | r < b^*_{h,s})$ player two then also wins her tract in which case player one's high bid induces her to drill with probability $\lambda^*(l, b^*_{l,s}, b^*_{h,s})$. With probability $\Pr(H|l,l)$ player two then discovers oil which allows player one to obtain $\delta(1-c)$. Suppose now that player one submits a low bid, that she wins her tract and that she waits. At time $\frac{1}{2}$ her gain of waiting equals

$$\Pr(h|l)\left\{\Pr(H|l,h)\lambda^*(h, b^*_{h,s}, b^*_{l,s})\delta(1-c) + (1-\lambda^*(\cdot))\delta \max\{\Pr(H|l,h) - c, 0\}\right\}. \tag{9}$$

To understand player one's incentives to deviate, compare 8 with 9. Observe that in 9 a high-type player drills with probability $\lambda^*(h, b^*_{h,s}, b^*_{l,s})$, while in 8 she drills with probability $\lambda^*(h, b^*_{h,s}, b^*_{h,s})$. Similarly, in 9, a low-type player does not drill at all, while in 8 she drills with probability $\lambda^*(l, b^*_{l,s}, b^*_{h,s})$. It is easy to show that $\lambda^*(h, b^*_{h,s}, b^*_{l,s}) \leq \lambda^*(h, b^*_{h,s}, b^*_{h,s})$. This is intuitive: by bidding $b^*_{h,s}$, player one succeeds to make her neighbor "more optimistic" about the prospect of finding oil. As explained in my previous section, this induces player two to drill with a (weakly) higher probability. It then follows from Lemma 1 that this (weakly) increases her gain of waiting.

Submitting a high bid, however, also involves a cost: Player one may have to pay "a lot" of money for a piece of sea below which she thinks there is no oil! If signals are sufficiently precise, the difference between $b^*_{h,s}$ and $b^*_{l,s}$ is very big. high-type players are very confident that there is oil underneath the sea and are prepared to submit a "very high" bid to secure the purchase of the tract. low-type players, however, are very skeptical concerning the probability of finding oil, and therefore refuse to bid as if they possess favorable private information (correctly anticipating that this will dent their neighbor's incentives to drill). If δ is low, player one has also little incentives to submit a high bid. For, any increase in her (undiscounted) gain of waiting (thanks to a higher probability of drilling) is then offset by the low discount factor. Perhaps more surprisingly, if δ is high, player one has also little incentives to submit a high bid. To see this, consider equation 5. The equation teaches us that in equilibrium the discounting cost of waiting must balance the gain of waiting. If the discount factor is high, the opportunity cost of waiting is low (even if player two became "very optimistic" about the prospects of finding oil). Player two's equilibrium drilling probability is then not very sensitive to her time-one posterior. This strongly reduces player one's gain of bidding as if she had favorable private information. Hence, in all these cases there exists a separating equilibrium.

It is worth stressing that the three conditions guaranteeing existence of a separating equilibrium are are not necessary ones. To see this, suppose signals are very imprecise and that player one is a low-type player. Suppose also that players focus on the separating equilibrium. As signals are imprecise, $b^*_{h,s}$ and $b^*_{l,s}$ (as computed in 7) are close to each other. Hence, player one's cost of bidding as if she possesses favorable private information (i.e. $b^*_{h,s} - b^*_{l,s}$) is low. Player one's gain of submitting bid $b^*_{h,s}$, however, is also low. For, if signals are imprecise player two's posterior (and thus also her drilling probability) is hardly influenced by her observation that player one bid "high". One can find values of (p, c, δ) (where p is sufficiently low), such that the benefit of bidding "high" is even lower than its cost.

Proposition 4 also states two sufficient conditions for uniqueness in my game. The logic behind the proof of this uniqueness result is straightforward. Consider candidate equilibrium strategies in which optimists and pessimists randomize their bids according to some distribution functions. Call $\underline{b}_h$, the lowest bid that can be submitted by an optimist in a candidate equilibrium strategy. Call $\overline{b}_l$, the highest bid that can be submitted by a pessimist in a candidate equilibrium strategy. It is easy to show that if signals are sufficiently precise (or if both tracts are marginal ones and if δ is sufficiently small) in any candidate equilibrium $\overline{b}_l < \underline{b}_h$. This is intuitive: if signals are sufficiently precise an optimist would never agree to submit a bid close to zero even if this guaranteed her the right to free-ride with probability one. In that case any bid will perfectly reveal a player's type. It then follows from the second paragraph following Proposition 4 that both players face a very simple (strictly) concave maximization problem, which possesses a unique equilibrium.

Proposition 5 *Suppose both tracts are marginal ones. Then, either there exists a separating or there exists a pooling equilibrium (in which both types bid $b^*_{h,p} = \frac{1}{2}(p-c)$). The equilibrium, however, need not be unique. In particular, there exist values of (p, c, δ) which support a separating, a pooling and a semi-separating equilibrium (in which high-type players bid $b^*_{h,\overline{ss}} \in (b^*_{h,p}, b^*_{h,s})$ while low-type players bid $b^*_{h,\overline{ss}}$ with probability $x \in (0,1)$ and zero with probability $(1-x)$).*

The proposition states a.o. that, as far as marginal tracts are concerned,[17] existence of an equilibrium is always guaranteed. Unfortunately (though not surprisingly) the equilibrium need not be unique.

In a pooling equilibrium a low-type player bids as if she possesses favorable private information. This "high" bid, however, does not succeed to make her neighbor more optimistic about the prospect of finding oil. Nonetheless, it is optimal for her to bid $b^*_{h,p}$, because if she were to submit a different bid instead, this would reveal that she possesses unfavorable private information. As the tract is a marginal one, this would eliminate her neighbor's incentives to drill (and any hope she had to free-ride on her neighbor's drilling cost). An optimist cannot gain by deviating either: If she bids $b_i \neq b^*_{h,p}$, she destroys he neighbor's incentives to drill. It is then optimal for

[17] I have not been able to prove existence of a strongly symmetric (perfect Bayesian) equilibrium $\forall \delta \in [0,1]$ and $\forall 1-p < c < \frac{1}{2}$. I have been able to show, however, the existence of a (not strongly symmetric) perfect Bayesian equilibrium $\forall \delta \in [0,1]$ and $\forall p > \frac{1}{2}$ and $\forall c \in (1-p, p)$. As the proof of this latter result lacks interest, I decided not to include it in this paper.

her to drill and $E_1(U|h, b_1 \neq b^*_{h,p}, b^*_{h,p}) = p - c$. If she bids $b_i = b^*_{h,p}$ (and wins her tract), she engages in a war-of-attrition with her neighbor (which implies that $E_1(U|h, b^*_{h,p}, b^*_{h,p}) = p - c$). Independently of her bid, her time-one payoff is thus equal to $p - c$. Correctly anticipating this, at time zero she maximizes $b_1(p - c - b_1)$, which possesses as unique solution $b^*_1 = \frac{1}{2}(p - c) \equiv b^*_{h,p}$.

It follows from 7 and from Proposition 5 that if both tracts are marginal ones

$$b^*_{h,s} = \frac{1}{2}\Pr(h|h)[\Pr(H|h,h) - c], \text{ while } b^*_{h,p} = \frac{1}{2}(p - c).$$

Observe that

$$b^*_{h,p} = \frac{1}{2}(\Pr(H|h) - c) = b^*_{h,s} + \frac{1}{2}\Pr(l|h)[\Pr(H|h,l) - c].$$

As the tract is a marginal one, $\Pr(H|h,l) = \frac{1}{2} < c$. Hence, a high-type player bids more aggressively in the separating than in the pooling equilibrium. This is intuitive: in the separating equilibrium a high-type player learns her neighbor's signal through her bid. As the tract is a marginal one, this information is very valuable to her. For, if she were to find out that her neighbor is a low-type player, she would refrain from drilling and save $c - \Pr(H|h,l)$. Stated differently, the separating equilibrium provides a high-type player with valuable information which increases her willingness to buy the tract (and thus to bid more aggressively). Hence, there exist values of (p, c, δ) which support multiple equilibria: If players focus on the separating equilibrium, high-type players bid aggressively and thereby discourage a low-type player from submitting the same bid. On the other hand if a high-type player anticipates that her neighbor will bid $\frac{1}{2}(p - c)$ independently of her private information, she values the tract less, bids less aggressively and thereby encourages a low-type player to submit the same bid as hers.

In the Appendix, I show that some values of my exogenous parameters also support a semi-separating equilibrium in which high-type players bid $b^*_{h,\overline{ss}} \in (b^*_{h,p}, b^*_{h,s})$, while low-type players bid $b^*_{h,\overline{ss}}$ with probability x and zero with probability $1 - x$. The intuition is identical to the one I explained above. An optimist knows that if her neighbor submits a high bid, she will be more confident about her prospects of finding oil. This increases her time-zero willingness to buy the tract (which explains why she now submits a bid between $b^*_{h,p}$ and $b^*_{h,s}$). If a low-type player bids $b^*_{h,\overline{ss}}$, from Proposition 2 she knows that this increases the likelihood that she will free-ride on her neighbor's drilling cost. The increase in her gain of waiting, however, is fully compensated by the fact that she has to bid more aggressively to hide her bad private information. Therefore, a low-type player is indifferent between bidding zero and $b^*_{h,\overline{ss}}$.[18]

[18]Proposition 5 establishes the existence of a semi-separating equilibrium when both tracts are marginal ones. I have been able to prove that a pooling equilibrium fails to exist when both tracts are not marginal ones. A semi-separating equilibrium (in which low-type players bid $b^*_{h,\overline{ss}}$ with probability x), however, also exists for some

The proposition below shows that my game may also be characterized by an equilibrium in which a high-type player bids as if she has "bad" private information.

Proposition 6 *There exist values of* (p, c, δ) *which support an equilibrium in which low-type players bid* $b^*_{l,\underline{ss}}$ *with probability one, while high-type players bid* $b^*_{h,\underline{ss}}$ $(> b^*_{l,\underline{ss}})$ *with probability* $x \in (0,1)$ *and* $b^*_{l,\underline{ss}}$ *with probability* $(1-x)$. *Such an equilibrium only exists if* $\delta \Pr(L|h,h)c > (1-\delta)[\Pr(H|h,h) - c]$.

In the appendix I prove the existence of such an equilibrium when both tracts are not marginal ones. I conjecture, however, that such an equilibrium also exists when both tracts are marginal ones. The equilibrium is supported by the continuation strategies summarized in Proposition 3. An optimist knows that if she bids "high" she will either engage in a war-of-attrition with her neighbor (in case her neighbor also submitted a high bid) or she will drill with probability one (in case her neighbor submitted a "low" bid). In either case her time-$\frac{1}{2}$ expected payoff equals $p - c$. Moreover, I also assume that any out-of-equilibrium bid is supposed to have been submitted by a high-type player. Hence, if player one submits any bid different from $b^*_{l,\underline{ss}}$, she gets: $E_{\frac{1}{2}}(U|h, b_1 \neq b^*_{l,\underline{ss}}) = p - c$. Given this time-$\frac{1}{2}$ expected payoff, player one's optimal non-$b^*_{l,\underline{ss}}$ bid equals $\frac{1}{2}(p-c) \equiv b^*_{h,\underline{ss}}$.

x^* and $b^*_{l,\underline{ss}}$ are determined on the basis of the following two equations in two unknowns:

$$E_0(U|h, b_1 = b^*_{l,\underline{ss}}) = E_0(U|h, b_1 = b^*_{h,\underline{ss}}), \text{ and}$$

$$b^*_{l,\underline{ss}} = \frac{1}{2} E_{\frac{1}{2}}(U|l, b^*_{l,\underline{ss}}).$$

The first equation ensures that a high-type player is indifferent between submitting either one of the two bids. The second equation ensures that a low-type player chooses her bid optimally. To gain some insight behind this system of simultaneous equations suppose player i is a high-type player. In this equilibrium she is indifferent between submitting both bids because the gain of bidding low (i.e. increasing the probability that she will be allocated the right to free-ride) is compensated by its cost (i.e. lower probability of winning the tract). Observe that a high-type player only values the right to free-ride if the discount rate is sufficiently high. For, if δ were low, player i would prefer to drill at time one even if she anticipates her neighbor to drill too! This explains why this semi-separating equilibrium only exists if δ is sufficiently high. If $x = 0$, player $-i$ never bids high. Hence, the right to free-ride is never allocated to player i, and there is no gain in bidding "low". Stated differently, if $x = 0$ (i.e. if player $-i$ always bids as if she

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$(p, c, \delta)'s$ when $c \in (1-p, \frac{1}{2})$. As the proofs of both results lack interest, I decided not to include them in this paper.

were a low-type player) it is a best reply for player i to bid $b^*_{h,\underline{ss}}$. The higher x, the higher the probability that she will be allocated the right to free ride (provided she bids "low"), and the higher player i's gain of bidding "low". Similarly, a low-type player values the tract more (and thus bids more aggressively) when x increases. As $b^*_{l,\underline{ss}}$ is increasing in x, this reduces player i's cost of bidding "low". Both reasons explain why $E_0(U|h, b_1 = b^*_{h,\underline{ss}})$ is increasing in x. It can easily be shown that for some (p, c, δ) it is a best response for player i to bid $b^*_{l,\underline{ss}}$ when x is close to one. It then follows from the intermediate value theorem that there exists a semi separating equilibrium of the type described in the Proposition.

According to HP it takes about three months to set up and complete an exploratory drilling program. Hence, if the outcome of a firm's exploratory drilling program is rapidly learnt by neighboring firms, one should expect the discount factor to be very high. HP therefore estimated a discount factor equal to 0.99 while Lin (2006) worked with a discount factor of 0.9. On the basis of those discount factors it is reasonable to assume that if player i anticipates her neighbor to drill, she prefers to wait. Hence, the necessary condition stated in Proposition 6 is most likely satisfied.

I assumed that both tracts are adjacent and that their values are perfectly correlated. This is a simplifying rather than a crucial assumption. To see this, suppose both tracts' values are independently drawn from the following distribution: if the state of the world is high, $\Pr(V_i = 1) = q \in (\frac{1}{2}, 1)$, where V_i $(\in \{0, 1\})$ denotes the value of tract i. If the state of the world is low, $\Pr(V_i = 0) = q$. Hence, if the state of the world is high, with probability $q(1-q)$ the value of tract one is equal to one, while the the second one is worthless. Both players receive a conditionally independent signal about the probability of finding oil, i.e. $\Pr(s_i = h|V_i = 1) = \Pr(s_i = l|V_i = 0) = p \in (\frac{1}{2}, 1)$. Consider a candidate separating equilibrium in which high-type players bid "high" while low-type players bid "low". Along the (candidate) equilibrium path, both players share the same posterior if they submitted the same bid. If q is sufficiently high, and for a sufficiently high discount factor, player i prefers to wait if she anticipates her (distant) "neighbor" to drill. It then transpires from the discussion following Proposition 4 that — for sufficiently unprecise signals — a low-type player may want to deviate from this candidate separating equilibrium. The equilibrium summarized in Proposition 6 does not rely on my assumption of perfect correlation either. As long as tract values are correlated, for a sufficiently high discount factor, it is optimal for firm i to wait if she anticipates her distant neighbor to drill. Hence, there should then exist an equilibrium in which firms use bids as a coordination device.

4 Positive and Normative Implications

Consider the following probit regression model:

$$\Pr(drill) = \beta_0 + \beta_1 \times bid + \beta_2 \times neighbor\_bid + ...$$

where $\Pr(drill)$ denotes the probability that player i drills at time one and where "..." indicates the presence of other explanatory variables. At the risk of stating the obvious, my model implies that this regression may suffer from endogeneity problems. In the equilibrium highlighted in Proposition 6, a low-type player bids "not low" because she knows that it is not unlikely that her neighbor will drill. Her neighbor's decision to drill, however, is also partly influenced by her bid. Hence, the coefficients in this model should be interpreted as (interesting) correlations. HP found that $\beta_1 > 0$ and that $\beta_2 < 0$. Both coefficients were significantly different from zero at the 5% level.

Consider the equilibrium summarized in Proposition 6. Fix player $-i$'s bid at $b^*_{l,\underline{ss}}$ and suppose player i increases her bid from $b^*_{l,\underline{ss}}$ to $b^*_{h,\underline{ss}}$. It then follows from the discussion following Proposition 6 that player i increases her drilling probability from

$$\Pr(s_i = l | b_i = b^*_{l,\underline{ss}}) \times 0 + \Pr(s_i = h | b_i = b^*_{l,\underline{ss}}) \times \lambda^*(h, b^*_{l,\underline{ss}}, b^*_{l,\underline{ss}}) \tag{10}$$

to one. Similarly, fix player $-i$'s bid at $b^*_{h,\underline{ss}}$ and suppose player i increases her bid from $b^*_{l,\underline{ss}}$ to $b^*_{h,\underline{ss}}$. The Proposition 6-equilibrium then predicts that player i should increase her drilling probability from zero to $\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}})$. Hence, this equilibrium predicts that $\beta_1 > 0$. Fix player i's bid at $b^*_{l,\underline{ss}}$ and suppose that her neighbor increases her bid from $b^*_{l,\underline{ss}}$ to $b^*_{h,\underline{ss}}$. The equilibrium then predicts that player i decreases her probability of drilling from the number computed in 10 to zero. Fix player i's bid at $b^*_{h,\underline{ss}}$ and suppose that player $-i$ increases her bid from $b^*_{l,\underline{ss}}$ to $b^*_{h,\underline{ss}}$. This equilibrium then predicts that player i also decreases her drilling probability from one to $\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}})$. Hence, this equilibrium predicts that $\beta_2 < 0$.

As mentioned in my introduction, the hazard rate of drilling features a U-shaped pattern. The equilibrium summarized in Proposition 6 is also consistent with this finding. Sometimes (i.e. if player i bids low while her neighbor bids high) players succeed to coordinate their drilling activities, which explains a high probability of drilling in year one. If players fail to coordinate their drilling decisions through their bids, they play a standard war-of-attrition, which explains why in years 2, 3, and 4 the hazard rate of drilling is "low". In year 5 the probability of drilling is "high" because of the end-game effect.

HP provided an alternative explanation behind the two empirical facts mentioned above. To understand their argument, suppose tract values are imperfectly correlated and that both players possess a signal about the value of their tracts. Suppose there are three signals (l, h, v) and that bids must be chosen from {low,high,very high}. Suppose (exogenously) that an l-type player bids "low", that an h-type player bids high, and that a v-type player bids "very high". Suppose player i did not bid "very high" and that player $-i$ increases her bid from "low" to "high". As a higher bid reveals a higher signal, player i must increase her drilling probability to make player $-i$ indifferent between drilling and waiting. Suppose now that player $-i$ increases her bid from "high" to "very high". This signals that she is so confident about the value of her tract that there is no point in waiting. Correctly anticipating that her neighbor will drill, it is then optimal for player i to wait. Hence, this candidate equilibrium predicts a non-linear relationship between $\Pr(drill)$ and *neighbor_bid*. If $\Pr(drill)$ is not very sensitive to *neighbor_bid* whenever *neighbor_bid* $<$ very high, then β_2 should turn out to be negative in the probit regression model presented above. Similarly, suppose player $-i$ did not bid "very high" and that player i increases her bid from "low" to "high". As this increases player $-i$'s posterior probability of finding oil (as both tract's values are correlated), player i must increase $\Pr(drill)$ to make her neighbor indifferent. Hence, this candidate equilibrium also predicts that $\beta_1 > 0$. More empirical and theoretical research is needed to discriminate my theory (that bids serve as a coordination device) from theirs. From an auction-design point-of-view, it is important to know which explanation prevails. To see this, consider the proposition below:

Proposition 7 *The semi-separating equilibrium summarized in Proposition 6 may yield more expected revenues than the separating one.*

Suppose (p, c, δ) supports both a semi-separating (in which high-type players bid $b^*_{l,\underline{ss}}$ with probability $1 - x$) and a separating equilibrium.[19] Recall that in the separating equilibrium a high-type player bids $b^*_{h,s} = \frac{1}{2}(p - c)$. In the semi-separating equilibrium she randomizes her bid between $b^*_{h,\underline{ss}} = \frac{1}{2}(p - c)$ and $b^*_{l,\underline{ss}}$. As $b^*_{l,\underline{ss}} < b^*_{h,\underline{ss}}$, it is immediate that a high-type player bids more aggressively in the separating equilibrium. Comparing $b^*_{l,\underline{ss}}$ with $b^*_{l,s}$, however, is a more delicate matter. As argued above, $b^*_{l,\underline{ss}}$ is computed out of a system of two equations in two unknowns. As x $(= \Pr(b_{-i} = b^*_{h,\underline{ss}} | s_{-i} = h))$ increases, it becomes more likely that player i's neighbor will drill and this increases her willingness to bid more aggressively. The proposition states that $b^*_{l,\underline{ss}} - b^*_{l,s}$ can become so big to compensate the government for any lost revenues due to a high-type player's strategic "low" bidding behavior.

The proposition implies that if players use bids as a coordination device, it may be a very good idea to use an auction in which bids are disclosed: it reduces the inefficiency associated with a mixed-strategy Nash equilibrium in a battle-of-the-sexes game, and it need not even hurt revenues! If, however, bids merely reflect private information, it is not clear whether the U.S.

[19] In the appendix I show that the set of (p, c, δ)'s which support both types of equilibria is non-empty.

government should use an auction in which players infer each others' private information through their bids. To see this, suppose player i is a low type. Player i knows that during (or after) the auction, her neighbor will infer her low type. This reduces her neighbor's incentives to drill. This causes i's valuation of the tract (and thus also her bid) to be "very low". Suppose now that the government uses an auction in which player $-i$ will not infer player i's type. Then player i knows that her bid will not reduce her neighbor's incentives to drill, which causes her valuation of the tract (and thus also her bid) to be "not very low". Hence, Milgrom and Weber's (1982) linkage principle need not hold in my set-up. More research is needed to shed light on this and related questions.

5 Conclusions

The findings of this paper suggest that the optimal auction format depends on the importance of the information externality at the drilling stage. As documented by HP, the information externality is important in drilling for oil in the outer continental shelf of the US. In other parts of the world the information externality is less important. For example in Libya the probability of finding oil is much higher than in the Gulf of Mexico. Similarly, offshore drilling (in the Gulf of Mexico) is more expensive than drilling in Libya. This might explain why the Libyan Government decided to auction their oil fields (predominantly) using first-price sealed royalty rate[20] bidding while the US (predominantly) used a standard first-price sealed bid auction (followed by bid disclosure). More research is needed to shed light on this and related questions.

Appendix

Proof of Proposition 4

I first state and prove:

Lemma 2

$$\Pr(H|h,h) - c \geq \delta[\Pr(H|h,h) - c + \Pr(L|h,h)\lambda^*(h, b^*_{h,s}, b^*_{l,s})c].$$

Proof: Suppose $b_{-i} = b^*_{h,s} \Leftrightarrow s_{-i} = h$ and $b_{-i} = b^*_{l,s} \Leftrightarrow s_{-i} = l$. Suppose player $-i$ expects player i to follow the same bidding behavior as herself. It follows from Proposition 1 that if

[20] Under royalty rate bidding firms bid percentage figures. If, for example, a firm bids 80%, this means that it is prepared to give 80% of the value of the extracted oil to the government if awarded the tract.

$b_i = b^*_{l,s}$, player $-i$ drills with probability

$$\lambda^*(s_{-i}, b_{-i}, b^*_{l,s}) = \min\left\{1, \max\left\{0, \frac{(1-\delta)(\Pr(H|s_{-i}, l) - c)}{\delta \Pr(L|s_{-i}, l)c}\right\}\right\},$$

while if $b_i = b^*_{h,s}$ she drills with probability

$$\lambda^*(s_{-i}, b_{-i}, b^*_{h,s}) = \min\left\{1, \max\left\{0, \frac{(1-\delta)(\Pr(H|s_{-i}, h) - c)}{\delta \Pr(L|s_{-i}, h)c}\right\}\right\}.$$

It is straightforward to see that the latter probability is (weakly) greater than the former one. It follows from the proof of Proposition 1 that

$$\Pr(H|h, h) - c \geq \delta[\Pr(H|h, h) - c + \Pr(L|h, h)\lambda^*(h, b^*_{h,s}, b^*_{h,s})c],$$

where the right-hand side denotes player i's gain of waiting given that $(s_i, s_{-i}) = (h, h)$ and that $(b_i, b_{-i}) = (b^*_{h,s}, b^*_{h,s})$. As $(\lambda^*(h, b^*_{h,s}, b^*_{l,s})) \leq (\lambda^*(h, b^*_{h,s}, b^*_{h,s}))$ and as $(\lambda^*(l, b^*_{l,s}, b^*_{l,s})) = (\lambda^*(l, b^*_{h,s}, b^*_{h,s})) = 0$, it follows from Lemma 1 that

$$\Pr(H|h, h) - c \geq \delta[\Pr(H|h, h) - c + \Pr(L|h, h)\lambda^*(h, b^*_{h,s}, b^*_{l,s})c].$$

■

The Lemma above together with the explanations following Proposition 4 prove that a high-type player cannot gain by setting $b_i \neq b^*_{h,s}$. It follows from Proposition 1 that if a low-type player bids $b^*_{l,s}$, she gets (at time $\frac{1}{2}$)

$$E_{\frac{1}{2}}(U|l, b^*_{l,s}) = \sum_{s_{-i}} \Pr(s_{-i}|l) \max\{0, \Pr(H|l, s_{-i}) - c\}. \tag{11}$$

If a low-type player bids $b^*_{h,s}$, she gets (at time $\frac{1}{2}$)

$$E_{\frac{1}{2}}(U|l, b^*_{h,s}) = \sum_{s_{-i}} \Pr(s_{-i}|l) \max\{\Pr(H|l, s_{-i}) - c, \delta W(l, b^*_{h,s}, b_{-i})\}, \tag{12}$$

where $\delta W(l, b^*_{h,s}, b_{-i})$ is given in equation 8. As a high-type player cannot gain by deviating, a separating equilibrium exists if and only if $E_0(U|l, b^*_{l,s}) \geq E_0(U|l, b^*_{h,s})$. Observe that if δ is close to zero

$$E_0(U|l, b^*_{h,s}) = b^*_{h,s}[E_{\frac{1}{2}}(U|l, b^*_{l,s}) - b^*_{h,s}] < E_0(U|l, b^*_{l,s}),$$

where the inequality follows from the fact that $b_i[E_{\frac{1}{2}}(U|l, b^*_{l,s}) - b_i]$ is a strictly concave function and reaches its maximum when $b_i = b^*_{l,s}$. Observe also that if δ is close to one, $\lambda^*(\cdot) = 0$ and $E_0(U|l, b^*_{h,s}) < E_0(U|l, b^*_{l,s})$ for the same reason. By continuity, there exists a $\underline{\delta}(c, p) > 0$ and a $\overline{\delta}(c, p) < 1$ such that all $\delta \leq \underline{\delta}(c, p)$ and all $\delta \geq \overline{\delta}(c, p)$ support a separating equilibrium. Finally, if p is close to one, it follows from 7, 11, 12 and 8 that $b^*_{l,s} = 0$, $b^*_{h,s} = \frac{1}{2}(1 - c)$ and

$E_{\frac{1}{2}}(U|l, b^*_{l,s}) = E_{\frac{1}{2}}(U|l, b^*_{h,s}) = 0$. Hence, if p is close to one, $E_0(U|l, b^*_{h,s}) < E_0(U|l, b^*_{l,s})$. By continuity, there exists a $\overline{p}(c, \delta) \in [\frac{1}{2}, 1)$ such that all $p \geq \overline{p}(c, \delta)$ support a separating equilibrium.

I now prove the uniqueness part of Proposition 4. Suppose $s_1 = h$ and consider candidate equilibrium strategies in which high-type players randomize their bids according to an arbitrary c.d.f. $\beta(h)$ and low-type players according to an arbitrary c.d.f. $\beta(l)$. Let $b_{min}(\beta(h)) \equiv \inf\{b : \Pr(b_1 = b|s_1 = h, \beta(h)) > 0\}$. Let $\underline{b}_h \equiv \min\{b_{min}(\beta(h)) : \beta(h)$ is part of an equilibrium strategy$\}$. Observe that, in any equilibrium, $E_0(U|h, b_1) \geq \frac{1}{4}(p - c)^2 > 0$ as player one always has the possibility to bid $\frac{1}{2}(p - c)$ and, if awarded the tract, to drill at time one independent of player two's bid. This implies that $\forall(p, c, \delta)$, $\underline{b}_h > 0$.

Suppose $s_1 = l$. One has: $E_0(U|l, b_1) = b_1 \left(E_{\frac{1}{2}}(U|l, b_1) - b_1\right)$, where

$$E_{\frac{1}{2}}(U|l, b_1) = \int \max\{\Pr(H|l, b_2) - c, \delta W(l, b_1, b_2)\} dF(b_2),$$

where $\delta W(l, b_1, b_2)$ denotes player 1's gain of waiting given her signal, her bid and her rival's bid and where $F(b_2) = \Pr(s_2 = h|l)\beta(h) + \Pr(s_2 = l|l)\beta(l)$ represents the c.d.f. of player two's bid conditional on s_1. Observe also that $E_{\frac{1}{2}}(U|l, b_1)$ is computed conditional upon whether $r < b_2$ or $r > b_2$. Observe that

$$E_{\frac{1}{2}}(U|l, b_1) \leq \int \sum_{s_2} \max\{\Pr(H|l, s_2) - c, \delta W(l, s_2, b_1, b_2)\} \Pr(s_2|l, b_2) dF(b_2)$$

The inequality above comes from the fact that player one may take the wrong time-one decision (e.g. she may drill at time one when, had she known player two's type, she would have preferred to wait). Observe also that $\delta W(l, s_2, b_1, b_2) \leq \delta \Pr(H|l, s_2)(1 - c)$. Hence,

$$\begin{aligned} E_{\frac{1}{2}}(U|l, b_1) &\leq \int \sum_{s_2} \max\{\Pr(H|l, s_2) - c, \delta \Pr(H|l, s_2)(1 - c)\} \Pr(s_2|l, b_2) dF(b_2) \\ &= \sum_{s_2} \max\{\Pr(H|l, s_2) - c, \delta \Pr(H|l, s_2)(1 - c)\} \Pr(s_2|l) \equiv \overline{b}_l. \end{aligned}$$

Note: if p is close to one, $\overline{b}_l$ is close to zero. Similarly, if $c > \frac{1}{2}$ and if δ is close to zero, $\overline{b}_l$ is also close to zero. Note also that a low-type player will never submit a bid higher than $\overline{b}_l$ as she would then get a negative payoff. Thus, for p close to one, or if $c > \frac{1}{2}$ and if δ is close to zero, $\overline{b}_l < \underline{b}_h$. As $\overline{b}_l$ is continuous in p and δ, there exists a $p^c \in [\frac{1}{2}, 1)$ and a $\delta^c \in (0, 1)$, such that if $p \geq p^c$ or if $c > \frac{1}{2}$ and $\delta \leq \delta^c$ in any equilibrium the highest bid of a low-type player is lower than the lowest bid of a high-type player.

Suppose p is sufficiently high such that $\overline{b}_l < \underline{b}_h$. It then follows from my two previous paragraphs that in any equilibrium

$$E_{\frac{1}{2}}(U|s_1, b_1) = \sum_{s_2} \Pr(s_2|s_1) \max\{\Pr(H|s_1, s_2) - c, \delta W(s_1, s_2, b_1, b_2), 0\}.$$

As a bid reveals a player's type, at time one both players possess the same posterior. As I focus on the class of the strongly symmetric strategies, this implies that both players must drill with the same probability (provided both players won their respective tracts). In particular, this implies that $\delta W(s_1, s_2, b_1, b_2) \leq \max\{\Pr(H|s_1, s_2) - c, 0\}$. Hence, at time zero player one chooses b_1 to maximize

$$E_0(U|h, b_1) = b_1 \left[\sum_{s_2} \Pr(s_2|s_1) \max\{\Pr(H|s_1, s_2) - c, 0\} - b_1 \right],$$

which yields as unique solution:

$$b_1^* = \frac{1}{2} \sum_{s_2} \Pr(s_2|s_1) \max\{\Pr(H|s_1, s_2) - c, 0\}.$$

■

Proof of Proposition 5

Let $\lambda_0 \equiv \frac{(1-\delta)(p^2(1-c)-(1-p)^2 c)}{\delta(1-p)^2 c}$, $\lambda_1 \equiv \frac{(1-\delta)(p-c)}{\delta(1-p)^2 c}$, and $\lambda_2 \equiv \frac{p-c}{\delta p^2(1-c)}$. Observe that

$$\lambda_0 < 1 \Leftrightarrow p^2(1-c) - (1-p)^2 c < \delta p^2(1-c). \tag{13}$$

I first state and prove the following Lemmas.

Lemma 3 *If $c > \frac{1}{2}$, $\lambda_0 > \lambda_1$.*

Proof: The stated inequality can be written as

$$\Pr(h|h)[\Pr(H|h, h) - c] > \Pr(h|h)[\Pr(H|h, h) - c] + \Pr(l|h)[\Pr(H|h, l) - c],$$

which is satisfied as $\Pr(H|h, l) = \frac{1}{2} < c$. ■

Lemma 4 $\lambda_0 < 1 \Leftrightarrow \lambda_1 < \lambda_2$

Proof:

$$\lambda_1 < \lambda_2 \Leftrightarrow (1-\delta)p^2(1-c) < (1-p)^2 c.,$$

which is identical to inequality 13. ■

Lemma 5 $\lambda^*(h, b_{h,p}^*, b_{h,p}^*) = \min\left\{1, \frac{(1-\delta)(p-c)}{\delta(1-p)^2 c}, \frac{p-c}{\delta p^2(1-c)}\right\}$.

The proof of this Lemma is identical to the one present in the proof of Proposition 2 (available upon request) and is therefore omitted.

Suppose $c > \frac{1}{2}$ and that beliefs are updated under the assumption that high-type players bid b_h with probability one, while low-type players bid b_h with probability x and zero with probability

$1-x$. Suppose also that out-of-equilibrium beliefs are computed under the assumption that $\Pr(s_i = l | b_i \notin \{b_l, b_h\}) = 1$. Let $b_h \equiv \frac{1}{2}\Pr(b_{-i} = b_h|h)(\Pr(H|h, b_h) - c)$.

Suppose $s_i = h$ and that she bids b_h. By definition, $E_{\frac{1}{2}}(U|h, b_h) \equiv \Pr(b_{-i} = b_h|h)E_1(U|h, b_h, b_h) + \Pr(b_{-i} = b_l|h)E_1(U|h, b_h, b_l)$. As $c > \frac{1}{2} = \Pr(H|h, b_{-i} = b_l)$, $E_1(U|h, b_h, b_l) = 0$. If $b_i = b_{-i} = b_h$, both players play a war-of-attrition. In the symmetric equilibrium (in which low-type players do not drill at time one while high-type players drill with some probability) high-type players either strictly prefer to drill at time one (i.e. when δ is "low") or they will be indifferent between drilling and waiting (i.e. when δ is not "low"). In both cases, $E_1(U|h, b_h, b_h) = \Pr(H|h, b_h) - c$, and

$$E_{\frac{1}{2}}(U|h, b_i = b_h) = \Pr(b_{-i} = b_h|h)(\Pr(H|h, b_h) - c). \tag{14}$$

Suppose now that $b_i \neq b_h$. As mentioned above, players compute their posteriors under the assumption that $\Pr(s_i = l|b_i \neq b_h) = 1$. Hence, $\Pr(H|s_{-i} = h, b_i \neq b_h) = \frac{1}{2} < c$. Player i knows that, if $b_i \neq b_h$, she will never free-ride on her neighbor's drilling cost and

$$E_{\frac{1}{2}}(U|h, b_i \neq b_h) = \Pr(b_{-i} = b_h|h)(\Pr(H|h, b_h) - c). \tag{15}$$

It follows from 14 and 15 that player i's maximization problem can be written as

$$\max_{b_i} E_0(U|h, b_i) = b_i[\Pr(b_{-i} = b_h|h)(\Pr(H|h, b_h) - c - b_i],$$

which yields the solution

$$b_i^* = \frac{1}{2}\Pr(b_{-i} = b_h|h)(\Pr(H|h, b_h) - c) = b_h. \tag{16}$$

Hence, a high-type player cannot gain by bidding differently than b_h.

Suppose now that $s_i = l$. Let

$$\Delta(l, x) = E_0(U|l, b_i = 0; x) - E_0(U|l, b_i = b_h; x).$$

Intuitively, $\Delta(l, x)$ measures player i's incentives to bid low (as opposed to bidding as if she had a high signal) given that player $-i$ computes her posterior under the assumption that $\Pr(b_i = b_h|s_i = l) = x$. If $\Delta(l, x) \geq 0$, player i prefers to bid low (despite the fact that she knows that this will reduce her neighbor's incentives to drill). From my preceding paragraphs we know that a high-type player cannot gain by setting $b_i \neq b_h$. Hence, a separating equilibrium exists if and only if $\Delta(l, 0) \geq 0$. A pooling equilibrium exists if and only if $\Delta(l, 1) \leq 0$, while a semi-separating equilibrium (in which low-type players bid b_h with probability $x^* \in (0, 1)$) exists if and only if $\Delta(l, x^*) = 0$.

Observe that $E_0(U|l, 0; x) = 0 \;\forall x$. Furthermore,

$$E_0(U|l, b_h; x) = b_h\{\Pr(H, s_{-i} = h|l)\lambda^*(h, b_h, b_h)\delta(1 - c) - b_h\}. \tag{17}$$

Observe that b_h and all the probabilities in the equation above are continuous in x. Moreover, it follows from the proof of Proposition 2 (available upon request) that $\lambda^*(h, b_h, b_h)$ is also continuous in x. Hence, $\Delta(l, x)$ is continuous in x, and there exists a strongly symmetric PBE.

It follows from equations 16 and 17 that

$$\Delta(l, 0) \geq 0 \Leftrightarrow \Pr(H, s_{-i} = h|l)\lambda^*(h, b^*_{h,s}, b^*_{h,s})\delta(1-c) \leq \frac{1}{2}\Pr(h|h)[\Pr(H|h,h) - c], \tag{18}$$

and that

$$\Delta(l, 1) \leq 0 \Leftrightarrow \Pr(H, s_{-i} = h|l)\lambda^*(h, b^*_{h,p}, b^*_{h,p})\delta(1-c) \geq \frac{1}{2}[\Pr(H|h) - c]. \tag{19}$$

Lemma 6 *If $c > \frac{1}{2}$, and if $p^2(1-c)-(1-p)^2c < \delta p^2(1-c)$ there either exists a separating or there exists a pooling equilibrium. No (p, c, δ) supports both a separating and a pooling equilibrium.*

Proof: From equation 13 we know that $p^2(1-c) - (1-p)^2c < \delta p^2(1-c) \Leftrightarrow \lambda_0 < 1$, and, thus, $\lambda^*(h, b^*_{h,s}, b^*_{h,s}) = \lambda_0$. This insight, combined with Lemmas 3, 4, 5 and with our assumption that $c > \frac{1}{2}$ allows me to conclude that $\lambda^*(h, b^*_{h,p}, b^*_{h,p}) = \lambda_1$. Hence, inequalities 18 and 19 can respectively be rewritten as $p(1-\delta)(1-c) \leq \frac{1}{2}(1-p)c$, and as $p(1-\delta)(1-c) \geq \frac{1}{2}(1-p)c$. Obviously, both inequalities cannot be simultaneously satisfied (except in the non-generic case in which $p(1-\delta)(1-c) = \frac{1}{2}(1-p)c$). ∎

Lemma 7 *If $c > \frac{1}{2}$ and if $p^2(1-c) - (1-p)^2c > \delta p^2(1-c)$, there either exists a separating or there exists a pooling equilibrium. Moreover there exists values of the parameters which support a separating, a pooling and a semi-separating equilibrium.*

Proof: As $p^2(1-c) - (1-p)^2c > \delta p^2(1-c)$, it follows from inequality 13 that $\lambda_0 > 1$, and, thus, that $\lambda^*(h, b^*_{h,s}, b^*_{h,s}) = 1$. This insight, combined with Lemma 4, allows me to conclude that $\lambda_1 > \lambda_2$. There are two possible cases: (i) $\lambda_2 \geq 1$, and (ii) $\lambda_2 < 1$.

In case (i), inequalities 18 and 19 boil down to

$$\delta p(1-p)(1-c) \leq \frac{1}{2}(p^2(1-c) - (1-p)^2c), \text{ and}$$

$$\delta p(1-p)(1-c) \geq \frac{1}{2}(p-c).$$

Observe that, if $c > \frac{1}{2}$, $p^2(1-c) - (1-p)^2c > p - c$. Hence, there are three different subcases: either $\delta p(1-p)(1-c) \leq \frac{1}{2}(p-c) < \frac{1}{2}(p^2(1-c)-(1-p)^2c)$, in which case there exists a separating, but no pooling equilibrium, or $\frac{1}{2}(p-c) < \delta p(1-p)(1-c) < \frac{1}{2}(p^2(1-c) - (1-p)^2c)$, in which case there exists a separating, a pooling and, by continuity, a semi-separating equilibrium, or $\frac{1}{2}(p-c) < \frac{1}{2}(p^2(1-c) - (1-p)^2c) \leq \delta p(1-p)(1-c)$, in which case there exists a pooling but no separating equilibrium. Note that in this case, there always exists either a separating or a pooling equilibrium.

In case (ii), inequalities 18 and 19 boil down to

$$\delta p(1-p)(1-c) \leq \frac{1}{2}(p^2(1-c)-(1-p)^2c), \text{ and} \tag{20}$$
$$p \leq \frac{2}{3}.$$

I now show that if $p > \frac{2}{3}$ (i.e. if there does not exist a pooling equilibrium), then there exists a separating one. Observe that $p > \frac{2}{3} \Leftrightarrow \frac{1-p}{p} < \frac{1}{2}$. Moreover in this case $\delta p^2(1-c) < p^2(1-c) - (1-p)^2c$. As $\frac{1-p}{p} < \frac{1}{2}$, $\frac{1-p}{p}\delta p^2(1-c) < \frac{1}{2}(p^2(1-c)-(1-p)^2c)$, which is equivalent to 20. Hence, as in the former case, there always exists either a pooling or a separating equilibrium. ■

Proof of Proposition 6

The equilibrium is supported by the continuation strategies summarized in Proposition 3. Moreover, I assume that off-the-equilibrium path, players compute their posteriors under the assumption that $\Pr(s_i = h|b_i \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) = 1$. I first show that a low-type player cannot gain by deviating (Step 1). Next, I show that a high-type player cannot gain by deviating (Step 2). Finally, I show that a $b^*_{l,\underline{ss}} < b^*_{h,\underline{ss}}$ (Step 3).

Step 1: Suppose $s_1 = l$ and that $b_1 = b^*_{l,\underline{ss}}$. As $b^*_{l,\underline{ss}} < b^*_{h,\underline{ss}}$, $\Pr(r < b_2|r < b_1) = 1$. It then follows from Proposition 3 that

$$\begin{aligned} E_{\frac{1}{2}}(U|l, b_1 = b^*_{l,\underline{ss}}) &= \Pr(b_2 = b^*_{h,\underline{ss}}|l)\Pr(H|l,h)\delta(1-c) \\ &\quad + \Pr(s_2 = h, b_2 = b^*_{l,\underline{ss}}|l)\Pr(H|l,h)\lambda^*(h, b^*_{l,\underline{ss}}, b^*_{l,\underline{ss}})\delta(1-c). \end{aligned} \tag{21}$$

Observe that $E_{\frac{1}{2}}(U|l, b_1 = b^*_{l,\underline{ss}})$ only depends on (p, c, δ, x^*) but not on player one's bid. Let $b^*_{l,\underline{ss}} \in \arg\max_{b_1} E_0(U|l, b_1) = b_1(E_{\frac{1}{2}}(U|l, b_1 = b^*_{l,\underline{ss}}) - b_1)$. Observe that $b^*_{l,\underline{ss}} = \frac{1}{2}E_{\frac{1}{2}}(U|l, b_1 = b^*_{l,\underline{ss}})$ and that

$$\lim_{x\to 1} b^*_{l,\underline{ss}} = \frac{1}{2}\Pr(H, h|l)\delta(1-c). \tag{22}$$

Suppose $s_1 = l$, $b_2 = b^*_{h,\underline{ss}}$ and $b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}$. If $b_2 = b^*_{h,\underline{ss}} < r$, player one's payoff (net of bidding costs) equals zero. Suppose that $r < b_2 = b^*_{h,\underline{ss}}$. As player two computes her posterior under the assumption that player one is a high-type player, she believes that her time-one posterior ($= \Pr(H|h,h)$) is equal to the one of player one ($= \Pr(H|h, b_2 = b^*_{h,\underline{ss}})$). As I focus on strongly symmetric strategies, I assume that player two believes that player one will drill with the same probability as herself. It follows from Proposition 3 that this implies that player two will drill at time one with probability $\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}}) = \frac{(1-\delta)(\Pr(H|hh)-c)}{\delta\Pr(L|h,h)c}$, which, by assumption, is less than one. Suppose now that $s_1 = l$, $b_2 = b^*_{l,\underline{ss}}$ and $b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}$. As player two believes that player one is a high-type player and as $b_2 = b^*_{l,\underline{ss}}$, I assume that she anticipates that player one will drill. As $\delta \Pr(L|h,h)c > (1-\delta)(\Pr(H|h,h)-c)$, it is then optimal for her to wait. Hence, $E_{\frac{1}{2}}(U|l, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) = \Pr(r < b^*_{h,\underline{ss}}, b_2 = b^*_{h,\underline{ss}}|l, r < b_1) \Pr(H|l,h)\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}})\delta(1-c)$. Observe that $E_{\frac{1}{2}}(U|l, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) \leq \overline{E}_{\frac{1}{2}}(U|l, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\})$, where

$$\overline{E}_{\frac{1}{2}}(U|l, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) \equiv \Pr(b_2 = b^*_{h,\underline{ss}}|l) \Pr(H|l,h)\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}})\delta(1-c), \tag{23}$$

which is independent of player one's bid.

Let $b_l^{\text{od}} \in \arg\max_{b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}} b_1 \left(\overline{E}_{\frac{1}{2}}(U|l, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) - b_1\right)$.[21] Observe that both b_l^{od} and $b^*_{l,\underline{ss}}$ were chosen to maximize $b_i(E_{\frac{1}{2}}(U|l,\cdot) - b_i)$, where both time-one expectations are independent of b_i. Observe that this is a strictly concave function which is symmetric around $b_i^* = \frac{1}{2}E_{\frac{1}{2}}(U|l,\cdot)$. In particular, this implies that there exists an increasing relationship between $E_{\frac{1}{2}}(U|l, b_i^*)$ and $E_0(U|l, b_i^*)$. Mere inspection of 21 and 23 reveals that $\overline{E}_{\frac{1}{2}}(U|l, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) < E_{\frac{1}{2}}(U|l, b_1 = b^*_{l,\underline{ss}})$. Thus, player one cannot gain by setting $b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}$.

Suppose now that $s_1 = l$ and that $b_1 = b^*_{h,\underline{ss}}$. Using an identical reasoning as in our previous paragraph, player two will drill with probability $\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}})$ if $b_2 = b^*_{h,\underline{ss}}$ and with probability zero if $b_2 = b^*_{l,\underline{ss}}$. As above, player one can then not gain by setting $b_1 = b^*_{h,\underline{ss}}$.

Step 2: Suppose $s_1 = h$ and that she bids $b^*_{h,\underline{ss}}$. Suppose she won her tract. It then follows from Proposition 3 that her payoff is equal to the one she gets if she were to drill at time one with probability one. Hence, $E_{\frac{1}{2}}(U|h, b^*_{h,\underline{ss}}) = p - c$. Let $b^*_{h,\underline{ss}} \in \arg\max_{b_1} E_0(U|h, b_1) = b_1(p - c - b_1)$. Observe that $b^*_{h,\underline{ss}} = \frac{1}{2}(p-c)$.

Suppose $s_1 = h$ and that she bids $b^*_{l,\underline{ss}}$. Suppose she wins her tract. As $b^*_{l,\underline{ss}} < b^*_{h,\underline{ss}}$, $\Pr(r < b_2 | r < b_1) = 1$. It then follows from Proposition 3 that

$$\begin{aligned} E_{\frac{1}{2}}(U|h, b_1 = b^*_{l,\underline{ss}}) \quad = \quad & \Pr(b_2 = b^*_{h,\underline{ss}}|h) \Pr(H|h,h)\delta(1-c) \\ & + \Pr(b_2 = b^*_{l,\underline{ss}}|h) \max\Big\{\Pr(H|h, b_2 = b^*_{l,\underline{ss}}) - c, \\ & \Pr(s_2 = h|h, b_2 = b^*_{l,\underline{ss}})\lambda^*(h, b^*_{l,\underline{s}}, b^*_{l,\underline{ss}}) \Pr(H|h,h)\delta(1-c) + \\ & \delta \Pr(a_{2,1} = wait|h, b_2 = b^*_{l,\underline{ss}}) \Big(\Pr(H|h, b_2 = b^*_{l,\underline{ss}}, a_{2,1} = wait) - c\Big)\Big\}. \end{aligned} \tag{24}$$

Observe that $\Pr(H|h, b_2 = b^*_{l,\underline{ss}}, a_{2,1} = wait) \geq \Pr(H|h,l) = \frac{1}{2} > c$. Observe also that if $b_1 = b_2 = b^*_{l,\underline{ss}}$ in a strongly symmetric equilibrium either player one strictly prefers to drill at time one, or she is indifferent between her two time-one actions. This insight allows me to rewrite the equation above as

$$E_{\frac{1}{2}}(U|h, b^*_{l,\underline{ss}}) = p - c + \Pr(b_2 = b^*_{h,\underline{ss}}|h)(\delta \Pr(L|h,h)c - (1-\delta)(\Pr(H|h,h) - c)). \tag{25}$$

[21] The superscript "od" stands for "optimal deviation".

It follows from the first paragraph of this step that $E_0(U|h, b^*_{h,\underline{ss}}; x \to 0) = \frac{1}{2}(p-c)(p-c-\frac{1}{2}(p-c))$. It follows from 25 that $E_0(U|h, b^*_{l,\underline{ss}}; x \to 0) = b^*_{l,\underline{ss}}(p-c-b^*_{l,\underline{ss}})$. As $b^*_{h,\underline{ss}} = \frac{1}{2}(p-c) \in \arg\max_{b_1} b_1(p-c-b_1)$, as $b^*_{l,\underline{ss}} < b^*_{h,\underline{ss}}$ and as $b_1(p-c-b_1)$ is strictly concave it follows that $E_0(U|h, b^*_{h,\underline{ss}}; x \to 0) > E_0(U|h, b^*_{l,\underline{ss}}; x \to 0)$. It follows from 22 and from 25 that

$$\begin{aligned} E_0(U|h, b^*_{l,\underline{ss}}; x \to 1) &= \frac{1}{2}p(1-p)\delta(1-c)\left(p-c+\delta(1-p)^2c-(1-\delta)(p^2(1-c)\right. \\ &\quad \left. -(1-p)^2c)-\frac{1}{2}p(1-p)\delta(1-c)\right). \end{aligned}$$

It follows from the first paragraph of this step that

$$E_0(U|h, b^*_{h,\underline{ss}}; x \to 1) = \frac{1}{4}(p-c)^2.$$

Let

$$\begin{aligned} \Omega_2 \quad &\equiv \left\{ (p,c,\delta) \mid \quad 1-p<c<\frac{1}{2}, \delta \Pr(L|h,h)c > (1-\delta)(\Pr(H|h,h)-c), \right. \\ &\qquad \left. E_0(U|h, b^*_{h,\underline{ss}}; x \to 1) < E_0(U|h, b^*_{l,\underline{ss}}; x \to 1) \right\}. \end{aligned} \tag{26}$$

Observe that Ω_2 is non-empty. For example, $(p,c,\delta) = (0.52, 0.49, 1) \in \Omega_2$. As $E_0(U|h,\ b^*_{h,\underline{ss}}\ ;\ x \to 0) > E_0(U|h, b^*_{l,\underline{ss}}; x \to 0)$, and as both expectations are continuous in x, it follows that $\forall (p,c,\delta) \in \Omega_2$, $\exists\ x \in (0,1)$ such that $E_0(U|h, b^*_{l,\underline{ss}}; x) = E_0(U|h, b^*_{h,\underline{ss}}; x)$.

Suppose $s_1 = h$, that $b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}$ and that she won the tract. If $r > b_2$, player one drills her tract at time one. If $r < b_2 = b^*_{l,\underline{ss}}$, player two believes that player one is a high-type player who will drill at time one. From Proposition 3 we know that it is a best reply then for player one to drill at time one. If $r < b_2 = b^*_{h,\underline{ss}}$, player two believes that player one possesses the same posterior as herself and drills at time one with probability $\lambda^*(h, b^*_{h,\underline{ss}}, b^*_{h,\underline{ss}}) = \frac{(1-\delta)(\Pr(H|hh)-c)}{\delta \Pr(L|h,h)c} < 1$. Hence, player one is, at best, indifferent between drilling and waiting and $E_{\frac{1}{2}}(U|h, b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}) = E_{\frac{1}{2}}(U|h, b_1 = b^*_{h,\underline{ss}}) = p-c$. As $b^*_{h,\underline{ss}} \in \arg\max_{b_1} E_0(U|h, b_1) = b_1(p-c-b_1)$, player one cannot gain by setting $b_1 \notin \{b^*_{l,\underline{ss}}, b^*_{h,\underline{ss}}\}$.

Step 3: By contradiction, suppose that $b^*_{h,\underline{ss}} < b^*_{l,\underline{ss}}$. As an optimal bid is always equal to $\frac{1}{2}E_{\frac{1}{2}}(U|\cdot)$, this implies that $E_{\frac{1}{2}}(U|h, b^*_{h,\underline{ss}}) < E_{\frac{1}{2}}(U|l, b^*_{l,\underline{ss}}; x^*)$. However, mere inspection of 21 and 24 reveals that $E_{\frac{1}{2}}(U|l, b^*_{l,\underline{ss}}; x^*) < E_{\frac{1}{2}}(U|h, b^*_{l,\underline{ss}}; x^*)$. Hence, $E_{\frac{1}{2}}(U|h, b^*_{h,\underline{ss}}) < E_{\frac{1}{2}}(U|h, b^*_{l,\underline{ss}}; x^*)$. Moreover, in step one I argued that there exists an increasing relationship between $E_{\frac{1}{2}}(U|\cdot)$ and $E_0(U|\cdot)$. Hence, $E_0(U|h, b^*_{h,\underline{ss}}) < E_0(U|h, b^*_{l,\underline{ss}}\ ; x^*)$, and a high-type player cannot be indifferent between the two bids. ■

Proof of Proposition 7

Let $E(R|s)$ denote the expected revenue if players focus on the separating equilibrium. One has:

$$E(R|s) = \Pr(s_i = h) \Pr(r < b^*_{h,s}) b^*_{h,s} + \Pr(s_i = l) \Pr(r < b^*_{l,s}) b^*_{l,s}.$$

Similarly,

$$\begin{aligned} E(R|\underline{ss}) &= \Pr(s_i = h) \left[x \Pr(r < b^*_{h,\underline{ss}}) b^*_{h,\underline{ss}} + (1-x) \Pr(r < b^*_{l,\underline{ss}}) b^*_{l,\underline{ss}} \right] \\ & \quad + \Pr(s_i = l) \Pr(r < b^*_{l,\underline{ss}}) b^*_{l,\underline{ss}}, \end{aligned}$$

where $E(R|\underline{ss})$ denotes the expected revenue if players focus on the semi-separating equilibrium. Taking into account that $\Pr(s_i = h) = \Pr(s_i = l) = \frac{1}{2}$ and that $r \sim [0,1]$, one has:

$$E(R|\underline{ss}) > E(R|s) \Leftrightarrow \left(b^*_{l,\underline{ss}}\right)^2 - \left(b^*_{l,s}\right)^2 > (1-x^*) \left[\left(b^*_{h,s}\right)^2 - \left(b^*_{l,\underline{ss}}\right)^2 \right]. \tag{27}$$

Suppose c is close to (but nonetheless strictly greater than) 0.4, δ is close to (but nonetheless strictly less than) 1, and $p = 0.6$. As δ is close to one, it follows from proposition 4 that there exists a separating equilibrium. It is also easy to check that $(p, c, \delta) = (0.6, 0.4 + \epsilon, 1 - \epsilon) \in \Omega_2$ (see 26). Thus $(p, c, \delta) = (0.6, 0.4 + \epsilon, 1 - \epsilon)$ also supports a semi-separating equilibrium in which high-type players bid $b^*_{l,\underline{ss}}$ with probability $1 - x$. Furthermore, it can be checked that if $(p, c, \delta) = (0.6, 0.4 + \epsilon, 1 - \epsilon)$, $x^* \simeq 0.72$ and that inequality 27 boils down to $0.00211 > 0.00205$, which is obviously satisfied. ■

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Appendix B (Not for Publication)

Proof of Lemma 1

Observe that equation 1 can be rewritten as:

$$\begin{aligned} W(s_i, b_i, b_{-i}) &= \Pr(H|s_i, b_{-i})(1-c) - (1-\mathcal{I})\Pr(H, a_{-i,1} = wait|s_i, b_i, b_{-i})(1-c) \\ &\quad -\mathcal{I}\Pr(L, a_{-i,1} = wait|s_i, b_i, b_{-i})c, \end{aligned}$$

where $\mathcal{I} = 1$ if $\Pr(H|s_i, b_{-i}, a_{-i,1} = wait) \geq c$ and $\mathcal{I} = 0$ otherwise. The Lemma then follows from the fact that $\Pr(\cdot, a_{-i,1} = wait|\cdot)$ is decreasing in $(\lambda(h, \cdot), \lambda(l, \cdot))$. ■

Proof of Proposition 2

Suppose $s_i = l$. At time 1/2, player i computes $\Pr(H|l, b_{-i}) \leq \frac{1}{2} < c$, and there does not exist a continuation equilibrium in which $\lambda^*(l, b_i, b_{-i}) \neq 0$. Suppose $s_i = h$ and that $b_{-i} = b_l$. As a "low" bid can only come from a low-type player, $\Pr(H|h, b_{-i} = b_l) = \frac{1}{2} < c$, and there does not exist a continuation equilibrium in which $\lambda^*(h, b_i, b_l) \neq 0$.

I now compute $\lambda^*(h, b_h, b_h)$. Suppose $s_i = s_{-i} = h$. $\hat{\lambda}(h, b_h, b_h)$ is defined as a real number with which player $-i$ must drill to equate player i's gain of drilling (at time one) with her gain of waiting. Formally, $\hat{\lambda}(h, b_h, b_h)$ is computed such that

$$\begin{aligned} \Pr(H|h, b_h) - c &= \delta\Pr(s_{-i} = h|h, b_h)\hat{\lambda}(h, b_h, b_h)\Pr(H|h, h)(1-c) \\ &\quad +\delta[\Pr(s_{-i} = h|h, b_h)(1-\hat{\lambda}(h, b_h, b_h)) + \Pr(s_{-i} = l|h, b_h)] \\ &\quad \times \max\left\{0, \frac{p(1-p)x + p^2(1-\hat{\lambda}(h, b_h, b_h))}{2p(1-p)x + (p^2 + (1-p)^2)(1-\hat{\lambda}(h, b_h, b_h))} - c\right\}. \end{aligned} \tag{28}$$

Suppose $\hat{\lambda}(h, b_h, b_h) < 1$. Then, both players will only drill at time one with the same probability if $\lambda^*(h, b_h, b_h) = \hat{\lambda}(h, b_h, b_h)$. Suppose $\hat{\lambda}(h, b_h, b_h) > 1$ (which is the case for sufficiently low values of δ). Then player i prefers to drill at time one even if she knows that her neighbor will drill with probability one. Thus, in any strongly symmetric equilibrium $\lambda^*(h, b_h, b_h) = \min\{1, \hat{\lambda}(h, b_h, b_h)\}$. I now show that there exists a unique value of $\hat{\lambda}(h, b_h, b_h)$ which satisfies 28.

If $\Pr(H|h, b_h, a_{-i,1} = wait) \geq c$, 28 boils down to $\Pr(H|h, b_h) - c = \delta(\Pr(H|h, b_h) - c) + \delta\Pr(L, s_{-i} = h|h, b_h)\hat{\lambda}(h, b_h, b_h)c \equiv RHS_1$. Denote by λ_1 the value of $\hat{\lambda}(\cdot)$ which equates the LHS of the above equation with RHS_1. Formally, $\lambda_1 \equiv \frac{(1-\delta)(\Pr(H|h,b_h)-c)}{\delta\Pr(L,h|h,b_h)c}$. If $\Pr(H|h, a_{2,1} = wait) < c$, 28 boils down to

$$\Pr(H|h, b_h) - c = \delta\Pr(H, s_{-i} = h|h, b_h)\hat{\lambda}(h, b_h, b_h)(1-c) \equiv RHS_2.$$

Denote by λ_2 the value of $\hat{\lambda}(\cdot)$ which equates the LHS of the above equation with RHS_2. Formally, $\lambda_2 \equiv \frac{\Pr(H|h,b_h)-c}{\delta \Pr(H,h|h,b_h)(1-c)}$.

Observe that $\Pr(H|h, b_h, a_{2,1} = wait)$ is decreasing in $\hat{\lambda}(\cdot)$. Call λ^c the value of $\hat{\lambda}(\cdot)$ such that $\Pr(H|h, b_h, a_{2,1} = wait) = c$. Observe that $\lambda^c < 1 \Leftrightarrow c > \frac{1}{2}$. If $\hat{\lambda}(\cdot) < \lambda^c$, RHS_1 is the relevant right-hand side of equation 28. If $\hat{\lambda}(\cdot) > \lambda^c$, RHS_2 is the relevant right-hand side of equation 28. If $\hat{\lambda}(\cdot) = \lambda^c$, $RHS_1 = RHS_2$. Observe also that

$$\frac{\partial RHS_1}{\partial \hat{\lambda}(\cdot)} < \frac{\partial RHS_2}{\partial \hat{\lambda}(\cdot)} \Leftrightarrow (1-p)^2 c < p^2(1-c) \Leftrightarrow \Pr(h|h)[\Pr(H|h,h) - c] > 0,$$

which is obviously satisfied.

I now show that $\hat{\lambda}(\cdot) = \min\{\lambda_1, \lambda_2\}$. Suppose $\hat{\lambda}(\cdot) = \lambda_2$ and that $\lambda_1 < \lambda_2$. $\hat{\lambda}(\cdot)$ will only be equal to λ_2 if

$$\Pr(H|h, b_h, a_{-i,1} = wait; \hat{\lambda}(\cdot) = \lambda_2) < c \Leftrightarrow \lambda_2 > \lambda^c. \quad (29)$$

As $\lambda_1 < \lambda_2$, and as $0 < \frac{\partial RHS_1}{\partial \hat{\lambda}(\Pr(H|h))} < \frac{\partial RHS_2}{\partial \hat{\lambda}(\Pr(H|h))}$, RHS_1 will only be equal to RHS_2 at a value $\lambda^c > \lambda_2$, which contradicts inequality 29. Using a similar reasoning, one can check that $\hat{\lambda}(\cdot)$ cannot be equal to λ_1 when $\lambda_2 < \lambda_1$. If $\hat{\lambda}(\cdot) > 1$, this means that player two cannot make player one indifferent between drilling and waiting (not even if she drills for sure if $s_2 = h$) and in that case player one strictly prefers to drill. Hence, $\lambda^*(h, b_h, b_h) = \min\{1, \lambda_1, \lambda_2\}$.■

Proof of Proposition 3

The proof proceeds in two steps. First I compute $\lambda^*(h, b_i, b_{-i})$ under the assumption that $\lambda^*(l, b_i, b_{-i}) = 0$. Next, I take $\lambda^*(h, b_i, b_{-i})$ as given and show that it is a best reply for low-type players to wait.

Step 1: Suppose $s_i = h$. There are then three different cases: (i) $(b_i, b_{-i}) = (b_h, b_h)$, (ii) $(b_i, b_{-i}) = (b_l, b_l)$ and (iii) $(b_i, b_{-i}) = (b_l, b_h)$.

Consider case (i). As only high-type players bid b_h, both players infer that their neighbor is a high-type player. This implies that $\Pr(H|h, b_h, a_{-i,1} = wait) = \Pr(H|h, h) > c$. Observe also that both players possess identical private information and face identical histories. As players use symmetric strategies, both of them drill at time one with the same probability. Player i is indifferent between drilling and waiting if

$$\Pr(H|h,h) - c = \delta \Pr(H, a_{-i,1} = drill|h,h)(1-c) + \delta \Pr(a_{-i,1} = wait|h,h)(\Pr(H|h,h) - c).$$

The equality above can be rewritten as $\lambda^*(h, b_h, b_h) = \frac{(1-\delta)(\Pr(H|h,h)-c)}{\delta \Pr(L|h,h)c}$, which, by assumption, is less than one.

Consider case (ii). Observe that $\Pr(H|h, b_l) \geq \Pr(H|h, b_l, a_{-i,1} = wait) \geq \frac{1}{2} > c$. If $s_{-i} = h$, both players possess identical private information and face identical histories. Hence, they drill at time one with the same probability. Define $\hat{\lambda}(h, b_l, b_l)$ as a real number with which player $-i$ must drill (provided she is a high-type player) to make player i indifferent between drilling and waiting (provided player i is a high-type player). Formally,

$$\begin{aligned}\Pr(H|h,b_l) - c &= \delta \Pr(H, a_{-i,1} = drill|h, b_l)(1-c) + \delta \Pr(a_{-i,1} = wait|h,h) \\ &\quad \times (\Pr(H|h, b_l, a_{-i,1} = wait) - c).\end{aligned}$$

The equality above can be rewritten as $\hat{\lambda}(h, b_l, b_l) = \frac{(1-\delta)(\Pr(H|h,b_l)-c)}{\delta \Pr(s_{-i}=h|h,b_l) \Pr(L|h,h)c}$. If $\hat{\lambda}(h, b_l, b_l) > 1$, this means that, due to a low discount factor (or to a low $\Pr(s_{-i} = h|h, b_l)$), player i strictly prefers to drill at time one despite the fact that player $-i$ will drill with probability one if she is a high-type player. Hence, if $\lambda^*(h, b_l, b_l)$ is defined as $\min\{1, \hat{\lambda}(h, b_l, b_l)\}$, player i cannot gain by drilling at time one with a different probability.

Consider case (iii). I show that $\lambda(h, b_l, b_h) = 0$ and $\lambda(h, b_h, b_l) = 1$ constitutes an equilibrium in the continuation game. As $b_{-i} = b_h$, player i knows that $s_{-i} = h$. Suppose player i expects player $-i$ to drill at time one with probability one. Player i prefers to wait if $\delta \Pr(H|h,h)(1-c) > \Pr(H|h,h) - c$. Rewriting the inequality above yields $\delta \Pr(L|h,h)c > (1-\delta)(\Pr(H|h,h) - c)$, which, by assumption, is satisfied. Player $-i$ expects player i to wait with probability one. As $\Pr(H|h, b_l) \geq \frac{1}{2} > c$, and as $\delta < 1$, it is a best reply for her to drill at time one with probability one.

Step 2: Suppose $s_i = l$. There are then two different cases: either $b_{-i} = b_l$ or $b_{-i} = b_h$. If $b_{-i} = b_l$, player i computes $\Pr(H|l, b_l) \leq \Pr(H|l) < c$ and she strictly prefers to wait. If $b_{-i} = b_h$, player i computes $\Pr(H|l, b_h) = \Pr(H|l, h) = \frac{1}{2}$, which is greater than c. As $x < 1$, player $-i$ computes $\Pr(H|h, b_l) > \frac{1}{2}$ = player i's posterior. As both players possess different posteriors, they do not have to drill at time one with the same probability. Player i strictly prefers to wait if and only if

$$(1-\delta)\left(\frac{1}{2} - c\right) < \delta\ \frac{1}{2}\ c. \tag{30}$$

By assumption $(1-\delta)(\Pr(H|h,h) - c) < \delta \Pr(L|h,h)c$. As $(1-\delta)(\frac{1}{2} - c) < (1-\delta)(\Pr(H|h,h) - c)$ and as $\delta \Pr(L|h,h)c < \delta \frac{1}{2} c$, I conclude that inequality 30 holds. Thus, in this continuation equilibrium a low-type player always waits with probability one. ■