

On Bid Disclosure in OCS Wildcat Auctions*

Nicolas Melissas[†]

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Abstract

Two players first bid for offshore tracts—below which oil and gas may be present—and next time their drilling decisions. High types bid more aggressively if the auctioneer discloses bids as this gives them useful information about the profitability of drilling. Low types may also prefer the auctioneer to disclose bids as it sometimes facilitates coordination: If Player two finds out that Player one bid “low”, she might conclude that Player one will not drill. This induces Player two to drill first. Low types, however, may also fear that the disclosure of their “low” bids reduces the other player’s incentives to drill. In that case they bid more aggressively if the auctioneer does not disclose bids. If players are sufficiently patient and if the ex ante profitability of drilling is “high”, it is optimal to disclose bids. Otherwise, it may be optimal not to disclose them.

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[†]Centro de Investigación Económica, Instituto Tecnológico Autónomo de México, Camino a Santa Teresa 930, México D.F. 10700, Mexico. E-mail nicolas.melissas@itam.mx.

1 Introduction

In recent years, some countries decided to put part of their offshore oil and gas reserves under the hammer. Brazil, Cuba, Libya, Nigeria, Russia, and the U.S., for example, organized offshore oil and gas auctions in the past decade. Those auctions often generate huge revenues and secure the supply of crucial energy resources. In a seminal paper, Hendricks and Porter (1996) argued that offshore drilling suffers from a public good problem: If a firm drills and finds oil (and gas), this is costlessly observed by other firms. They also argued that firms do *not* coordinate their drilling decisions, i.e. firms typically play a war of attrition to determine who will drill first. Comparing auctions in this context is thus a delicate matter as the post-auction war of attrition should influence bidding behavior (and vice versa).

In this paper I develop a two-unit, two-player bidding-waiting model to compare the performance of two different oil and gas auctions. In particular, my model tackles the following questions: Should oil and gas auctions be designed such that players learn each others' private information through their bids? Or should one use an auction in which a player's private information is only partially revealed after the auction? I consider a set-up in which two players possess private information (i.e. the result of a seismic test) about the value of a tract they are interested in acquiring. Player two possesses a weakly more informative signal than Player one. In the first stage, both players participate in a first-price auction. After this first stage, both players might end up owning tracts that are relatively close to each other. In that case they play a waiting game to determine who will drill first. To be more specific, players can drill in two periods. If Player i drills in the first investment period, both players observe the value of Tract i . Tract values are assumed to be "very" (though not perfectly) correlated: If Player $-i$ learns that Player i drilled a dry hole, his payoff from drilling becomes negative—even if he received a favorable signal. Similarly, if Player i found oil, Player $-i$'s payoff from drilling becomes positive—even if he received an unfavorable signal. Waiting thus yields an informational benefit but comes at the cost of discounting. I consider two different auction designs. In the first one (which closely matches the one used by the U.S. government), the auctioneer uses a first-price sealed bid auction and, prior to the waiting game, discloses both players' bids. In the second one, the auctioneer also uses a first-price sealed bid auction but does not disclose each player's bid. Instead, the auctioneer merely announces which tracts were won by which players.

I assume that players focus on the class of the non-coordinating strategies if the auctioneer discloses bids. A strategy is said to be non-coordinating if both players—whenever possible—play a war of attrition to determine who will drill first. The qualification "whenever possible" must be added as a particular posterior profile may arise in which one player faces a positive payoff

from drilling while the other one doesn't. Recall that tract values are imperfectly correlated and that Player two may possess a more informative signal than Player one. Hence, at the start of the waiting game both players may compute different posterior probabilities that their tracts contain oil. Suppose that Player one faces a negative payoff from drilling while Player two's payoff is positive. Player two then knows that Player one will not drill. Hence, it is a best response for her to drill at the start of the waiting game. In this case both players thus succeed to coordinate their drilling activities,¹ and Player one prefers the auctioneer to disclose bids. Had bids not been disclosed, Player two would not know that Player one faces a negative payoff from drilling. Instead, she would believe that perhaps Player one is a type-one player who might drill in the first period. This, in turn, reduces her incentives to drill (and hampers Player one's possibilities to free-ride). In this case, Player one thus bids more aggressively in the auction with bid disclosure. Another advantage of disclosing bids is related to Blackwell's (1951) value of information theorem. Player two's observation that Player one bid "low", for example, may convince her not to drill her tract: Had bids not been disclosed, she might have drilled and would have incurred an expected loss. Hence, Player two, anticipating that she will receive useful information prior to drilling, values "winning" more and, thus, bids more aggressively in the auction with bid disclosure. Disclosing bids, however, also suffers from one drawback. To see this, consider Player one's incentives to bid in my previous example. Anticipating that her low bid will destroy the other player's incentives to drill, Player one is then not interested in acquiring a tract (i.e. she bids zero in the auction with bid disclosure). Had bids not been disclosed, her "low" bid would not have adversely affected the other player's incentives to drill. Anticipating this, she would have valued "winning" more and, thus, would have bid more aggressively. If the discount factor is "sufficiently high" and if the profitability of drilling is "not low", it is optimal to disclose bids. Otherwise it may be optimal not to disclose them. I explain that this insight implies that an open ascending auction possesses a previously unnoticed disadvantage in this context. Finally, I argue that disclosing bids may either promote or hamper total drilling activity.

This is not the first paper to address issues of auction design when pre- or post-auction considerations are important. Haile (2000) considers a game in which players can resell after the auction took place.² He shows that the possibility of reselling affects bids in two opposing

¹I assume, however, that this successful coordination only occurs for this specific posterior profile. In particular, if both players possess a positive payoff from drilling at the start of the waiting game, it is assumed that Player i chooses her drilling probability to make Player $-i$ indifferent between drilling and waiting.

²Bikhchandani and Huang (1989) also compare the revenue properties of different auction designs in the presence of a resale market.

ways. On the one hand, some types have an incentive to bid more aggressively in order to extract seller's surplus at the reselling stage. On the other hand, some types have an incentive to bid less aggressively to extract buyer's surplus at the reselling stage. In general, either effect can dominate and an English auction (followed by resale) does not necessarily yield higher expected revenues than a second-, or a first-price one (followed by resale). Goeree (2003) analyzes an auction followed by some downstream interaction. In contrast to this paper, downstream interaction is not modelled explicitly.³ Instead he takes a reduced-form approach in which a player's payoff depends (i) on whether she won the object or not, (ii) on her true type and (iii) on her perceived type. He characterizes bidding strategies in first-price, second-price and English auctions and shows that an English auction may yield higher expected revenues than a sealed-bid auction when bidders have an incentive to understate their private information. If bidders have an incentive to overstate their private information, the three auctions yield the same expected revenues.⁴ In Haile and Goeree signaling motives are important at the auction stage. As incentives to signal are affected by the auction design, the revenue equivalence theorem fails to go through. In section 4, I show that if signals (instead of bids) were disclosed after the auction, both types would still bid the same amount as in the auction with bid disclosure. Furthermore, in the auction without bid disclosure, players trivially have no signaling motive. Signaling is thus not the driving force behind my results. Arozamena and Cantillon (2004) analyze incentives to invest in a cost reducing technology prior to a procurement auction. They argue that a firm has more incentives to lower her costs if the procurement contract is offered via a second-price auction (as opposed to a first-price auction). Their results are driven by their finding that if a firm reduces her costs, this induces the other firms to bid more aggressively (in the subsequent first-price auction).

2 Some institutional features

In this section I explain some important institutional features of the U.S. offshore leasing program. This will help the reader to understand better the game I will study. It also provides a justification for some of my simplifying assumptions.

I focus on wildcat tracts. Such a tract is situated in an offshore geographical area where no

³Das varma (2003) models post-auction (Bertrand and Cournot) competition explicitly and obtains essentially the same results as Goeree (2003).

⁴Molnár and Virág (2008) consider a set-up very similar to Goeree's. They show that the sign of the second derivative of the profit function (with respect to the firm's perceived type) crucially determines the optimal auction format.

exploratory drilling has occurred in the past. Hence, in those auctions no firm should possess superior information about the value of a tract.⁵ A tract covers an area not exceeding 5,760 acres ($\approx 23.3km^2$ or 9 square miles). Prior to bidding, firms perform seismic tests to assess the likelihood of finding oil (and gas). The seismic tests which prevailed 25 years ago only provided a noisy statistic about the value of the oil (and gas) underneath a tract. Fortunately, at the end of the 80' new technologies (such as 3D seismic mapping) were invented which permitted firms to drastically reduce the risk associated with offshore drilling. General Electric, for example, nowadays offers “intelligent drilling” technology which helps firms to locate oil and gas deposits and to extract them in the most economical way. Forbes and Zampelli (2000) estimated that the introduction of those new technologies explains why the success rate (measured as $\frac{P}{1-P}$, where $P = \Pr(\text{Firm } i \text{ finds oil (and gas) | Firm } i \text{ drills})$) more than doubled between 1986 and 1995.

A bid is a dollar figure that the firm must pay if it wins the tract. Apart from the bid, firms must also pay a royalty fee which—depending on water depth—typically lies between one sixth and one eighth of the value of the extracted oil. Firms submit their bids simultaneously.

Firms bid on a small subset of the tracts offered for sale. For example, between 1998 and 2005 (inclusive) the U.S. government organized 22 auctions. On average 3,145 tracts were offered in each one of them. On average only 305 of them received at least one bid.⁶ Hence, in those auctions the number of tracts offered for sale by far exceeds total demand. As a result of this, few of the tracts offered for sale receive more than one bid. Summed over all those 22 auctions, for example, 6,705 tracts received at least one bid and 5,255 received exactly one bid. Stated differently, conditional on the event that a tract received at least one bid, there is a 78.4% probability that that tract received only one bid.⁷

If a tract happens to possess only one bid, the U.S. government decides whether or not to reject the bid. To do so, it estimates the “fair market value” of the tract. Henceforth, this fair-market-value estimate will be called the (government’s) reservation price. A tract which received only one bid is sold if the bid exceeds the reservation price. The reservation price is computed after all bids were submitted. Hence, ex-ante bidders don’t know what the realization of the reservation price will be. This insight, combined with my earlier finding that few tracts

⁵The U.S. government sometimes auctions tracts that are situated next to already developed ones (those ones are called drainage tracts). Hendricks and Porter (1988) showed that the neighbor firm then possess an informational advantage over the value of those tracts.

⁶Source: own computations based on data taken from <http://www.boemre.gov/econ/econHIST.htm>.

⁷Solo bidding, however, has not always been the norm in OCS auctions. In particular, Hendricks, Porter and Boudreau (1987) documented that $\Pr(\text{tract } i \text{ receives only one bid} | \text{tract } i \text{ receives at least one bid})$ was approximately 32% for wildcat auctions held during the period 1954-1969.

receive more than one bid, indicate that a player's bidding strategy is primarily determined by her desire to "beat" the reservation price rather than to "beat" a hypothetical competing bid. So far, only Hendricks, Porter and Spady (HPS, 1989) analyzed the government's rejection decision on offshore tracts. They focussed on drainage and development tracts that were sold during the period 1959 - 1979.⁸ Wildcat tracts were unfortunately not included in their sample. They found that the rejection decision on drainage tracts was positively correlated with a tract's size, with the price of oil and with the identity of the highest bidder (i.e. the government was more likely to reject a given high bid submitted by a neighbor firm than by a non-neighbor one). The rejection decision was also negatively correlated with the value of the highest bid. The decision, however, was *not* significantly correlated with the amount of oil extracted *nor* with the bidding history of the neighboring tract. As the reservation price on drainage tracts did not depend on the expected quantity of oil (of the neighboring tract) nor on the neighbors' bids, there is no reason to assume that the contrary situation would prevail on wildcat tracts. After firms submitted their bids, but before the first drilling date, the government releases the identity of all bidders along with their bids. The government also releases—after a few years—the reservation price for tracts with a rejected high bid.⁹ The reservation price for tracts with an accepted high bid, however, is not made public.

After winning her tract, a firm is given five years to initiate an exploratory drilling program. If after five years it has not initiated such a program, its lease expires and the tract is returned to the government which may decide to resell it in some future auction. The tracts are usually smaller than the sizes of the deposits. Lin (2009), for instance, documents that the largest petroleum field in the Gulf of Mexico spans 23 tracts. Depending on water depth, 57% to 67% of all deposits spanned more than one tract. Furthermore, even if two adjacent tracts do not share the same deposit, this does not mean that their tract values are uncorrelated. As adjacent tracts possess almost the same geological characteristics, their values should still be significantly correlated. More generally, one would expect the correlation of tract values gradually to decrease with distance.

Drilling an exploratory well can be very costly. According to Forbes and Zampelli (2000) in 1996 the average exploratory well had a depth of 11,203 feet (3,414 meters) and cost 3.3 million USD. This cost, however, dramatically increases with well depth: A 15,000 feet (4,572 meters) exploratory well cost 10 million USD. As tract values are correlated and as drilling is costly, a

⁸The definition of a drainage tract is provided in footnote 5. A development tract is a tract re-offered for sale as its past owner let her lease expire without drilling any well. See below for more details.

⁹Those reservation prices can be downloaded from <http://www.boemre.gov/econ/econHIST.htm>.

firm has an incentive to postpone its exploratory drilling in the hope that a (not-too-distant) neighbor drills first.¹⁰ This plausible strategic behavior is not inconsistent with the available empirical evidence. Hendricks and Porter (1996) documented that the hazard rate of drilling (i.e. the probability to drill at time t given that the tract has not been drilled before) features a U-shaped pattern. A tract is most likely to be drilled at the start or at the end of her lease term. In years 2, 3 and 4, however, the hazard rate is significantly lower. If a firm drills its tract during the final year of her lease, this indicates that it must hold sufficiently optimistic beliefs about her prospects of finding oil. The fact that it postponed its drilling decision indicates that there was a positive option value of waiting. A plausible explanation behind this option value of waiting is that the firm hoped to learn from its (not-too-distant) neighbor's drilling outcomes. Furthermore, Hendricks and Porter (1996) also found that the probability to drill during the second and the third year of the lease term is positively influenced by the number of past successful drilling outcomes on other tracts in the same area-cohort.

3 The Model

Two risk-neutral players are interested in acquiring one of two offshore tracts. The seller offers them in two simultaneously-held sealed-bid first-price auctions. Each of the players bid in one of the two auctions.¹¹

$V_i \in \{0, 1\}$ denotes the value of tract i ($i = 1, 2$). If $V_i = 1$, tract i is said to possess oil (and gas). If $V_i = 0$, tract i does not possess any oil nor gas. It is assumed that $\Pr(V_i = 1) = \nu \in (0, 1) \forall i$. Tract values are correlated. In particular, it is assumed that $\Pr(V_i = 0 | V_{-i} = 0) = \rho^0 \in (1 - \nu, 1]$ and that $\Pr(V_i = 1 | V_{-i} = 1) = \rho^1 \in (\nu, 1]$. If ρ^0 is close to $1 - \nu$ and if ρ^1 is close to ν , tract

¹⁰Hendricks and Kovenock (1989) were the first to analyze incentives to wait in the oil exploration industry. In particular, they showed that if Player i 's neighbor does not drill, this signals that he is not very optimistic about his prospects of finding oil. In its turn, this induces Player i to not drill her lease either. They do not, however, analyze how the interaction between the bidding and the waiting game affects issues of auction design.

¹¹Implicitly, I am making two assumptions here. First, I assume that bidders have unit demand. Second, I assume that there is only one bidder per tract. The first assumption can be defended on the grounds that firms may not want to bid on all the tracts offered for sale (recall that in the period 1998-2005 on average 3,145 tracts were simultaneously offered for sale!) either because of bidding constraints, or limited refining capacity, or because of a bottleneck in the supply of drilling rigs or because of risk-aversion. None of those reasons, however, are explicitly modeled here. Next, because of the information externality, a firm's valuation of a particular tract is nondecreasing in the number of neighboring tracts it wins in the auction. Recall, however, that I study how the information externality at the drilling stage affects some issues of auction design. Introducing supermodular utility functions in the analysis would therefore unnecessarily complicate matters. The second assumption considerably simplifies computations and is consistent with the recent U.S. experience as explained in Section 2.

values are almost uncorrelated. This is most realistic when the distance between both tracts is very large. If $\rho^0 = \rho^1 = 1$, my model boils down to a common value problem. This corresponds to the case where both tracts are neighbors who share a common pool. For intermediate values of ρ^0 and ρ^1 , tract values are imperfectly correlated. In general, one would expect ρ^0 and ρ^1 to decrease with the distance between both tracts.¹²

Player i receives an informative but imperfect signal $s_i \in \{0, 1\}$ concerning the value of her tract. Player two possesses a weakly more informative signal than Player one. Formally, Player one receives the correct signal with probability p_1 , while Player two receives the correct signal with probability $p_2 \geq p_1$, i.e.

$$\begin{aligned} \Pr(s_1 = 1|V_1 = 1) &= \Pr(s_1 = 0|V_1 = 0) = p_1 \in \left(\frac{1}{2}, 1\right), \text{ and} \\ \Pr(s_2 = 1|V_2 = 1) &= \Pr(s_2 = 0|V_2 = 0) = p_2 \in [p_1, 1). \end{aligned}$$

Signals are (conditionally) independent. I denote the common drilling cost by c . $\xi_i(s_i)$ denotes Player i 's posterior probability that $V_i = 1$ conditional on her signal s_i . i.e. $\xi_i(s_i) \equiv \Pr(V_i = 1|s_i)$. Henceforth, a player who received signal zero (one) is called a type-zero (type-one) player. I assume that

ASSUMPTION 1 $\Pr(V_1 = 1|s_1 = s_2 = 0) \leq c \leq \xi_1(1)$.

The first inequality states that if Player one finds out that Player two also possesses signal zero, she faces a non-positive payoff from drilling. Observe that $\Pr(V_1 = 1|s_1 = s_2 = 0) < \xi_1(0)$: If Player one finds out that Player two also possesses signal zero, she becomes less confident in her prospects of finding oil. Hence, I allow the drilling cost c to be sufficiently low (i.e. lower than $\xi_1(0)$) such that even type-zero players may face a positive payoff from drilling after the auction. As Player two possesses a weakly more informative signal, she infers more bad news out of “ $s_2 = 0$ ” than Player one does out of “ $s_1 = 0$ ”. Therefore, $\Pr(V_2 = 1|s_1 = s_2 = 0) \leq \Pr(V_1 = 1|s_1 = s_2 = 0) \leq c$ and Player two also faces a non-positive payoff from drilling if both players possess signal zero. The second inequality states that if Player one possesses signal one, drilling yields a non-negative expected payoff. As Player two possesses a weakly more informative signal, she also faces a non-negative payoff from drilling, i.e. $c \leq \xi_1(1) \leq \xi_2(1)$. Tract values are assumed to be “very” correlated such that

ASSUMPTION 2 $\Pr(V_i = 1|s_i = 1, V_{-i} = 0) < c < \Pr(V_i = 1|s_i = 0, V_{-i} = 1) \forall i$.

¹²Note that my assumptions imply that $(1-\nu)(1-\rho^0) = \nu(1-\rho^1)$. (The latter equality actually directly follows from my assumption that $\Pr(V_i = 1) = \nu \forall i$.) In words, the latter equality implies that the probability that both tracts possess different values is independent of their identities, i.e. $\Pr(V_1 = 0, V_2 = 1) = \Pr(V_1 = 1, V_2 = 0)$.

In words, Assumption 2 ensures that my model possess a meaningful information externality: A negative drilling outcome on Tract $-i$ completely removes Player i 's incentives to explore her tract, even if she is a type-one player. Conversely, a positive drilling outcome on Tract $-i$ induces Player i to explore her tract, even if she is a type-zero player.

The government's reservation price on Tract i is denoted by r_i . Recall from section 2 that the government's rejection decision was only correlated with (i) the tract size, (ii) the winning bid, (iii) the identity of the winning bidder and (iv) the price of oil. The quantity of oil (extracted in the past underneath the neighboring tract) and a neighbor's bid were *not* significant in HPS's regression equation. Hence, there is no reason to assume that r_i is contingent on the bid on Tract $-i$, on V_i or on V_{-i} . Nor is there any reason to assume that r_i is correlated with r_{-i} . Furthermore, in my model both tracts have the same size and both bidders do not own a neighboring tract. Hence, there is no reason to assume that r_1 is drawn from a different distribution than r_2 . On the basis of these findings, I assume that r_1 and r_2 are independently drawn from an identical distribution with support $[0, 1]$. To simplify computations, I assume that r_i is drawn from a uniform distribution. Observe that the upper bound of the distribution of r_i is "very high". If signals are almost uninformative, for example, $\xi_1(1)$ and $\Pr(V_1 = 1 | s_1 = s_2 = 0)$ are both close to the prior ν . It then follows from Assumption 1 that the cost of drilling c is also close to ν . In that case Player one's valuation of her tract is close to zero! More generally if signals are "sufficiently noisy", one would expect a Player's valuation to lie "well below" one. If signals are almost perfect, however, $\xi_1(1)$ is close to one while $\Pr(V_1 = 1 | s_1 = s_2 = 0)$ is close to zero. It then follows from Assumption 1 that the drilling cost c can also be close to zero. In that case Player i 's valuation of her tract is close to one. Setting the maximal value of r_i equal to one therefore ensures that—for all values of my exogenous parameters—Player i knows that by shading her bid she strictly reduces her chances of winning her tract. Furthermore, the high upper bound of r_i also ensures that, in any undominated strategy, Player i 's bid will be rejected with positive probability. As explained in section 5, this implies that there are no off-the-equilibrium information sets when the auctioneer does not disclose bids. Finally, players discount the future at a rate $\delta \in (0, 1)$.

4 The auction with bid disclosure

In this section, I consider the following sequencing of events:

- 1 Nature draws the state of the world, the reservation prices and players receive their signals.
- 0 Player one bids on tract one, Player two bids on tract two.

$\frac{1}{2}$ The auctioneer publicly announces all bids and whether they were higher or lower than the reservation price.¹³

1 If Player i wins her tract, she¹⁴ decides whether to drill or wait.

2 In case Player $-i$ drilled, Player i observes the value of Tract $-i$. If Player i waited, she decides whether to drill or not.

3 Players receive their payoffs and the game ends.

When solving my game, I rely on two equilibrium selection criteria. First, I require a candidate equilibrium to belong to the class of the perfect Bayesian equilibria. Second, I restrict attention to the class of the *non-coordinating* strategies. To understand this restriction suppose both players win their tracts. Suppose also that the following three conditions are satisfied: (i) at the start of the waiting game both players hold degenerate beliefs about the other player's type, (ii) both players face a positive payoff from drilling at time one, and (iii) both players face a positive option value of waiting. Condition (i) is satisfied when both players use invertible bidding strategies. In that case, Player i perfectly infers his type out of his bid. Condition (iii) means that if Player i expects Player $-i$ to drill (at time one), she prefers to wait. The continuation game then possesses three equilibria. In the first one, Player one drills (at time one) while Player two waits. In the second one, Player two drills while Player one waits. In the third one, Player i drills with probability λ_i to make Player $-i$ indifferent between drilling and waiting. A strategy is said to be non-coordinating if players focus on the mixed-strategy Nash equilibrium when both players win their tracts and when conditions (i), (ii), and (iii) are satisfied.

The reader might wonder why both players cannot reach an efficient post-auction drilling agreement. Both players could, for instance, exchange their seismic information after the auction and then bargain over a course of action. This "solution", however, still leaves scope for disagreement. First, it is not clear whether players have the correct incentives to truthfully reveal how they interpret the seismic data. Player i , for example, may be worried that if she were to reveal her interpretation to Player $-i$, this will increase Player $-i$'s expertise knowledge in interpreting seismic data and induce him to bid more aggressively in future auctions. Second, players need to decide on when to drill. (Firms may possess different private information about future oil

¹³As mentioned in section 2, the reservation price for tracts with a rejected high bid can be downloaded (after some time) from the Bureau of Ocean Energy Management, Regulation and Enforcement website (which is the division in the Department of the Interior responsible for organizing those auctions). The reservation price for tracts with an accepted high bid, however, is not made public.

¹⁴Henceforth, I assume Player i to be female, and Player $-i$ to be male.

and gas prices.) Third, firms need to agree on where to drill. (The value of the informational externality obviously depends on where players drill and, if players do not truthfully reveal how they interpret seismic data, disagreement about this value should still persist.) Fourth, firms need to agree on how to share costs and benefits (if any). Finally, Firm A may be reluctant to come to a drilling agreement with Firm B unless other firms also pay part of the drilling cost. Hence, even if firms were to disclose their seismic data, post-auction bargaining is unlikely to yield efficient drilling outcomes. While in theory one could come up with clever mechanisms to reduce inefficiencies in the post-auction game, the drilling data (see Hendricks and Porter (1996) and section 2) is more consistent with the view that players are not able to avoid playing a war of attrition in the continuation game.¹⁵

To understand equilibrium strategies in this game it is illuminating to analyze first the hypothetical case in which signals instead of bids are disclosed at time $\frac{1}{2}$. In this hypothetical case, I first analyze the continuation game in which both players win their tracts. Let $W_i(s_i, s_{-i})$ denote Player i 's undiscounted payoff from waiting given both players' signals and given that both players win their tracts. Let λ_i denote the probability with which Player i drills at time one. To understand how $W_i(s_i, s_{-i})$ is computed, observe that three time-one outcomes may occur. Either Player $-i$ drills at time one but does not find oil (i.e. $V_{-i} = 0$), or Player $-i$ finds oil (i.e. $V_{-i} = 1$) or Player $-i$ does not drill. Recall from Assumption 2 that Player i does not drill if Player $-i$'s drilling effort is unsuccessful. If Player $-i$ finds oil, however, Player i drills at time two. Player i 's payoff from waiting can therefore be written as:

$$W_i(s_i, s_{-i}) = \Pr(V_{-i} = 1 | s_i, s_{-i}) \lambda_{-i} \left\{ \Pr(V_i = 1 | s_i, V_{-i} = 1) - c \right\} + (1 - \lambda_{-i}) \max \left\{ 0, \Pr(V_i = 1 | s_i, s_{-i}) - c \right\}. \quad (1)$$

Observe that the terms between curly brackets of 1 are independent of Player $-i$'s drilling probability. Hence, $W_i(s_i, s_{-i})$ is increasing in λ_{-i} whenever $\Pr(V_i = 1 | s_i, s_{-i}) < c$. Suppose now that $\Pr(V_i = 1 | s_i, s_{-i}) > c$. Player i 's payoff from waiting can then be rewritten as

$$W_i(s_i, s_{-i}) = \Pr(V_i = 1 | s_i, s_{-i}) - c + \Pr(V_{-i} = 0 | s_i, s_{-i}) \lambda_{-i} \left(c - \Pr(V_i = 1 | s_i, V_{-i} = 0) \right). \quad (2)$$

¹⁵Pre-auction cooperation is also problematic: Prior to bidding, firms must invest in geophysical surveys and in a team of experts to interpret the seismic data. Understandably, firms are reluctant to share this private information with firms which are uninterested in drilling (and bidding) in their geographical area. (Hendricks and Porter (1992) provide empirical evidence which is consistent with this explanation.) If firms were to know the identity of their (not-too-distant) neighbor, they might decide to create a joint venture prior to bidding. Unfortunately, the identity of a firm's (not-too-distant) neighbor only becomes available after the auction. Furthermore, in 1975 the U.S. Congress passed a law banning the eight largest firms to form bidding consortia on the grounds that they reduced competition.

The above equation is intuitive: If $\Pr(V_i = 1|s_i, s_{-i}) > c$, Player i always drills at time two unless Player $-i$ drills at time one and finds no oil. Note that this event happens with probability $\Pr(V_{-i} = 0|s_i, s_{-i})\lambda_{-i}$ and Player i then saves $c - \Pr(V_i = 1|s_i, V_{-i} = 0)$. Under Assumption 2 the term between brackets is positive. As signals are imperfect (i.e. as $p_1 \leq p_2 < 1$), $\Pr(V_{-i} = 0|s_i, s_{-i}) > 0$. Hence, $W_i(s_i, s_{-i})$ is also increasing in λ_{-i} in this case. I conclude:

Lemma 1 *Player i 's payoff from waiting $W_i(s_i, s_{-i})$ is increasing in Player $-i$'s drilling probability λ_{-i} .*

Lemma 1 is intuitive: the higher λ_{-i} , the greater the probability that Player $-i$ drills at time one and, thus, the greater the probability that Player i learns V_{-i} . As tract values are correlated, this increases Player i 's payoff from waiting.

Let

$$\bar{W}_i(s_i, s_{-i}) \equiv \Pr(V_{-i} = 1|s_i, s_{-i}) \left[\Pr(V_i = 1|s_i, V_{-i} = 1) - c \right]. \quad (3)$$

$\bar{W}_i(s_i, s_{-i})$ represents Player i 's maximal undiscounted payoff from waiting, i.e. the value taken by $W_i(s_i, s_{-i})$ when Player $-i$ drills with probability one at time one. Player i is said to face a positive option value of waiting in the auction with bid disclosure if she prefers to wait when she anticipates the other player to drill with probability one at time one, i.e. if $\Pr(V_i = 1|s_i, s_{-i}) - c \leq \delta \bar{W}_i(s_i, s_{-i})$.

Lemma 2 *If $\delta \in [\tilde{\delta}, 1]$, Player i always (i.e. $\forall(s_i, s_{-i})$) faces a positive option value of waiting in the auction with bid disclosure.*

If the discount factor δ is very low, Player i 's opportunity cost of waiting is very high. She may then prefer to drill at time one even if she anticipates the other player to also drill at time one. If the discount factor δ is sufficiently high, however, she will prefer to wait. In what follows, I will assume that $\delta \geq \tilde{\delta}$. As documented by Hendricks and Porter (1996), the relevant time period in my model is about three months. (This is the time that elapses between building an exploratory well and finding out whether sufficient quantities of oil are present to warrant production.) Suppose that the (annual) opportunity cost of capital is 10%. This implies a three-monthly interest rate of about 2.41%. This also implies a three-monthly discount factor of 0.976. Hence, in any reasonable calibration of my model, one should work with a very high discount factor. In that sense, it is reasonable to assume that players—for all signal realizations—prefer to wait if they expect the other player to drill.

Suppose $\Pr(V_i = 1|s_i, s_{-i}) \geq c$ and that Player $-i$ drills with probability zero. In that case waiting entails a discounting cost but no information externality. Hence, in that case Player i prefers to drill at time one. Recall from Lemma 1 that $W_i(\cdot)$ is increasing in λ_{-i} . The lemma below then straightforwardly follows from the intermediate value theorem.

Lemma 3 *Suppose $\delta \in [\tilde{\delta}, 1]$ and that $\Pr(V_i = 1|s_i, s_{-i}) \geq c$. There exists then a unique $\lambda_{-i}^* \in [0, 1]$ such that Player i is indifferent between drilling and waiting.*

Suppose $\delta = 1$ and that $\Pr(V_i = 1|s_i, s_{-i}) \geq c$. It then follows from 1 that Player i is indifferent between drilling and waiting if

$$\begin{aligned} \Pr(V_i = 1|s_i, s_{-i}) - c &= \Pr(V_{-i} = 1|s_i, s_{-i})\lambda_{-i} \left[\Pr(V_i = 1|s_i, V_{-i} = 1) - c \right] \\ &+ (1 - \lambda_{-i}) \left[\Pr(V_i = 1|s_i, s_{-i}) - c \right]. \end{aligned} \quad (4)$$

Observe that the equality is trivially satisfied if $\lambda_{-i} = 0$. It also follows from Lemma 1 that, whenever $\lambda_{-i} > 0$, Player i prefers to wait. I conclude:

COROLLARY 1 *Suppose $\delta = 1$ and that $\Pr(V_i = 1|s_i, s_{-i}) \geq c$. Player i is then indifferent between drilling and waiting only if $\lambda_{-i} = 0$.*

The corollary is very intuitive: If there is no discounting cost, Player i is only indifferent between drilling and waiting if there is no gain from waiting either. As explained above, this corresponds to the case in which Player $-i$ drills with probability zero.

As stated previously, I restrict attention to the class of the non-coordinating strategies. This restriction, however, does *not* imply that players never successfully coordinate their drilling plans. Recall that—as tract values are imperfectly correlated and as Player two possesses a more informative signal—both players need not possess the same time-one posterior. Suppose now that $\Pr(V_i = 1|s_i = 0, s_{-i} = 1) < c < \Pr(V_{-i} = 1|s_i = 0, s_{-i} = 1)$. In this case, Player i faces a negative payoff from drilling, and, thus, never drills at time one. As Player $-i$ faces a positive payoff from drilling and as he knows that Player i refrains from drilling, it is a best reply for him to drill at time one. In this case, both players thus succeed to coordinate their drilling activities.¹⁶ In the light of this comment, let

$$\mathcal{I}_i(s_i, s_{-i}) = \begin{cases} 1 & \text{if } \Pr(V_i = 1|s_i, s_{-i}) < c < \Pr(V_{-i} = 1|s_i, s_{-i}), \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

Hence, if the indicator function $\mathcal{I}_i(s_i, s_{-i})$ takes value 1, Player i gets $\delta \overline{W}_i(s_i, s_{-i})$. $\mathcal{I}_i(s_i, s_{-i})$ takes value 0 if the drilling cost is “very high”. In that case both players face a negative payoff

¹⁶Recall from Section 2 that the hazard rate of drilling features a U-shaped pattern. Hendricks and Porter (1996) provide two different explanations for the initial high hazard rate. First, they argue that some firms may be very optimistic about their drilling prospects. For them, the opportunity cost of waiting is too high and they prefer to drill their tracts at the start of their lease term. Second, they argue that asymmetric lease holdings induces firms to coordinate their drilling activities. In my model—depending on the drilling cost c and on signal realizations—both players may coordinate their drilling activities, despite both of them possessing the same number of tracts, and despite both of them possessing a positive option value of waiting.

from drilling and, therefore, get zero. $\mathcal{I}_i(s_i, s_{-i})$ also takes value 0 if the drilling cost is very low. In that case both players face a positive payoff from drilling and, as stressed in Lemma 3, are indifferent between drilling and waiting. In that case Player i gets $\Pr(V_i = 1|s_i, s_{-i}) - c$. Finally, $\mathcal{I}_i(s_i, s_{-i})$ also takes value zero if Player i is more confident about her prospects of finding oil than Player $-i$. Hence, conditional on both players winning their tracts and conditional on their signals, Player i gets

$$\mathcal{I}_i(s_i, s_{-i})\delta\bar{W}_i(s_i, s_{-i}) + \left(1 - \mathcal{I}_i(s_i, s_{-i})\right) \max\left\{0, \Pr(V_i = 1|s_i, s_{-i}) - c\right\} \quad (6)$$

Consider now the continuation game in which Player i wins her tract but not Player $-i$. As signals are disclosed, Player i takes an optimal time-one drilling decision and gets $\max\{0, \Pr(V_i = 1|s_i, s_{-i}) - c\}$. Recall that if Player i does not win her tract, she gets zero.

Let $u_i^<(s_i)$ denote Player i 's *interim* payoff.¹⁷ It represents her expected utility conditional on her signal, conditional on the event that she wins her tract but before she finds out whether Player $-i$ wins his tract or not. Suppose $s_i = 1$. Recall from Assumption 1 that Player i faces a positive payoff from drilling, i.e. $c < \xi_i(1)$.¹⁸ As tract values are correlated, $\xi_i(1) \leq \Pr(V_i = 1|s_i = s_{-i} = 1)$. Hence, if Player $-i$ is also a type-one player, both players face a positive payoff from drilling. It then follows from 5 that $\mathcal{I}_i(1, 1) = 0$. Suppose now that Player $-i$ is a type-zero player. In that case, it is easy to check that Player i 's posterior is higher than the one of Player $-i$. Formally,

$$\Pr(V_{-i} = 1|s_i = 1, s_{-i} = 0) \leq \Pr(V_i = 1|s_i = 1, s_{-i} = 0).$$

If tract values are perfectly correlated, both players possess two contradictory signals about the realization of the same random variable. Unsurprisingly, they then possess the same time-one posterior. If tract values are imperfectly correlated, however, the above inequality becomes strict: As s_i is more informative about V_i than s_{-i} , the type-one player is more confident about the value of her tract than the type-zero player. It then follows from 5 that $\mathcal{I}_i(1, 0)$ also equals zero in this case. Using 6, I thus conclude that, if Player $-i$ wins his tract, Player i gets $\max\{0, \Pr(V_i = 1|s_i, s_{-i}) - c\}$. Recall from my previous paragraph that, if Player $-i$'s bid is rejected, she also gets $\max\{0, \Pr(V_i = 1|s_i, s_{-i}) - c\}$. Hence, Player i gets the same payoff as the one she would get if she were to take a once-and-for-all decision at time one, i.e.

$$u_i^<(1) = \sum_{s_{-i}} \Pr(s_{-i}|s_i = 1) \max\left\{0, \Pr(V_i = 1|s_i, s_{-i}) - c\right\}. \quad (7)$$

¹⁷The “<” superscript refers to the event “ $r_i < b_i$ ”, i.e. Player i wins her tract.

¹⁸Henceforth, I will sometimes refer to $\xi_i(s_i)$ ($\equiv \Pr(V_i = 1|s_i)$) as Player i 's *time-zero* posterior.

Suppose now that Player i is a type-zero player. It then follows from my previous paragraphs that

$$\begin{aligned}
u_i^{\leq}(0) &= \sum_{s_{-i}} \Pr(s_{-i}, r_{-i} < b_{-i} | s_i = 0) \left[\mathcal{I}_i(0, s_{-i}) \delta \bar{W}_i(0, s_{-i}) + \left(1 - \mathcal{I}_i(0, s_{-i}) \right) \max \left\{ 0, \Pr(V_i = 1 | s_i = 0, s_{-i}) - c \right\} \right] \\
&+ \sum_{s_{-i}} \Pr(s_{-i}, r_{-i} > b_{-i} | s_i = 0) \max \left\{ 0, \Pr(V_i = 1 | s_i = 0, s_{-i}) - c \right\} \tag{8}
\end{aligned}$$

We now know enough to analyze Player i 's optimal bidding strategy. Recall that $r_i \sim U[0, 1]$. Hence, $\Pr(r_i < b_i) = b_i$. Recall also that the U.S. government uses a first-price auction. Player i thus chooses b_i to maximize $b_i(u_i^{\leq}(s_i) - b_i)$. Henceforth, I will refer to this as Player i 's time-zero maximization problem. Recall that I work under the assumption that *signals* instead of *bids* are disclosed at time one. Player i 's interim payoff $u_i^{\leq}(s_i)$ is therefore independent of b_i . Player i 's time-zero maximization problem is thus very simple: If she increases her bid, this increases her chances of winning her tract. This benefit, however, comes at the cost of having to put more money on the table. The solution to this problem is given by $b_i^* = \frac{1}{2}u_i^{\leq}(s_i)$. Henceforth, I will refer to $u_i^0(s_i, b_i) (\equiv b_i(u_i^{\leq}(s_i) - b_i))$ as Player i 's *time-zero utility*: It represents her expected payoff evaluated at time zero given that she bids b_i . Let $b_i^*(0)$ denote Player i 's optimal bid if she possesses signal zero. Similarly, $b_i^*(1)$ denotes the optimal bid of a type-one player.

Suppose now that bids instead of signals are disclosed. (I still assume that players face a positive option value of waiting, i.e. $\delta \geq \tilde{\delta}$.) A type-one player has the possibility to drill at time one, independent of Player $-i$'s bid. From an ex ante point of view, this strategy yields an interim payoff equal to $\xi_i(1) - c$. This interim payoff represents a lower bound: If Player i were to infer information out of his bid and to take an optimal time-one decision, she would obtain an even higher interim payoff. Recall that Player i chooses her bid b_i to maximize $b_i(u_i^{\leq}(1) - b_i)$. As her interim payoff is bounded below by $\xi_i(1) - c$, I conclude that, in any undominated strategy, type-one players bid a strictly positive amount, i.e. $b_i(1) \geq \underline{b} > 0$. If signals are very precise, type-one players are very confident about their prospects of finding oil. The lower bound \underline{b} will then be “high”. Observe also that the interim payoff of a type-zero player is bounded above by

$$u_i^{\leq}(0) < \delta \sum_{s_{-i}} \Pr(s_{-i} | s_i = 0) \bar{W}_i(0, s_{-i}) \equiv \bar{u}_i^{\leq}(0).$$

The upper bound is computed under the assumption that Player $-i$ —independent of his type—bids one and drills at time one with probability one. Hence, in any undominated strategy, type zero players never bid above $\bar{u}_i^{\leq}(0)$. If signals are very precise, type-zero players believe that the other player will not find oil anyway. The upper bound will then be “low”. Hence, for sufficiently precise signals, in any undominated strategy, type-zero players bid strictly less than type-one players. In this case in any equilibrium players must thus use invertible bidding strategies. It then

follows from my previous paragraphs that, within the class of the non-coordinating strategies, there only exists one equilibrium outcome. I conclude:

PROPOSITION 1 *Suppose $\delta \in [\tilde{\delta}, 1]$. If, additionally, $p_1 \in [\tilde{p}_1, 1)$, there exists a unique equilibrium outcome in which Player i bids $b_i^*(0)$ if $s_i = 0$ and $b_i^*(1)$ if $s_i = 1$. Furthermore, $b_i^*(0) < b_i^*(1)$.*

5 The auction without bid disclosure

In this section I consider the same game as above, except that at time $\frac{1}{2}$ the auctioneer only announces whether or not Player i 's bid is rejected. Bids are kept secret. Recall that $u_i^<(s_i)$ denotes Player i 's interim payoff: In this section it represents Player i 's expected utility given her signal, given that she wins her tract and prior to finding out whether Player $-i$ wins his tract or not. $\mu_i^+(s_i)$ denotes Player i 's time-one posterior conditional on her signal and on the event that Player $-i$ wins his tract, i.e. $\mu_i^+(s_i) \equiv \Pr(V_i = 1 | s_i, r_{-i} < b_{-i})$. Similarly, $\mu_i^-(s_i)$ denotes Player i 's time-one posterior conditional on her signal and on the event that Player $-i$'s bid is rejected by the U.S. government, i.e. $\mu_i^-(s_i) \equiv \Pr(V_i = 1 | s_i, r_{-i} > b_{-i})$. In general, I will refer to Player i 's bid with b_i . I will, however, sometimes refer to her bid with $b_i(s_i)$ to stress that I only consider her bid when her signal s_i takes a particular value.

As argued in section 3, Player i 's valuation is bounded above by $1 - c$. In any undominated strategy, type-one players therefore bid less than one. As the upper bound of the distribution of the reservation price is equal to one, this implies that, in any undominated strategy, Player i 's bid is rejected with positive probability. Furthermore, a type-one player has the possibility to drill at time one, independent of whether Player $-i$ wins his tract or not. Player i 's interim payoff is therefore bounded below by $\xi_i(1) - c > 0$. Hence, in any undominated strategy type-one players bid strictly positive amounts. As the lower bound of the distribution of the reservation price is zero, this implies that, in any undominated strategy, Player i 's bid is accepted with positive probability. Hence, in this game players can always use Bayes's rule. Any Bayesian equilibrium is therefore consistent and sequentially rational.

In what follows, I restrict attention to the class of the *non-coordinating* strategies. To understand this restriction, suppose that the following three conditions are satisfied: (ci) both players win their tracts, (cii) both players face a positive payoff from drilling provided they possess signal one, i.e. $\mu_i^+(1) > c$, and (ciii) both players prefer to wait if they anticipate the other player to drill with probability one if he possesses signal one. The continuation game is then characterized by multiple equilibria. In one equilibrium, Player one drills with probability one (provided $s_1 = 1$) while Player two waits. In another equilibrium, Player two drills (provided $s_2 = 1$) while Player one waits. A strategy is said to be non-coordinating if Players do *not* focus on any of

those two equilibria whenever conditions (ci), (cii) and (ciii) are satisfied. Henceforth, I will refer to a Bayesian, non-coordinating equilibrium simply as a *non-coordinating* equilibrium.

The lemma below states that, without loss of generality, I can restrict attention to pure strategies at the bidding stage. In the lemma $b_i^{BR}(s_i)$ denotes Player i 's optimal bid given her signal s_i and the other player's strategy. (*BR* stands for "best response".)

Lemma 4 *Suppose Player $-i$ randomizes his bid according to an arbitrary c.d.f. denoted by $\beta_{-i}(s_{-i})$. Suppose also that Player $-i$, conditional on both players winning their tracts, drills (at time one) with probability $\lambda_{-i}(s_{-i}, b_{-i})$. It is then optimal for Player i to bid $b_i^{BR}(0)$ (with probability one) if $s_i = 0$ and to bid $b_i^{BR}(1)$ (with probability one) if $s_i = 1$. Furthermore, $b_i^{BR}(0) < b_i^{BR}(1)$.*

To gain some intuition, suppose Player $-i$ follows the strategy described in the lemma. Player i chooses her bid to maximize $\max_{b_i} b_i(u_i^<(s_i) - b_i)$. To compute Player i 's interim payoff, I need to distinguish between two different cases: Either Player $-i$ wins his tract (i.e. $r_{-i} < b_{-i}$) or his bid is rejected by the U.S. government (i.e. $r_{-i} > b_{-i}$). In the former case, Player i either drills immediately or she waits in the hope to learn from Player $-i$'s drilling activity. In the latter case she drills only if it is profitable to do so. Hence:

$$\begin{aligned} u_i^<(s_i) &= \Pr(r_{-i} < b_{-i} | s_i) \max \left\{ \mu_i^+(s_i) - c, \delta W_i(s_i) \right\} \\ &+ \Pr(r_{-i} > b_{-i} | s_i) \max \left\{ 0, \mu_i^-(s_i) - c \right\}, \end{aligned} \quad (9)$$

where $W_i(s_i)$ represents Player i 's payoff from waiting (net of discounting costs), conditional on her signal and on the event that both players win their tracts. To understand how $W_i(s_i)$ is computed, let $a_{-i} \in \{drill, wait\}$ denote Player $-i$'s time-one action. Observe that with probability $\Pr(V_{-i} = 1, a_{-i} = drill | s_i, r_{-i} < b_{-i})$, Player $-i$ drills and finds oil underneath his tract. Recall from Assumption 2 that Player i faces a positive payoff from drilling if Player $-i$ finds oil. Hence, in this case Player i drills at time two and gets $\Pr(V_i = 1 | s_i, V_{-i} = 1) - c$. Recall also from Assumption 2 that Player i does not drill in case Player $-i$ does not find oil. If Player $-i$ waits, Player i drills only if it is profitable to do so. Hence,

$$\begin{aligned} W_i(s_i) &= \Pr(V_{-i} = 1, a_{-i} = drill | s_i, r_{-i} < b_{-i}) \left[\Pr(V_i = 1 | s_i, V_{-i} = 1) - c \right] \\ &+ \Pr(a_{-i} = wait | s_i, r_{-i} < b_{-i}) \max \left\{ 0, \Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = wait) - c \right\}. \end{aligned} \quad (10)$$

At the risk of stating the obvious, observe that Player $-i$'s "complicated" strategy does not impede Player i to "easily" compute all the above probabilities. For example, $\Pr(a_{-i} = drill | s_{-i} = 1) = \int_0^1 \lambda_{-i}(1, b_{-i}) d\beta_{-i}(1)$ and $\Pr(r_{-i} < b_{-i} | s_i) = \sum_{s_{-i}} \Pr(s_{-i} | s_i) \int_0^1 b_{-i} d\beta_{-i}(s_{-i}) = E(b_{-i} | s_i)$,

where E denotes the expectation operator. Furthermore, as r_1 is independently drawn from r_2 ,¹⁹ all the probabilities computed in Equation 9 are independent of Player i 's bid b_i . It then follows from Player i 's time-zero maximization problem that she should bid $\frac{1}{2}u_i^<(s_i)$ with probability one, i.e. Player i 's best response to any arbitrary strategy of her rival is to bid according to a deterministic bidding function. In the Appendix, I show that $u_i^<(0) < u_i^<(1)$, thereby establishing the last claim of the lemma. The intuition is straightforward: As a type-one player is more confident about the value of her tract, she values “winning” more than a type-zero player.

It follows from Lemma 4 that $\mu_i^-(s_i) < \xi_i(s_i) < \mu_i^+(s_i)$. This is intuitive: As type-zero players bid less aggressively than type-one players, observing that Player $-i$'s bid is rejected indicates that he is more likely to possess signal zero. As tract values are correlated, this decreases Player i 's posterior probability of finding oil. Similarly, observing Player $-i$ winning his tract indicates that he is more likely to possess signal one. This leads to an upward revision of Player i 's posterior probability of finding oil.

Let $\lambda_i(s_i)$ denote the probability with which Player i drills at time one, conditional on her signal s_i and conditional on the event that both players win their tracts. The lemma below establishes an intuitive property of Player i 's payoff from waiting.

Lemma 5 *Suppose type-zero and type-one players bid positive amounts.²⁰ $W_i(s_i)$ is then increasing in $(\lambda_{-i}(0), \lambda_{-i}(1))$.*

The intuition behind my last lemma is identical to the one of Lemma 1: The higher $(\lambda_{-i}(0), \lambda_{-i}(1))$, the higher the probability that Player i learns the value of Tract $-i$. As tract values are correlated, this increases Player i 's payoff from waiting. The result rests on the assumption that players bid positive amounts. This is also intuitive: If, for example, $b_{-i}(0) = 0$, $W_i(s_i)$ is independent of $\lambda_{-i}(0)$ as Player i then believes that Player $-i$ never wins his tract if $s_{-i} = 0$.

Let

$$\Delta_i(s_i) \equiv \mu_i^+(s_i) - c - \delta W_i(s_i). \quad (11)$$

$\Delta_i(s_i)$ measures Player i 's incentives to drill at time one as opposed to waiting when both players win their tracts. If $\Delta_i(s_i) > 0$, Player i prefers to drill.

¹⁹If, for example, r_1 were equal to r_2 , then $\mu_i^+(s_i)$ would depend on b_i . To see this, suppose both players bid b_0 if they are type-zero players and $b_1 (> b_0)$ if they are type-one players. Then $\mu_i^+(s_i) = \xi_i(s_i)$ if $b_i = b_0$ while $\mu_i^+(s_i) > \xi_i(s_i)$ if $b_i = b_1$. In the former case Player i knows that Player $-i$ wins his tract because the common reservation price is low. In the latter case, she believes that perhaps Player $-i$ wins his tract because he bid high. Recall, however, that my independence assumption is consistent with HPS's empirical findings (see Section 2).

²⁰Recall that, in any undominated strategy, type-one players bid positive amounts.

Lemma 6 *A type-zero player has more incentives to wait than a type-one player, i.e. $\Delta_i(0) < \Delta_i(1)$.*

Lemma 6 states a common property of waiting games studied in the literature.²¹ To gain some intuition, observe that Player i 's payoff from waiting can be rewritten as:²²

$$\begin{aligned} W_i(s_i) &= \mu_i^+(s_i) - c + \Pr(V_{-i}=0, a_{-i}=drill | s_i, r_{-i} < b_{-i}) \left[c - \Pr(V_i=1 | s_i, V_{-i}=0) \right] \\ &+ \Pr(a_{-i}=wait | s_i, r_{-i} < b_{-i}) \max \left\{ 0, c - \Pr(V_i=1 | s_i, r_{-i} < b_{-i}, a_{-i}=wait) \right\}. \end{aligned} \quad (12)$$

Recall from Assumption 2 that the term between square brackets is positive. The equation above teaches us that Player i has two good reasons to wait. First, with probability $\Pr(V_{-i} = 0, a_{-i} = drill | s_i, r_{-i} < b_{-i})$ Player $-i$ drills and finds no oil. Player i then refrains from drilling and avoids an expected loss equal to $c - \Pr(V_i = 1 | s_i, V_{-i} = 0)$. Second, with probability $\Pr(a_{-i} = wait | s_i, r_{-i} < b_{-i})$ Player $-i$ waits. If type-zero players are more likely to wait, this is bad news and leads to a downward revision of Player i 's posterior probability. In case $c - \Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = wait)$ is positive, Player i then refrains from drilling and avoids this expected loss. Henceforth, I will refer to the last two terms of 12 as Player i 's *informational benefit* of waiting. It follows from Lemma 5 that her informational benefit of waiting is increasing in Player $-i$'s drilling probability. Equation 12 allows me to rewrite $\Delta_i(s_i)$ as:

$$\begin{aligned} \Delta_i(s_i) &= \underbrace{(1-\delta) \left(\mu_i^+(s_i) - c \right)}_{\text{Opportunity cost of waiting}} - \delta \Pr(V_{-i}=0, a_{-i}=drill | s_i, r_{-i} < b_{-i}) \left[c - \Pr(V_i=1 | s_i, V_{-i}=0) \right] \\ &- \delta \Pr(a_{-i}=wait | s_i, r_{-i} < b_{-i}) \max \left\{ 0, c - \Pr(V_i=1 | s_i, r_{-i} < b_{-i}, a_{-i}=wait) \right\}. \end{aligned} \quad (13)$$

Player i is thus indifferent between drilling and waiting if her opportunity cost of waiting is equal to her discounted informational benefit of waiting.

The higher Player i 's time-one posterior $\mu_i^+(s_i)$, the higher her opportunity cost of waiting. Suppose $(\lambda_{-i}(0), \lambda_{-i}(1))$ is computed to balance Player i 's opportunity cost of waiting with her informational benefit of waiting (provided that $s_i = 1$). Recall from Assumption 1 and from Lemma 4 that type-one players face a positive payoff from drilling after observing the other player winning his tract, i.e. $c < \xi_i(1) < \mu_i^+(1)$. As $\mu_i^+(1)$ is “high”, it follows from Lemma 5 that Player $-i$ must drill with a “relatively high” probability. As $\mu_i^+(0) < \mu_i^+(1)$, a type-zero player faces a lower opportunity cost of waiting. As Player $-i$ drills with a “relatively high” probability, type-zero players prefers to wait. This explains the intuition behind Lemma 6.

²¹See, for example, Hendricks and Kovenock (1989) and Chamley (2004b, Lemma 6.1, p. 124).

²²See proof of Lemma 6 for more details.

In the light of my last lemma, I first focus on a non-coordinating equilibrium in which type-zero players wait while type-one players are indifferent between drilling and waiting. In the Appendix (see proof of Proposition 3) I prove that, if the discount factor δ is sufficiently high, there does not exist a non-coordinating equilibrium in which type-zero players drill with positive probability.

Suppose $\delta = 1$ and that $s_i = 1$. It then follows from 13 that Player i is indifferent between drilling and waiting if Player $-i$ drills with probability zero. This is easy to check: If $\delta = 1$, Player i 's opportunity cost of waiting equals zero. If Player $-i$ drills with probability zero, the first term of her informational benefit of waiting also equals zero. The second term is equal to zero as well as

$$\Pr(V_i = 1 | s_i = 1, r_{-i} < b_{-i}, a_{-i} = \text{wait}; \lambda_{-i}(1) = 0) = \Pr(V_i = 1 | s_i = 1, r_{-i} < b_{-i}) \equiv \mu_i^+(1) > c.$$

The equality is intuitive: If Player $-i$ drills with probability zero, Player i does not infer any bad news out of his waiting decision. Recall that Assumption 1 states that the investment cost c is less than the time-zero posterior $\xi_i(1)$. Furthermore, in Lemma 4 I have proven that type-zero players bid less aggressively than type-one players. Hence, $\xi_i(1) < \mu_i^+(1)$, which explains the above inequality. I conclude:

COROLLARY 2 *Suppose that $\delta = 1$, that $s_i = 1$, and that both players win their tracts. Suppose type-zero players wait. Player i is then indifferent between drilling and waiting only if $\lambda_{-i}(1) = 0$.*

Suppose Player one possesses signal one and that—in case both players win their tracts—she is indifferent between drilling and waiting. I can rewrite her interim payoff then as:

$$u_1^<(1) = \Pr(r_2 < b_2 | s_1 = 1) \left(\mu_1^+(1) - c \right) + \Pr(r_2 > b_2 | s_1 = 1) \max \left\{ 0, \mu_1^-(1) - c \right\}.$$

Recall from above that the term between round brackets is positive. Recall also that players shade their bids in first-price auctions. Player two's bid can therefore be rejected for two different reasons: Either he is unlucky and his reservation price r_2 is just simply “too high” (i.e. higher than $b_2(1)$), or r_2 is not “too high” but he just simply bids “too low” (this corresponds to the case in which $b_2(0) < r_2 < b_2(1)$). In the former case, Player one should not infer any bad information out of the event that $r_2 > b_2$ and should still possess a positive payoff from drilling at time one. In the latter case, Player one should revise her posterior probability of finding oil downwards. If signals are very precise, however, Player one primarily attributes the event “Player two's bid is rejected” to the former explanation. This is easiest to understand in the limit case when signals are perfect. Player one then knows that there is oil underneath her tract—independent of whether Player two's bid is accepted or not. Hence, there exists a $\hat{p}_1 < 1$ such that if the

precision of Player one's signal p_1 lies above \hat{p}_1 , $\mu_1^-(1)$ is greater than c . The same remark applies when the drilling cost c is “not high”. To see this, observe that Player i 's posterior $\mu_i^-(1)$ is a weighted average between her posterior when the other player also possesses signal one and her posterior when the other player possesses signal zero. Formally,

$$\begin{aligned}\mu_i^-(1) &\equiv \Pr(V_i = 1 | s_i = 1, r_{-i} > b_{-i}) \\ &= \Pr(s_{-i} = 1 | s_i = 1, r_{-i} > b_{-i}) \Pr(V_i = 1 | s_i = s_{-i} = 1) \\ &+ \Pr(s_{-i} = 0 | s_i = 1, r_{-i} > b_{-i}) \Pr(V_i = 1 | s_i = 1, s_{-i} = 0).\end{aligned}\tag{14}$$

The equality above implies that if the drilling cost c is not high—i.e. if c is less than $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$ —then Player i also faces a positive payoff from drilling even if the other player does not win his tract. Hence, if Player one's signal is sufficiently precise or if the drilling cost c is sufficiently low, she gets a payoff equal to the one she would get if she were to always—i.e. independent of whether or not the other player wins his tract—drill at time one. This implies that $u_1^<(1) = \xi_1(1) - c$. Hence, at time zero she chooses b_1 to maximize $b_1(\xi_1(1) - c - b_1)$ which yields as optimal solution $b_1^*(1) = \frac{1}{2}(\xi_1(1) - c)$. The same reasoning also applies to Player two. As she possesses a more informative signal than Player one, she also drills in the event that Player one's bid is rejected by the U.S. government. Therefore, $b_2^*(1) = \frac{1}{2}(\xi_2(1) - c)$. To summarize:

Lemma 7 *Suppose type-one players are indifferent between drilling and waiting (in case both players win their tracts). If, additionally, the drilling cost c is less than $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$, Player i 's bid $b_i^*(1) = \frac{1}{2}(\xi_i(1) - c)$. If $c > \Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$, there exists a $\hat{p}_1 < 1$ such that if Player one's precision $p_1 \in [\hat{p}_1, 1)$, $b_i^*(1)$ also equals $\frac{1}{2}(\xi_i(1) - c)$.*

In the remainder of this section, I restrict attention to the case in which type-one players bid $\frac{1}{2}(\xi_i(1) - c)$. This assumption, however, is solely done for pedagogical purposes. I have been able to show that Propositions 2 and 3 below hold even if Player i faces a negative payoff from drilling when the other player's bid is rejected, i.e. if $\mu_i^-(1) < c$. This proof is available upon request. It follows from Lemma 4 that, if $b_i^*(1) = \frac{1}{2}(\xi_i(1) - c)$, without loss of generality $b_i(0)$ can be restricted to lie in $[0, \frac{1}{2}(\xi_i(1) - c)]$.

Player i is said to face a positive option value of waiting in the auction without bid disclosure if $\mu_i^+(1) - c \leq \delta W_i(1; \lambda_{-i}(1) = 1)$. In words, Player i faces a positive option value of waiting if she prefers to wait even if she expects Player $-i$ to drill with probability one in case $s_{-i} = 1$.

Lemma 8 *If $\delta \in [\hat{\delta}, 1]$, type-one players face a positive option value of waiting in the auction without bid disclosure.*

This is very intuitive: If the discount factor δ is sufficiently high, Player i 's opportunity cost of waiting is low. Hence, if she expects the other player to drill with probability one (in case

$s_{-i} = 1$), she prefers to wait. In what follows, I will assume that $\delta \in [\hat{\delta}, 1]$. Recall from Section 4 that it only takes three months to complete an exploratory well. In that sense, it seems reasonable to assume that players possess a positive option value of waiting.

Suppose that $s_i = 1$. If Player $-i$ drills with probability zero, Player i prefers to drill as waiting bears an opportunity cost and no informational benefit, i.e. $\Delta_i(1; \lambda_{-i}(1) = 0, \delta \in [\hat{\delta}, 1]) \geq 0$. Trivially, if $\delta \in [\hat{\delta}, 1]$, Player i prefers to wait if she expects Player $-i$ to drill with probability one (provided, of course, that he is a type-one player). Formally, $\Delta_i(1; \lambda_{-i}(1) = 1, \delta \in [\hat{\delta}, 1]) \leq 0$. Observe that Lemma 5 implies that $\Delta_i(s_i)$ is decreasing in $\lambda_{-i}(1)$. The lemma below then straightforwardly follows from the intermediate value theorem.

Lemma 9 *Suppose $\delta \in [\hat{\delta}, 1]$ and that $b_{-i}(1) = \frac{1}{2}(\xi_{-i}(1) - c)$. Fix $b_{-i}(0) \in [0, \frac{1}{2}(\xi_{-i}(1) - c)]$. There exists then a unique $\lambda_{-i}^*(1) \in [0, 1]$ such that $\Delta_i(1) = 0$.*

It follows from the explanations provided before Corollary 2 that the drilling probability $\lambda_{-i}^*(1)$ is strictly positive when the discount factor is less than one. This is intuitive: If the discount factor $\delta < 1$, Player i 's opportunity cost of waiting is positive. To make her indifferent between drilling and waiting, Player $-i$ must thus drill with positive probability. In its turn, this implies that in the event that Player i possesses signal zero, with positive probability she learns that the value of Tract $-i$ equals one. Hence, Player i 's interim payoff $u_i^<(0)$ is positive. In its turn, this implies that when the discount factor δ is less than one, she bids a positive amount, i.e. $b_i^*(0) > 0$. For future reference, I summarize this insight in my next corollary.

COROLLARY 3 *Suppose that the discount factor $\delta \in [\hat{\delta}, 1)$. In equilibrium, type-zero players then bid positive amounts, i.e. $b_i^*(0) > 0$.*

We now know enough to prove existence of a non-coordinating equilibrium. Recall from my discussion after Lemma 4 that $b_i^*(0) = \frac{1}{2}u_i^<(0)$. I focus on an equilibrium in which type-zero players wait. Player i 's interim payoff $u_i^<(0)$ therefore positively depends on her beliefs about the probability that Player $-i$ drills (i.e. on $\lambda_{-i}(1)$). Furthermore, Equation 10 reveals that her payoff from waiting is also influenced by her posterior in case Player $-i$ wins his tract and waits. In particular, as type-zero players are more likely to wait, Player i becomes more pessimistic about her prospects of finding oil after observing Player $-i$ waiting, i.e.

$$\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}) \leq \mu_i^+(0).$$

The extent to which she revises her posterior downwards depends on her beliefs about $b_{-i}(0)$.²³ If, for example, she believes that $b_{-i}(0) = 0$, she knows that Player $-i$ is a type-one player and

²³It also depends on her beliefs about $b_{-i}(1)$. Lemma 7, however, shows that if signals are sufficiently precise or if the drilling cost is sufficiently low, $b_{-i}(1)$ is equal to some constant.

infers no bad information out of his waiting decision, i.e.

$$\begin{aligned} \Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}; b_{-i}(0) = 0) &= \mu_i^+(0; b_{-i}(0) = 0) \\ &= \Pr(V_i = 1 | s_i = 0, s_{-i} = 1). \end{aligned}$$

I conclude that Player i 's interim payoff $u_i^<(0)$ depends on her beliefs about $\lambda_{-i}(1)$ and $b_{-i}(0)$. To stress this dependency, in what follows I will sometimes write $u_i^<(0; b_{-i}(0), \lambda_{-i}(1))$ instead of $u_i^<(0)$. Observe that $\frac{1}{2}u_i^<(0; b_{-i}(0), \lambda_{-i}(1))$ maps $[0, \frac{1}{2}(\xi_{-i}(1) - c)] \times [0, 1]$ into $[0, \frac{1}{2}(\xi_i(1) - c)]$.²⁴

Let $h : [0, \frac{1}{2}(\xi_1(1) - c)] \rightarrow [0, 1]$ with the interpretation that $h(b_1(0))$ is the probability with which Player one must drill in case she is a type-one player to ensure that Player two is indifferent between drilling and waiting (i.e. $\Delta_2(1) = 0$) given that $b_1(1) = \frac{1}{2}(\xi_1(1) - c)$, and given that Player one bids $b_1(0)$ if $s_1 = 0$. The higher $b_1(0)$, the lower Player two's posterior probability that Player one is a type-one player after observing her winning her tract. This makes Player two less optimistic about her prospects of finding oil and reduces her opportunity cost of waiting. Recall, however, that only type-one players drill at time one. The reduction in Player two's posterior probability that Player one is a type-one player, thus also makes Player two less optimistic about her chances of free-riding on Player one's drilling effort. In its turn, this reduces Player two's payoff from waiting. Hence, $\lambda_1(1)$ depends in a non-trivial way on $b_1(0)$. Recall from Lemma 9, however, that—for all values of $b_1(0)$ — $h(b_1(0)) \in [0, 1]$, i.e. if the discount factor is high enough, Player one can always choose some drilling probability to make Player two indifferent between drilling and waiting. Similarly, let $j : [0, \frac{1}{2}(\xi_2(1) - c)] \rightarrow [0, 1]$ with the interpretation that $j(b_2(0))$ is the probability with which Player two must drill (in case $s_2 = 1$) to make Player one indifferent between drilling and waiting given that Player two bids $b_2(0)$ if $s_2 = 0$. I conclude that a non-coordinating equilibrium exists only if $(b_1^*(0), b_2^*(0), \lambda_1^*(1), \lambda_2^*(1))$ are computed such that

$$b_1^*(0) = \frac{1}{2}u_1^<(0; b_2^*(0), \lambda_2^*(1)), \quad (15)$$

$$b_2^*(0) = \frac{1}{2}u_2^<(0; b_1^*(0), \lambda_1^*(1)), \quad (16)$$

$$\lambda_1^*(1) = h(b_1^*(0)), \text{ and} \quad (17)$$

$$\lambda_2^*(1) = j(b_2^*(0)). \quad (18)$$

Let $F \equiv (\frac{1}{2}u_1^<(0), \frac{1}{2}u_2^<(0), h, j)$ and $S \equiv [0, \frac{1}{2}(\xi_1(1) - c)] \times [0, \frac{1}{2}(\xi_2(1) - c)] \times [0, 1]^2$. Observe that, if $\delta \in [\hat{\delta}, 1]$, $F : S \rightarrow S$. As F is continuous in $(b_1(0), b_2(0), \lambda_1(1), \lambda_2(1))$ and as S is compact, it follows from Brouwer's theorem that a fixed point exists. I conclude:

²⁴Observe also that I slightly abuse notation here: I originally defined Player i 's interim payoff $u_i^<(0)$ as a number. In what follows, I define it as a function. This, however, should not cause confusion.

PROPOSITION 2 Suppose $\delta \in [\hat{\delta}, 1]$. There exists then a non-coordinating equilibrium in which Player i drills with probability $\lambda_i^*(1) \in [0, 1]$ if $s_i = 1$ and if both players win their tracts. If, additionally, signals are sufficiently precise or if the drilling cost c is sufficiently low, type-one players drill at time one when the other player's bid is rejected and bid $b_i^*(1) = \frac{1}{2}(\xi_i(1) - c)$.

The proof of the proposition when $\delta \in [\hat{\delta}, 1]$ and when $b_i^*(1) = \frac{1}{2}(\xi_i(1) - c)$ appears above. The proof of the proposition when at least one type-one player refrains from drilling when the other player did not win his tract, is available upon request. (That proof, however, is very similar to the one I detailed above in the sense that it also uses Brouwer's fixed point theorem.)

I now tackle uniqueness. I first show that, if the discount factor is sufficiently high, $u_i^<(0)$ is not very sensitive to changes in $b_{-i}(0)$. Formally, I argue that $\frac{du_i^<(0)}{db_{-i}(0)}$ is then $\in [-1, 1]$. As argued above, $u_i^<(0)$ only depends on the bids of Player $-i$ (i.e. $b_{-i}(0)$ and $b_{-i}(1)$) and on $\lambda_{-i}(1)$. Recall from Lemma 7 that, if signals are sufficiently precise or if the drilling cost c is sufficiently low, $b_{-i}(1)$ is equal to some constant. Therefore,

$$\frac{du_i^<(0)}{db_{-i}(0)} = \underbrace{\frac{\partial u_i^<(0)}{\partial b_{-i}(0)}}_{\text{Direct effect}} + \underbrace{\frac{d\lambda_{-i}(1)}{db_{-i}(0)} \frac{\partial u_i^<(0)}{\partial \lambda_{-i}(1)}}_{\text{Indirect effect}}.$$

The direct effect measures how a change in $b_{-i}(0)$ affects $u_i^<(0)$ keeping $\lambda_{-i}(1)$ fixed. The indirect effect measures how a change in $b_{-i}(0)$ affects $u_i^<(0)$ via an induced change in $\lambda_{-i}(1)$.

It can be shown that the absolute value of the direct effect,

$$\left| \frac{\partial u_i^<(0)}{\partial b_{-i}(0)} \right| \leq \Pr(s_{-i} = 0 | s_i = 0) \left[c - \Pr(V_i = 1 | s_i = s_{-i} = 0) \right] < 1.^{25} \quad (19)$$

To see this, suppose Player $-i$ possesses signal zero and that $b_{-i}(0)$ is either equal to x or equal to $x + \epsilon$. Suppose also that $x < r_{-i} < x + \epsilon$. If $b_{-i}(0) = x$, Player $-i$ loses his tract and Player i only drills if it is profitable for her to do so, i.e. if $\mu_i^-(0) \geq c$. If $b_{-i}(0) = x + \epsilon$, Player $-i$ wins his tract and, as he is a type-zero player, waits at time one. Player i then only drills if her time-two posterior $\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait})$ exceeds the drilling cost c . If both her time-two posterior $\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait})$ and her time-one posterior $\mu_i^-(0)$ are less than the drilling cost c , the direct effect is zero. This is intuitive: In this case Player i only drills if Player $-i$ wins his tract, drills, and finds oil. As only type-one players drill, her interim payoff is therefore solely influenced by $b_{-i}(1)$ and $\lambda_{-i}(1)$. Suppose now that

$$\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}) < c < \mu_i^-(0).$$

If he chooses $b_{-i}(0) = x$, he loses his tract, Player i drills at time one and loses $c - \Pr(V_i = 1 | s_i = s_{-i} = 0)$. Had he chosen $b_{-i}(0) = x + \epsilon$, however, he would have won his tract, both

²⁵The proof of this inequality is available upon request.

players would have waited and Player i would correctly have decided not to drill at time two. In this example, the increase in $b_{-i}(0)$ thus increases her interim payoff. Observe, however, that her interim payoff does not increase by “much”: She only avoids an expected loss equal to $c - \Pr(V_i = 1 | s_i = s_{-i} = 0)$ in case he is a type-zero player (which happens with probability $\Pr(s_{-i} = 0 | s_i = 0)$). This intuitively explains Inequality 19.

As argued prior to Proposition 2, the sign of $\frac{d\lambda_{-i}(1)}{db_{-i}(0)}$ is ambiguous. (On the one hand, an increase in $b_{-i}(0)$ reduces her time-one posterior $\mu_i^+(1)$. As explained in Equation 13, this reduces her opportunity cost of waiting. On the other hand, it also reduces her payoff from waiting as she realizes that it is less likely that the other player possesses signal one given that $r_{-i} < b_{-i}$.) The sign of $\frac{\partial u_i^<(0)}{\partial \lambda_{-i}(1)}$, however, is always positive: The higher $\lambda_{-i}(1)$, the higher the likelihood that the other player will drill at time one. This increases Player i 's payoff from waiting, and, by extension, her interim payoff when $s_i = 0$. Recall from Corollary 2, however, that if discount factor δ is equal to one, $\lambda_{-i}^*(1) = 0 \forall b_{-i}(0)$. (If the opportunity cost of waiting is zero, the informational benefit of waiting must be zero too, independent of Player i 's time-one posterior $\mu_i^+(1)$.) Hence, if $\delta = 1$, the indirect effect is equal to zero. By continuity, there exists a $\underline{\delta} < 1$ such that $\forall \delta \in [\underline{\delta}, 1]$, $\frac{du_i^<(0)}{db_{-i}(0)} \in [-1, 1]$.

I now use my last result to prove that, if type-one players bid $\frac{1}{2}(\xi_i(1) - c)$ and if $\delta > \max\{\underline{\delta}, \hat{\delta}\}$, Equations 15 to 18 admit a unique solution. Suppose $\mathbf{x}' \equiv (b_1(0)', b_2(0)', \lambda_1(1)', \lambda_2(1)')$ and $\mathbf{x}'' \equiv (b_1(0)'', b_2(0)'', \lambda_1(1)'', \lambda_2(1)'')$ are two different fixed points of 15 to 18. One has

$$\begin{aligned} |b_1(0)' - b_1(0)''| &= \frac{1}{2} |u_1^<(0)' - u_1^<(0)''| \leq \frac{1}{2} |b_2(0)' - b_2(0)''| \\ &= \frac{1}{4} |u_2^<(0)' - u_2^<(0)''| \leq \frac{1}{4} |b_1(0)' - b_1(0)''|, \end{aligned}$$

where the equalities follow from the fact that $b_i(0) = \frac{1}{2}u_i^<(0)$ and where the inequalities follow from my previous paragraph. Hence, \mathbf{x}' and \mathbf{x}'' are fixed points only if $b_1(0)' = b_1(0)''$. Mirroring this argument, $b_2(0)' = b_2(0)''$. Focus now on the continuation game in which both players win their tracts. As Player one uses the same bidding strategy in both fixed points, Player two's time-one posterior probability $\mu_2^+(1)$ is identical in both fixed points. Furthermore, in both fixed points Player two holds the same beliefs about Player one's type. Hence, the drilling probability which ensures that Player two is indifferent between drilling and waiting must be the same in both fixed points (i.e. $\lambda_1(1)' = \lambda_1(1)''$). Mirroring this argument $\lambda_2(1)' = \lambda_2(1)''$. This, however, contradicts my assumption that \mathbf{x}' and \mathbf{x}'' are two different fixed points. I conclude:

PROPOSITION 3 *There exists a critical discount factor $\delta' < 1$, such that if $\delta \geq \delta'$, there exists only one non-coordinating equilibrium.*

The proof of the proposition when type-one players bid $\frac{1}{2}(\xi_i(1) - c)$ appears above. The critical

discount factor δ' is then defined as $\delta' \equiv \max\{\hat{\delta}, \underline{\delta}\}$. As mentioned previously, I have been able to show that the above result also holds when at least one type-one player refrains from drilling when the other player does not win his tract. The critical discount factor δ' , however, is then different from $\max\{\hat{\delta}, \underline{\delta}\}$. (That proof is available upon request.) Finally, in the Appendix I also prove that, if the discount factor δ is sufficiently high, there does not exist a non-coordinating equilibrium in which type-zero players drill with positive probability at time one.

6 Revenues, welfare, and total expected drilling

I will sometimes use the symbol \mathcal{D} ($\mathcal{N}\mathcal{D}$) to refer to bids, time-one drilling probabilities, interim payoffs and auction design when the auctioneer discloses (does not disclose) bids. In section 4, λ_i denoted the probability with which Player i drills at time one conditional on the event that both players won their tracts. As λ_i depends on both player's signals (which are correctly inferred out of their bids) in this section $\lambda_i^*(s_i, s_{-i}; \mathcal{D})$ thus denotes Player i 's drilling probability given that the auctioneer discloses bids, given both players' signals and given that both of them win their tracts. Similarly, $b_i^*(s_i; k)$ ($k \in \{\mathcal{D}, \mathcal{N}\mathcal{D}\}$) denotes Player i 's equilibrium bid when her signal is s_i and when the auction design is k . In this section, I assume that $p_1 > \tilde{p}_1$ and that $\delta \geq \max\{\delta', \tilde{\delta}\}$, i.e. I focus on the parameter range in which equilibrium is unique (within some class of strategies) in both auction designs and in which players possess positive option values of waiting.

Suppose Player i is a type-one player and that the auctioneer discloses bids. Recall from Equation 7 that her interim payoff in this case equals

$$u_i^<(1; \mathcal{D}) = \sum_{x^{\mathcal{D}}} \Pr(x^{\mathcal{D}} | s_i = 1) \max\left\{0, \Pr(V_i = 1 | s_i, x^{\mathcal{D}}) - c\right\}, \quad (20)$$

where $x^{\mathcal{D}} \in \{“s_{-i} = 1”, “s_{-i} = 0”\}$. The equation above teaches us that Player i 's interim payoff in the bid-disclosure case is equivalent to the one she gets in the following decision problem: Player i first observes a statistic $x^{\mathcal{D}}$ which reveals the other player's type. Next, she decides whether to drill or not.

Suppose now that the auctioneer does not disclose bids. Recall from Lemma 4 that, in case Player $-i$ wins his tract, she faces a positive payoff from drilling (i.e. $\mu_i^+(1) > c$). Recall also that, in equilibrium, she is then indifferent between drilling and waiting. Hence, using Equation 9 Player i 's interim payoff in the no-disclosure case can be rewritten as:

$$u_i^<(1; \mathcal{N}\mathcal{D}) = \sum_{x^{\mathcal{N}\mathcal{D}}} \Pr(x^{\mathcal{N}\mathcal{D}} | s_i = 1) \max\left\{0, \Pr(V_i = 1 | s_i = 1, x^{\mathcal{N}\mathcal{D}}) - c\right\}, \quad (21)$$

where $x^{\mathcal{N}\mathcal{D}} \in \{“r_{-i} < b_{-i}”, “r_{-i} > b_{-i}”\}$. Player i 's interim payoff in the auction without bid disclosure is thus equivalent to the one she gets in the following decision problem: First Player i observes a statistic $x^{\mathcal{N}\mathcal{D}}$ which provides her with some noisy information about Player $-i$'s type; next she decides whether to drill or not. Observe that if

$$x^{\mathcal{D}} = “s_{-i} = 1”, \text{ then } x^{\mathcal{N}\mathcal{D}} = \begin{cases} “r_{-i} < b_{-i}” & \text{with prob. } b_{-i}^*(1; \mathcal{N}\mathcal{D}), \text{ and} \\ “r_{-i} > b_{-i}” & \text{with prob. } 1 - b_{-i}^*(1; \mathcal{N}\mathcal{D}). \end{cases}$$

Similarly, if

$$x^{\mathcal{D}} = “s_{-i} = 0”, \text{ then } x^{\mathcal{N}\mathcal{D}} = \begin{cases} “r_{-i} < b_{-i}” & \text{with prob. } b_{-i}^*(0; \mathcal{N}\mathcal{D}), \text{ and} \\ “r_{-i} > b_{-i}” & \text{with prob. } 1 - b_{-i}^*(0; \mathcal{N}\mathcal{D}). \end{cases}$$

Stated differently, $x^{\mathcal{N}\mathcal{D}}$ is created by “adding noise” to $x^{\mathcal{D}}$. It then follows from Blackwell's (1951) theorem that type-one players prefer the auction with bid disclosure, i.e. $u_i^<(1; \mathcal{N}\mathcal{D}) \leq u_i^<(1; \mathcal{D})$.

I now analyze when Player i strictly prefers the auctioneer to disclose bids. Clearly, if the drilling cost c is lower than $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$, Player i always—i.e. independent of $x^{\mathcal{D}}$'s realization—drills in the decision problem summarized by Equation 20. Recall from my discussion prior to Lemma 7 that if the drilling cost c is lower than $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$, Player i also always—i.e. independent of $x^{\mathcal{N}\mathcal{D}}$'s realization—drills in the decision problem summarized by Equation 21. Hence, in this case Player i is indifferent between receiving statistic $x^{\mathcal{D}}$ or receiving statistic $x^{\mathcal{N}\mathcal{D}}$. Suppose now that the drilling cost c is higher than $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$ and that the discount factor δ is less than one. Suppose that Player $-i$ possesses signal zero and that his reservation price r_{-i} is “very low”, i.e. lower than $b_{-i}^*(0; \mathcal{N}\mathcal{D})$. Hence, in this case $x^{\mathcal{D}} = “s_{-i} = 0”$ and $x^{\mathcal{N}\mathcal{D}} = “r_{-i} < b_{-i}”$. It follows from Corollary 3 that these events happen with positive probability. If the auctioneer discloses bids, Player i thus never drills. In the decision problem summarized by Equation 21, however, she drills and gets an expected loss equal to $c - \Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$. Hence, with positive probability Player i may incur a loss in the auction without disclosure which she would have avoided had bids been disclosed. If the drilling cost c is higher than $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$, her interim payoff is thus higher in the auction with bid disclosure, i.e. $u_i^<(1; \mathcal{N}\mathcal{D}) < u_i^<(1; \mathcal{D})$. As bids and time-zero payoffs are increasing in interim payoffs, I conclude:

Lemma 10 *Suppose that $p_1 > \tilde{p}_1$, that $\delta \in [\max\{\delta', \tilde{\delta}\}, 1)$ and that $s_i = 1$.*

1. *If, additionally, $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0) < c$, Player i strictly prefers the auctioneer to disclose bids, i.e. $u_i^0(1; \mathcal{N}\mathcal{D}) < u_i^0(1; \mathcal{D})$. She also bids more aggressively in this case, i.e. $b_i^*(1; \mathcal{N}\mathcal{D}) < b_i^*(1; \mathcal{D})$.*

2. If $c < \Pr(V_i = 1 | s_i = 1, s_{-i} = 0)$, however, she is indifferent between both auction designs and bids the same amount.

The lemma below summarizes how type-zero players rank both auction designs.

Lemma 11 *Suppose that $p_1 > \tilde{p}_1$, that $\delta > \max\{\delta', \tilde{\delta}\}$ and that $s_i = 0$.*

1. *If $c < \Pr(V_i = 1 | s_i = 0, s_{-i} = 1)$ and if the discount factor δ is sufficiently high, she prefers the auctioneer to disclose bids, i.e. $u_i^0(0; \mathcal{N}\mathcal{D}) < u_i^0(0; \mathcal{D})$. She also bids more aggressively in the auction with bid disclosure, i.e. $b_i^*(0; \mathcal{N}\mathcal{D}) < b_i^*(0; \mathcal{D})$.*
2. *If $\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) < c < \Pr(V_{-i} = 1 | s_i = 0, s_{-i} = 1)$, and if the discount factor δ is less than one, she also prefers the auctioneer to disclose bids, and bids more aggressively in the auction with bid disclosure.*
3. *If $\Pr(V_{-i} = 1 | s_i = 0, s_{-i} = 1) < c$, and if the discount factor δ is less than one, she prefers the auctioneer not to disclose bids. Consequently, she bids more aggressively in the auction without bid disclosure.*

To understand Point one of the lemma, suppose that drilling is “very” profitable, i.e. that $c < \Pr(V_i = 1 | s_i = 0, s_{-i} = 1)$ and that the discount factor δ is equal to one. In words, the inequality states that Player i faces a positive payoff from drilling if the other player possesses signal one, even if she possesses signal zero. It is straightforward to show that her interim payoffs then boil down to:

$$u_i^<(0; \mathcal{D}) = \sum_{x^{\mathcal{D}}} \Pr(x^{\mathcal{D}} | s_i = 0) \max \left\{ 0, \Pr(V_i = 1 | s_i = 0, x^{\mathcal{D}}) - c \right\}, \quad (22)$$

and

$$u_i^<(0; \mathcal{N}\mathcal{D}, \delta = 1) = \sum_{x^{\mathcal{N}\mathcal{D}}} \Pr(x^{\mathcal{N}\mathcal{D}} | s_i = 0) \max \left\{ 0, \Pr(V_i = 1 | s_i = 0, x^{\mathcal{N}\mathcal{D}}) - c \right\}. \quad (23)$$

To see this, suppose the auctioneer discloses bids and that Player $-i$'s bid is rejected. After inferring his signal out of his bid, Player i then takes an optimal time-one drilling decision. Suppose now that he wins his tract. If Player $-i$ is also a type-zero player, she gets zero as drilling yields a negative expected payoff. Suppose now that Player $-i$ is a type-one player. As tract values are imperfectly correlated, Player i 's signal is more informative about the value of Tract i than about the value of Tract $-i$. Player i 's posterior probability of finding oil is therefore lower than the one of Player $-i$, i.e.

$$\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) \leq \Pr(V_{-i} = 1 | s_i = 0, s_{-i} = 1). \quad (24)$$

As drilling is very profitable (i.e. as $c < \Pr(V_i = 1 | s_i = 0, s_{-i} = 1)$), both players thus face a positive payoff from drilling at time one. As explained in section 4, I then assume that both players play a mixed-strategy in the drilling game. As usual, in a mixed-strategy Nash equilibrium Player i is indifferent between drilling and waiting. It is thus without loss of generality to write that she then gets $\max\{0, \Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c\}$. Hence—independent of whether or not Player $-i$ wins his tract—Player i gets $\max\{0, \Pr(V_i = 1 | s_i = 1, s_{-i}) - c\}$. This explains equation 22. Suppose now that the auctioneer does not disclose bids. If the U.S. government rejects Player $-i$'s bid, she decides whether or not to drill at time one. Suppose now that the U.S. government does not reject his bid. As argued in section 5, in equilibrium type-zero players prefer to wait. Recall, however, that the discount factor δ is equal to one. From Corollary 2 we know that type-one players then also wait. As no one drills at time one, Player i does not infer any bad news out of the event “Player $-i$ waited”. Formally,

$$\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}; \lambda_{-i}(1) = 0) = \mu_i^+(0).$$

Player i thus drills at time two if and only if $\mu_i^+(0)$ is greater than the drilling cost c . This explains Equation 23. Equations 22 and 23 thus show that—if drilling is very profitable and if the discount factor δ equals one—her interim payoffs are also equivalent to the ones she would get in two similar decision problems: In the first one, she receives statistic x^D and then decides whether to drill or not; in the second one, she receives a more noisy statistic about the other player's type ($x^{\mathcal{N}D}$) and then decides whether to drill or not. It then follows from Blackwell's value of information theorem that she weakly prefers to receive statistic x^D .

Observe that $(x^D, x^{\mathcal{N}D})$ can take the following realization: Player $-i$ is a type-one player whose bid is rejected (i.e. $x^D = “s_{-i} = 1”$, and $x^{\mathcal{N}D} = “r_{-i} > b_{-i}”$).²⁶ As players shade their bids, this case happens with positive probability. In this case Player i strictly prefers to receive statistic x^D instead of statistic $x^{\mathcal{N}D}$: With the former statistic, she drills at time one and receives $\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c > 0$; with the second one, she (wrongly) chooses not to drill. Hence, type-zero players enjoy a (strictly) higher interim payoff when the auctioneer discloses bids and when the discount factor δ is equal to one. As bids are increasing in interim valuations, she also bids (strictly) more if the auctioneer discloses bids. Point one of the lemma then follows from the fact that, if $c < \Pr(V_i = 1 | s_i = 0, s_{-i} = 1)$, Player i 's interim payoffs are continuous in the discount factor δ , and from the fact that Player i 's time-zero payoff is increasing in her interim payoff.

I now explain Point two of the lemma. Suppose that Player i possesses type zero and that

²⁶It follows from Blackwell's theorem that in the other realizations of $(x^D, x^{\mathcal{N}D})$, Player i either prefers to receive statistic x^D or she is indifferent between both statistics.

Player $-i$ possesses type one. Recall from Equation 24 that Player i 's posterior probability is then lower than Player $-i$'s. Suppose also the profitability of drilling is “intermediate”, i.e. that $\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) < c < \Pr(V_{-i} = 1 | s_i = 0, s_{-i} = 1)$. As drilling is not “very profitable”, Player i —independent of Player $-i$'s type—faces a negative payoff from drilling at time one. As the profitability of drilling is not “low”, however, Player $-i$ —independent of Player i 's type—faces a positive payoff from drilling. If the auctioneer discloses bids, Player $-i$ learns that Player i will not drill at time one. As waiting is costly, he then prefers to drill at time one. This, in turn, increases her payoff from waiting. If the auctioneer does not disclose bids, however, Player $-i$ believes that she might be a type-one player who chooses her drilling probability $\lambda_i^*(1; \mathcal{N}\mathcal{D})$ to make him indifferent between drilling and waiting. This induces him to choose $\lambda_{-i}^*(1; \mathcal{N}\mathcal{D})$ to make her—in case $s_i = 1$ —indifferent between drilling and waiting. As explained prior to Lemma 9, if $\delta > \hat{\delta}$, $\lambda_{-i}^*(1; \mathcal{N}\mathcal{D}) < 1$. Hence, in this case Player i prefers the auctioneer to disclose bids as this increases her chances of free-riding on Player $-i$'s drilling cost.

To understand Point three, suppose that Player i is a type-zero player and that the profitability of drilling is “low”, i.e. that $\Pr(V_{-i} = 1 | s_i = 0, s_{-i} = 1) < c$. As explained previously, the inequality states that if Player $-i$ finds out that Player i is a type-zero player, he faces a negative payoff from drilling even if he is a type-one player. The inequality also implies that Player i only drills if Player $-i$ wins his tract, drills at time one and finds oil. Hence, if the auctioneer discloses bids, no one ever drills and her interim payoff is equal to zero. This also implies that, in the bid disclosure case, she bids zero. Suppose now that the auctioneer does not disclose bids and that Player i won her tract. Player $-i$ now does not know that she is a type-zero player who was lucky to have her “low” bid accepted by the U.S. government. As explained prior to Lemma 9, he then drills with some probability $\lambda_{-i}^*(1; \mathcal{N}\mathcal{D})$ at time one. Recall from Lemma 9 that Player $-i$'s drilling probability $\lambda_{-i}^*(1; \mathcal{N}\mathcal{D})$ is positive whenever the discount factor $\delta < 1$. Hence, if the discount factor $\delta < 1$, Player i 's interim payoff in the auction without bid disclosure is positive.²⁷ This also implies that she bids a positive amount. Player i thus bids more aggressively in this case if the auctioneer does not disclose bids.

We now know enough to rank both auctions in terms of revenues and welfare. It can be shown that the expected welfare in auction k ($k \in \{\mathcal{D}, \mathcal{N}\mathcal{D}\}$), $W(k)$, can be written as a weighted sum of the type-zero and type-one time-zero utilities $u_i^0(s_i; k)$ of both players. Formally,

$$W(k) = \sum_i \left[\Pr(s_i = 0) u_i^0(0; k) + \Pr(s_i = 1) u_i^0(1; k) \right]. \quad (25)$$

(The proof of this result is available upon request.) Similarly, expected revenues in auction k ,

²⁷If the discount factor δ is equal to one, we know from Corollary 2 that Player $-i$ waits (i.e. $\lambda_{-i}^*(1; \mathcal{N}\mathcal{D}) = 0$). Player i is then indifferent between both auction designs as she can never hope to learn out of his drilling effort.

$R(k)$, are computed as

$$R(k) = \sum_i \left[\Pr(s_i = 0) \Pr\left(r_i < b_i^*(0; k)\right) b_i^*(0; k) + \Pr(s_i = 1) \Pr\left(r_i < b_i^*(1; k)\right) b_i^*(1; k) \right].$$

Suppose first that the profitability of drilling is either “high” or “intermediate”, i.e. that $c < \Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$. The inequality implies that type-one players face a positive payoff from drilling—even if the other player possesses signal zero.²⁸ It then follows from Point two of Lemma 10 that type-one players are indifferent between both auction designs. Recall from Points one and two of Lemma 11 that if, additionally, the discount factor δ is sufficiently high but less than one, type-zero players then strictly prefer the auctioneer to disclose bids. Hence, in this case the auctioneer raises welfare and revenues by disclosing bids. To summarize:

PROPOSITION 4 *Suppose that signals are sufficiently precise, i.e. that $p_1 > \tilde{p}_1$. Suppose also that the profitability of drilling is not “low”, i.e. that $c < \Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$. If, additionally, the discount factor δ is sufficiently high but less than one, disclosing bids raises welfare and revenues.*

The cost of an exploratory well should often be small in comparison to the value of the oil (in case oil is found in sufficient quantities to warrant production). Hence, after normalizing the value of the oil to one, it is often realistic to assume that the drilling cost c is less than $\Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$. Furthermore, as argued in Section 4, in any reasonable calibration of my model the discount factor δ should be very high. Proposition 4 thus provides some theoretical justification for the U.S. government’s practice of disclosing bids in those auctions.

This justification, however, need not hold if at least one tract is “marginal”, i.e. if the ex ante profitability of one tract is low. To see this, suppose that $c > \Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$. In words, the inequality implies that at least one type-one player (i.e. Player one for sure and possibly Player two) faces a negative payoff from drilling if she finds out that the other player possesses signal zero. It follows from Point one of Lemma 10 that Player one (in case $s_1 = 1$) then prefers the auction with bid disclosure: If she were to find out that Player two possesses signal zero, she would refrain from drilling and thereby avoid an expected loss. Point three of Lemma 11, however, establishes that Player two (in case $s_2 = 0$) then prefers the auction without bid disclosure: If bids are disclosed, she correctly predicts that her low bid removes Player one’s incentives to drill. In turn, this “kills” any hope she has to free-ride on her drilling cost. An auction designer thus faces a trade-off in this case: If she discloses bids, the loss of revenues due

²⁸Recall that, as Player two possesses a weakly more informative signal, $\Pr(V_1 = 1 | s_1 = 1, s_2 = 0) \leq \Pr(V_2 = 1 | s_1 = 0, s_2 = 1)$. Hence, if Player one still faces a positive payoff from drilling after inferring that $s_2 = 0$, so does Player two after inferring that $s_1 = 0$.

to less aggressive bidding by at least one type-zero player needs to be weighed against the gain due to more aggressive bidding by at least one type-one player.

It is easy to show that the auction without bid disclosure may—depending on the values of the exogenous parameters—generate more expected revenues than the one with bid disclosure. To see this, suppose both players have the same signal precision, i.e. $p_1 = p_2$. In this case it is straightforward to show that $\Pr(V_1 = 1|s_1 = 1, s_2 = 0) = \Pr(V_2 = 1|s_1 = 0, s_2 = 1)$. In both probabilities Player i possesses signal one, while the other player possesses signal zero. As both players possess equally informative signals, Player one’s inference that player two possesses signal zero is equivalent to Player two inferring that Player one possesses signal zero. They therefore compute the same posterior probability of finding oil. Suppose now that if $s_i = 1$ and if $s_{-i} = 0$, Player i ’s payoff from drilling is equal to zero, i.e.

$$\Pr(V_i = 1|s_i = 1, s_{-i} = 0) = c \quad \forall i. \quad (26)$$

As Player i ’s payoff from investing is equal to zero, I assume that she does not invest.²⁹ Suppose that the auctioneer discloses bids. Recall from Equation 20 that her interim payoff is then equivalent to the one she gets in a decision problem in which she first observes Player $-i$ ’s type and next decides whether to drill or not. Observe that Equality 26 implies that, in this decision problem, she gets the same payoff as the one she would get if she were to drill at time one— independent of his type. It follows from Equation 14 that, in the auction without bid disclosure, one can think of a type-one player as someone who also drills at time one— independent of whether Player $-i$ wins his tract or not.³⁰ Hence, under Equality 26, type-one players get the same interim payoffs in both auction designs. This also implies that they bid the same amount and derive the same time-zero utility from both auction designs.

Suppose now that Player i is a type-zero player. If the auctioneer discloses bids, she never learns the value of Tract $-i$. Hence, she bids zero in the auction with bid disclosure. If the auctioneer does not disclose bids, however, we know from Corollary 3 that she bids a positive amount. Hence, in this example the auctioneer can raise welfare and revenues by not disclosing bids. Observe that this result holds even if the discount factor δ is close to one. Observe also that this result holds generically as—whenever $c > \Pr(V_1 = 1|s_1 = 1, s_2 = 0)$ —payoffs and drilling probabilities are continuous in the drilling cost c and in the signal precisions p_1 and p_2 . I therefore conclude:

²⁹This assumption is innocuous: All the results presented in this paragraph hold as long as the drilling cost is slightly above $\Pr(V_i = 1|s_i = 1, s_{-i} = 0)$.

³⁰To be more precise, she drills at time one if the auctioneer rejects Player $-i$ ’s bid and is indifferent between drilling and waiting if Player $-i$ also wins his tract. This precision, however, is immaterial for what follows.

PROPOSITION 5 *There exists a $\bar{c} > \Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$ and a $\bar{p}_2 > p_1$, such that if the drilling cost $c \in [\Pr(V_1 = 1 | s_1 = 1, s_2 = 0), \bar{c}]$, if Player two's precision $p_2 \in [p_1, \bar{p}_2]$ and if the discount factor $\delta < 1$, not disclosing bids raises welfare and revenues.*

So far, I defined welfare as the sum of the ex ante utilities of both bidders. Drilling, however, may also entail a positive externality: More drilling, for example, should lead to lower oil prices and less reliance on imported oil. To analyze this issue in greater detail, I define total expected drilling as the number of exploratory wells that are expected to be build in both periods. In the Appendix,³¹ I show that, if the discount factor is sufficiently high, an increase in a player's bid cannot decrease total expected drilling. This is intuitive: The higher a player's bid, the higher the probability that her bid will be accepted and, thus, that she will eventually drill. Recall from Point one of Lemma 10 that if the profitability of drilling is low (i.e. if $\Pr(V_1 = 1 | s_1 = 1, s_2 = 0) < c$), Player one (in case $s_1 = 1$) bids more aggressively if the auctioneer discloses bids. Point three of Lemma 11, however, states that Player two (in case $s_2 = 0$) bids more aggressively if the auctioneer does not disclose bids. Hence, *even if one disregards the effect of auction design on the drilling probabilities*, comparing total expected drilling in both auction designs when drilling is a low-profitability activity is not straightforward.

It follows from Point 2 of Lemma 10 and from Points one and two of Lemma 11 that, if the profitability of drilling is not low and if the discount factor is sufficiently high, all types bid weakly more in the auction with bid disclosure. If the drilling probabilities were independent of the auction design, disclosing bids would then promote total drilling. Bid disclosure, however, may sometimes reduce the equilibrium drilling probabilities. To see this, suppose both players possess signal zero and that the discount factor δ is less than one. If the auctioneer discloses bids, it follows from Assumption 1 that no one drills. Recall from Corollary 3 that, without bid disclosure, type-zero players bid positive amounts. Furthermore, without bid disclosure, Player i does not know that Player $-i$ also possesses signal zero. Instead, she may even observe him winning his tract which increases her posterior probability of finding oil. If the drilling cost c is low enough, she might then drill at time two.³² Comparing total expected drilling when

³¹See proof of Proposition 6.

³²The idea that not disclosing bids may promote drilling is not solely confined to the case in which both players possess signal zero, nor to the case in which $c < \Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$. Suppose, for example, that $(s_i, s_{-i}) = (1, 0)$ and that $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0) - c = -\epsilon$, where ϵ represents a very small positive number. If the auctioneer discloses bids, Player i does not drill. Without bid disclosure, however, Player i does not know that the expected payoff from drilling is negative and drills with positive probability. In the Appendix, I analyze such an example in greater detail and I argue that the auctioneer could increase welfare, revenues and total expected drilling by not disclosing bids. To be more precise, it is shown that total expected drilling is 73% higher in the

$c < \Pr(V_1 = 1|s_1 = 1, s_2 = 0)$ is thus not straightforward either.

Suppose that the drilling cost c lies between $\Pr(V_1 = 1|s_1 = 0, s_2 = 1)$ and $\Pr(V_1 = 1|s_1 = 1, s_2 = 0)$ and that the discount factor δ is sufficiently high.³³ It is easy to show that bid disclosure, besides raising revenues, then also raises total drilling. Recall that, as Player two possesses a more informative signal than Player one, she infers more bad news out of “ $s_2 = 0$ ” than Player one does out of “ $s_1 = 0$ ”. Therefore, in this case $\Pr(V_2 = 1|s_1 = 1, s_2 = 0) \leq \Pr(V_1 = 1|s_1 = 0, s_2 = 1) < c$. Hence, in this case type-zero players only drill if the other player drills his tract at time one and finds oil. This insight implies that, if Player i is a type-zero player and if the other player does not win his tract, she refrains from drilling—independent of the auction design. Similarly, as Player two possesses a more informative signal, in this case $c < \Pr(V_1 = 1|s_1 = 1, s_2 = 0) \leq \Pr(V_2 = 1|s_1 = 0, s_2 = 1)$. Hence, in this case type-one players always drill unless the other player drills at time one and finds no oil. This insight implies that, if Player i is a type-one player and if the other player does not win his tract, she drills at time one—independent of the auction design. Hence, both auction designs generate the same amount of drilling when only one player wins her tract. Suppose thus that both players win their tracts. If both players possess signal zero, no one drills—independent of the auction design. If both players possess signal one, they play a war-of attrition. In the auction with bid disclosure, Player i drills at time one with probability $\lambda_i^*(s_i = s_{-i} = 1; \mathcal{D})$. Without bid disclosure, she drills with probability $\lambda_i^*(1; \mathcal{N}\mathcal{D})$. Recall from Corollaries 1 and 2 that, if the discount factor δ is equal to one, both players wait, i.e. $\lambda_i^*(s_i = s_{-i} = 1; \mathcal{D}) = \lambda_i^*(1; \mathcal{N}\mathcal{D}) = 0$. They thus both drill at time two—independent of the auction design. Suppose now that Player i possesses signal one while the other player possesses signal zero. With bid disclosure, Player i drills while the other player only drills if Player i finds oil. Total expected drilling in this case thus equals

$$1 + \Pr(V_i = 1|s_i = 1, s_{-i} = 0). \quad (27)$$

Without bid disclosure, Player i drills at time one with probability $\lambda_i^*(1; \mathcal{N}\mathcal{D})$ and at time two with probability $1 - \lambda_i^*(1; \mathcal{N}\mathcal{D})$. Total expected drilling then equals

$$\lambda_i^*(1; \mathcal{N}\mathcal{D}) \left[1 + \Pr(V_i = 1|s_i = 1, s_{-i} = 0) \right] + 1 - \lambda_i^*(1; \mathcal{N}\mathcal{D}),$$

which is less than 27. As the drilling probabilities $\lambda_i^*(s_i = s_{-i} = 1; \mathcal{D})$ and $\lambda_i^*(1; \mathcal{N}\mathcal{D})$ are

auction without bid disclosure.

³³As both tract values are not perfectly correlated, signal s_1 may—provided that p_1 is sufficiently high—be more informative about the value of Tract one than s_2 . In that case $\Pr(V_1 = 1|s_1 = 0, s_2 = 1) < \Pr(V_1 = 1|s_1 = 1, s_2 = 0)$. This is the case that I consider in this paragraph. If, however, signal one is very uninformative or if signal two is very informative, $\Pr(V_1 = 1|s_1 = 1, s_2 = 0) < \Pr(V_1 = 1|s_1 = 0, s_2 = 1)$.

continuous in δ , this result holds generically, i.e. for a sufficiently high discount factor δ , the auction with bid disclosure generates more expected drilling than the one without disclosure.

Finally, it is also easy to show that bid disclosure may promote revenues but hurt total expected drilling. Suppose, for example, that both players' signals are equally precise (i.e. $p_1 = p_2$) and that

$$c = \Pr(V_i = 1 | s_1 = s_2 = 0) + \epsilon,$$

where ϵ represents an arbitrarily small positive number. Recall from Point two of Lemma 10 that type-one players bid the same amount in both auction designs. It follows from Point 1 of Lemma 11 that—if the discount factor δ is sufficiently high—type-zero players bid more aggressively if the auctioneer discloses bids. Hence, the auction with bid disclosure generates more revenues than the one without disclosure. Suppose now that the discount factor δ is equal to one and that both players win their tracts. Recall from Corollaries 1 and 2 that no one then drills at time one. If the auctioneer discloses bids Player i may infer that the other player also possesses signal zero. As drilling then yields an expected loss of ϵ , both players refrain from drilling. Had bids not been disclosed, she would not have inferred that the other player possesses signal zero and would have drilled at time two. To summarize:

PROPOSITION 6 *If the discount factor δ is sufficiently high and if*

$$\Pr(V_1 = 1 | s_1 = 0, s_2 = 1) < c < \Pr(V_1 = 1 | s_1 = 1, s_2 = 0), \quad (28)$$

disclosing bids raises revenues and total expected drilling. If 28 is not satisfied, the effect of bid disclosure on total expected drilling is ambiguous. Disclosing bids may—depending on the values of my exogenous parameters—decrease revenues and total expected drilling. Disclosing bids may even increase revenues but reduce total expected drilling.

7 Final Remark

My results are *not* driven by my assumption that bids are disclosed between the auction and the drilling stage. Instead, they are driven by the fact that bids are disclosed *before* the drilling game. An open ascending auction should thus suffer from the same weakness as the auction with bid disclosure: If i observes that $-i$ releases his button early, this affects her incentives to drill negatively. Anticipating this, $-i$ may bid less aggressively in an open ascending auction than in a first-price sealed bid one (not followed by bid disclosure).

Appendix

Proof of Lemma 2

Let $\tilde{\delta}_i(s_i, s_{-i})$ be defined as the critical discount factor such that

$$\max \left\{ 0, \Pr(V_i = 1 | s_i, s_{-i}) - c \right\} = \tilde{\delta}_i(s_i, s_{-i}) \overline{W}_i(s_i, s_{-i}). \quad (29)$$

Observe that $\tilde{\delta}_i(s_i, s_{-i}) = 0$ when $\Pr(V_i = 1 | s_i, s_{-i}) < c$. If $\delta > \tilde{\delta}_i(s_i, s_{-i})$, Player i prefers to wait if she anticipates Player $-i$ to drill at time one with probability one. Let

$$\tilde{\delta}_i \equiv \max \{ \tilde{\delta}_i(0, 0), \tilde{\delta}_i(0, 1), \tilde{\delta}_i(1, 0), \tilde{\delta}_i(1, 1) \}. \quad (30)$$

Player i is said to face a positive option value of waiting if $\delta \geq \tilde{\delta}_i$.

Trivially, $\tilde{\delta}_i(s_i, s_{-i}) < 1$ when $\Pr(V_i = 1 | s_i, s_{-i}) < c$. Hence, suppose that $\Pr(V_i = 1 | s_i, s_{-i}) > c$ and that Player $-i$ drills with probability one. As $\delta = 1$, there is no (discounting) cost of waiting. As $\lambda_{-i} = 1$, Player i 's gain of waiting is positive as with probability $\Pr(V_{-i} = 0 | s_i, s_{-i})$ Player $-i$ will find out that there is no oil (underneath his tract). Player i then refrains from drilling at time two and avoids an expected loss equal to $c - \Pr(V_i = 1 | s_i, V_{-i} = 0)$. Hence, if $\delta = 1$, Player i prefers to wait if she expects Player $-i$ to drill with probability one. Therefore, $\tilde{\delta}_i(s_i, s_{-i}) < 1 \forall (s_i, s_{-i})$. Using 30, I conclude that $\tilde{\delta}_i < 1 \forall i$.

Let $\tilde{\delta} \equiv \max \{ \tilde{\delta}_1, \tilde{\delta}_2 \}$. By construction, both players then always face a positive option value of waiting. ■

Proof of Proposition 1

I first state and prove the following Lemma.

Lemma 12 *Suppose p_1 is close to one and that Assumption 2 holds. ρ^0, ρ^1 , and $\tilde{\delta}$ must then also be close to one.*

Proof: Recall that $p_1 \leq p_2$. Hence, if p_1 is close to one, so is p_2 . Recall also from Assumption 2 that $\Pr(V_i = 1 | s_i = 1, V_{-i} = 0) < c$. Using Bayes's rule this inequality is equivalent to $\frac{1-c}{c} < \frac{1-p_i}{p_i} \frac{\rho^0}{1-\rho^0}$. If p_i is close to one, the inequality is only satisfied if ρ^0 is close to one too. As $\nu(1 - \rho^1) = (1 - \nu)(1 - \rho^0)$, ρ^1 must then also be close to one.

Recall from Assumption 1 that $c < \xi_i(1)$. As tract values are correlated, $\xi_i(1) < \Pr(V_i = 1 | s_i = s_{-i} = 1)$. Thus, if both players possess signal one, they face a positive payoff from drilling. It then follows from 3 and 29 that $\tilde{\delta}_i(1, 1)$ is computed such that

$$\Pr(V_i = 1 | s_i = s_{-i} = 1) - c = \tilde{\delta}_i(1, 1) \Pr(V_{-i} = 1 | s_i = s_{-i} = 1) \left[\Pr(V_i = 1 | s_i = 1, V_{-i} = 1) - c \right].$$

Observe that, if p_1 is close to one, the left-hand side is close to $1 - c$ while the right-hand side is close to $\tilde{\delta}_i(1, 1)(1 - c)$. Hence, $\lim_{p_1 \rightarrow 1} \tilde{\delta}_i(1, 1) = 1$. Recall from 30 that $\tilde{\delta}_i(1, 1) \leq \tilde{\delta}_i$. Hence, $\lim_{p_1 \rightarrow 1} \tilde{\delta}_i = 1$. $\tilde{\delta}$ must then also be close to one as, $\tilde{\delta} \equiv \max\{\tilde{\delta}_1, \tilde{\delta}_2\}$. ■

As argued in the body of the text, $u_i^c(1)$ is bounded below by $\xi_i(1) - c$. Hence, type-one players, by bidding $\frac{1}{2}(\xi_i(1) - c)$, guarantee that their time-zero payoff $u_i^0(1)$ is bounded below by $\frac{1}{4}(\xi_i(1) - c)^2$. Let \underline{b}_i be the lowest bid such that

$$\frac{1}{4}(\xi_i(1) - c)^2 = \underline{b}_i(\bar{u}_i^c(1) - \underline{b}_i),$$

where $\bar{u}_i^c(1) \equiv \Pr(V_{-i} = 1 | s_i = 1)(\Pr(V_i = 1 | s_i = 1, V_{-i} = 1) - c)$. Observe that $\underline{b}_i > 0$ and that, in any undominated strategy, $b_i(1) \geq \underline{b}_i$. Let $\underline{b} \equiv \min\{\underline{b}_1, \underline{b}_2\}$. Observe that $\lim_{p_1 \rightarrow 1} \underline{b} = \frac{1}{2}(1 - c)$.

In the body of the text, I also argued that, in any undominated strategy, type-zero players bid below

$$\delta \sum_{s_{-i}} \Pr(s_{-i} | s_i = 0) \bar{W}_i(0, s_{-i}) \equiv \bar{b}_i.$$

Let $\bar{b} \equiv \max\{\bar{b}_1, \bar{b}_2\}$. It follows from Lemma 12 that $\lim_{p_1 \rightarrow 1} \bar{b} = 0 < \lim_{p_1 \rightarrow 1} \underline{b}$. By continuity, there exists a $\tilde{p}_1 < 1$ such that $\forall p_1 \in [\tilde{p}_1, 1)$, $\bar{b} < \underline{b}$. Hence, if $\tilde{p}_1 < p_1$, in any undominated strategy, type-zero players bid strictly less than type-one players. In this case in any equilibrium players must thus use invertible bidding strategies.

Suppose now that $\tilde{p}_1 \leq p_1$ and that beliefs are updated as follows: If $b_i \leq \bar{b}$, Player i is supposed to be a type-zero player. If $b_i \in (\bar{b}, \underline{b})$, beliefs don't matter and Player i is supposed to be a type-zero player with any arbitrary probability. If $b_i \geq \underline{b}$, Player i is supposed to be a type-one player. These beliefs are consistent with the bids and with the continuation play computed in Section 4 when signals instead of bids were disclosed. To see this, suppose that $s_i = 0$ and that she expects Player $-i$ to bid $b_{-i}^*(1)$ if $s_{-i} = 1$ and $b_{-i}^*(0)$ if $s_{-i} = 0$. Suppose she bids $b_i \in [0, \bar{b}]$ and $b_i \neq b_i^*(0)$. Given the assumed out-of-equilibrium beliefs, his beliefs about her type would still be degenerate. As I restrict attention to the class of the non-coordinating strategies, this implies that Player $-i$'s continuation play is equivalent to the one he would follow had she bid $b_i^*(0)$. Hence, her interim payoff would then be equal to the one computed in Equation 8. As $b_i^*(0)$ denotes the optimal bid given that interim payoff, and as her objective function is concave, Player i loses by bidding $b_i \neq b_i^*(0)$. Similarly, a type-one player loses by bidding $b_i \in [\underline{b}, 1]$ and $b_i \neq b_i^*(1)$, as by assumption, this would not affect Player $-i$'s beliefs about her type, nor would it affect his continuation play. Hence, her interim payoff would then be equal to the one computed in Equation 7. As $b_i^*(1)$ is the optimal bid given that interim payoff, and as her objective function is concave, Player i loses by bidding $b_i \neq b_i^*(1)$. ■

Proof of Lemma 4

Observe that equation 9 can be rewritten as $u_i^<(s_i) = \max\{A_i(s_i), B_i(s_i)\} + \max\{0, C_i(s_i)\}$, where

$$\begin{aligned}
A_i(s_i) &\equiv \Pr(V_i = 1, r_{-i} < b_{-i} | s_i)(1 - c) - \Pr(V_i = 0, r_{-i} < b_{-i} | s_i)c & (31) \\
&= \Pr(r_{-i} < b_{-i} | s_i) \left(\Pr(V_i = 1 | s_i, r_{-i} < b_{-i}) - c \right), \\
B_i(s_i) &\equiv \delta \Pr(r_{-i} < b_{-i} | s_i) W_i(s_i), \text{ and} \\
C_i(s_i) &\equiv \Pr(V_i = 1, r_{-i} > b_{-i} | s_i)(1 - c) - \Pr(V_i = 0, r_{-i} > b_{-i} | s_i)c \\
&= \Pr(r_{-i} > b_{-i} | s_i) \left(\Pr(V_i = 1 | s_i, r_{-i} > b_{-i}) - c \right).
\end{aligned}$$

Observe also that $A_i(0) < A_i(1)$ is equivalent to

$$0 < \left[\xi_i(1) - \xi_i(0) \right] \left[\Pr(r_{-i} < b_{-i} | V_i = 1)(1 - c) + \Pr(r_{-i} < b_{-i} | V_i = 0)c \right]. \quad (32)$$

Under Assumption 1 the first term between square brackets is positive. The second term between square brackets is positive too as, in any undominated strategy, type-one players bid positive amounts. Hence, inequality 32 holds.

In a similar vein, $C_i(0) < C_i(1)$ can be rewritten as

$$0 < \left[\xi_i(1) - \xi_i(0) \right] \left[\Pr(r_{-i} > b_{-i} | V_i = 1)(1 - c) + \Pr(r_{-i} > b_{-i} | V_i = 0)c \right],$$

which also holds as, in any undominated strategy, players bid less than one.

Observe that $\Pr(r_{-i} < b_{-i} | s_i) W_i(s_i) = D_i(s_i) + \max\{0, E_i(s_i)\}$, where

$$\begin{aligned}
D_i(s_i) &\equiv \Pr(V_{-i}=1, r_{-i} < b_{-i}, a_{-i} = \text{drill} | s_i) \left[\Pr(V_i=1 | s_i, V_{-i}=1) - c \right] \text{ and,} \\
E_i(s_i) &\equiv \Pr(V_i=1, r_{-i} < b_{-i}, a_{-i} = \text{wait} | s_i)(1 - c) - \Pr(V_i=0, r_{-i} < b_{-i}, a_{-i} = \text{wait} | s_i)c.
\end{aligned}$$

Observe also that $E_i(0) \leq E_i(1)$ is equivalent to

$$0 \leq \left[\xi_i(1) - \xi_i(0) \right] \left[\Pr(r_{-i} < b_{-i}, a_{-i} = \text{wait} | V_i=1)(1 - c) + \Pr(r_{-i} < b_{-i}, a_{-i} = \text{wait} | V_i=0)c \right],$$

which is satisfied. Observe also that the inequality is strict when $(\lambda_{-i}(0), \lambda_{-i}(1)) \neq (1, 1)$.

$D_i(0) \leq D_i(1)$ is equivalent to

$$0 \leq \left[\xi_i(1) - \xi_i(0) \right] \left[\Pr(V_{-i}=1, r_{-i} < b_{-i}, a_{-i} = \text{drill} | V_i=1)(1 - c) + \Pr(V_{-i}=1, r_{-i} < b_{-i}, a_{-i} = \text{drill} | V_i=0)c \right],$$

which is satisfied. The inequality is strict when $(\lambda_{-i}(0), \lambda_{-i}(1)) \neq (0, 0)$.

It follows from my preceding paragraphs that $u_i^<(0) < u_i^<(1)$ when $(\lambda_{-i}(0), \lambda_{-i}(1)) \neq (0, 0)$.

Suppose $(\lambda_{-i}(0), \lambda_{-i}(1)) = (0, 0)$. Interim utilities can then be written as:

$$\begin{aligned}
u_i^<(0) &= \max\{0, A_i(0)\} + \max\{0, C_i(0)\}, \text{ and} \\
u_i^<(1) &= \max\{0, A_i(1)\} + \max\{0, C_i(1)\}.
\end{aligned}$$

Observe that $0 < \xi_i(1) - c = A_i(1) + C_i(1)$. Hence, either $A_i(1) > 0$ or $C_i(1) > 0$. This insight, combined with my earlier result that $A_i(0) < A_i(1)$ and that $C_i(0) < C_i(1)$, implies that in this case $u_i^<(0)$ is also strictly less than $u_i^<(1)$. As bids are increasing in interim payoffs, $b_i(0) < b_i(1)$. ■

Proof of Lemma 5

As type-zero and type-one players bid positive amounts, and as signals are imperfect, $\Pr(V_{-i} = 1, s_{-i} | s_i, r_{-i} < b_{-i}) > 0 \forall (s_i, s_{-i})$. Hence, if $\Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) \leq c$, it follows from Equation 10 that $W_i(s_i)$ is increasing in $(\lambda_{-i}(0), \lambda_{-i}(1))$. Suppose now that $\Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) > c$. In that case $W_i(s_i)$ can be rewritten as

$$W_i(s_i) = \mu_i^+(1) - c + \sum_{s_{-i}} \Pr(V_{-i} = 0, s_{-i} | s_i, r_{-i} < b_{-i}) \lambda_{-i}(s_{-i}) \left(c - \Pr(V_i = 1 | s_i, V_{-i} = 0) \right).$$

Recall that, under Assumption 2, $c > \Pr(V_i = 1 | s_i, V_{-i} = 0)$. As players bid positive amounts and as signals are not perfect, $\Pr(V_{-i} = 0, s_{-i} = 1 | s_i, r_{-i} < b_{-i}) > 0 \forall s_i$ and $\forall i$. Hence, in this case $W_i(s_i)$ is also increasing in $(\lambda_{-i}(0), \lambda_{-i}(1))$. ■

Proof of Lemma 6.

In this lemma I write μ_i instead of $\mu_i^+(s_i)$ to denote Player i 's time-one posterior given that Player $-i$ won his tract. This should not cause confusion. Similarly, in this lemma I use $\Delta_i(\mu_i)$ instead of $\Delta_i(s_i)$ to denote the difference between Player i 's payoff from drilling and her payoff from waiting given that she believes at time one that $V_i = 1$ with probability μ_i .

I first explain how to derive equation 12. Observe that $W_i(s_i)$ can be rewritten as

$$\begin{aligned} W_i(s_i) &= \Pr(V_{-i} = 1, a_{-i} = \text{drill} | s_i, r_{-i} < b_{-i}) \left[\Pr(V_i = 1 | s_i, V_{-i} = 1) - c \right] \\ &+ \Pr(a_{-i} = \text{wait} | s_i, r_{-i} < b_{-i}) \max \left\{ 0, \Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) - c \right\} \\ &+ \Pr(V_{-i} = 0, a_{-i} = \text{drill} | s_i, r_{-i} < b_{-i}) \left[\Pr(V_i = 1 | s_i, V_{-i} = 0) - c \right] \\ &+ \Pr(V_{-i} = 0, a_{-i} = \text{drill} | s_i, r_{-i} < b_{-i}) \left[c - \Pr(V_i = 1 | s_i, V_{-i} = 0) \right] \\ &+ \Pr(a_{-i} = \text{wait} | s_i, r_{-i} < b_{-i}) \left[\Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) - c \right] \\ &- \Pr(a_{-i} = \text{wait} | s_i, r_{-i} < b_{-i}) \left[\Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) - c \right]. \end{aligned}$$

The sum of the first, the third, and the fifth line is equal to $\mu_i - c$. Equation 12 then follows from the fact that the sum of the second and the sixth line equals

$$\Pr(a_{-i} = \text{wait} | s_i, r_{-i} < b_{-i}) \max \{ 0, c - \Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) \}.$$

Using 10, $\Delta_i(\mu_i)$ can be written as

$$\begin{aligned}\Delta_i(\mu_i) &= \mu_i - c - \delta \left[\mu_i \Pr(V_{-i}=1, a_{-i}=drill | V_i=1, r_{-i} < b_{-i})(1-c) - (1-\mu_i) \Pr(V_{-i}=1, a_{-i}=drill | V_i=0, r_{-i} < b_{-i})c \right] \\ &\quad - \delta \max \left\{ 0, \mu_i \Pr(a_{-i}=wait | V_i=1, r_{-i} < b_{-i})(1-c) - (1-\mu_i) \Pr(a_{-i}=wait | V_i=0, r_{-i} < b_{-i})c \right\}.\end{aligned}$$

Let $\mathcal{K}_i(\mu_i) = 1$ if the second term of the right-hand side is positive and $\mathcal{K}_i(\mu_i) = 0$ otherwise. In words, $\mathcal{K}_i(\mu_i) = 1$ if Player i faces a positive payoff from drilling after observing that Player $-i$ waited given that her time-one posterior equals μ_i . One has $\frac{\partial \Delta_i(\mu_i)}{\partial \mu_i} = 1 - \delta \frac{\partial W_i(\mu_i)}{\partial \mu_i}$, where

$$\begin{aligned}\frac{\partial W_i(\mu_i)}{\partial \mu_i} &= \Pr(V_{-i}=1, a_{-i}=drill | V_i=1, r_{-i} < b_{-i})(1-c) + \Pr(V_{-i}=1, a_{-i}=drill | V_i=0, r_{-i} < b_{-i})c \\ &\quad + \mathcal{K}_i(\mu_i) \Pr(a_{-i}=wait | V_i=1, r_{-i} < b_{-i})(1-c) + \mathcal{K}_i(\mu_i) \Pr(a_{-i}=wait | V_i=0, r_{-i} < b_{-i})c.\end{aligned}$$

The right-hand side is maximal when $\mathcal{K}_i(\mu_i) = 1$ in which case $\frac{\partial W_i(\mu_i)}{\partial \mu_i} \in (0, 1)$. Hence, $\frac{\partial \Delta_i(\mu_i)}{\partial \mu_i} > 0$.

Using the second fundamental theorem of calculus, one has

$$\Delta_i(1) = \Delta_i(0) + \int_{\mu_i(0)}^{\mu_i(1)} \frac{\partial \Delta_i(\mu_i)}{\partial \mu_i} \Big|_{\mu_i=x} dx.$$

The lemma then follows from my finding that $\frac{\partial \Delta_i(\mu_i)}{\partial \mu_i} > 0$ and from the fact that $\mu_i(0) < \mu_i(1)$. ■

Proof of Lemma 7

I am left to show that, if signals are sufficiently precise, $\mu_i^-(1) > c$. Using Bayes's law, one has

$$\mu_i^-(1) = \frac{1}{1 + \frac{1-\xi_i(1)}{\xi_i(1)} \frac{\Pr(r_{-i} > b_{-i} | V_i=0)}{\Pr(r_{-i} > b_{-i} | V_i=1)}}. \quad (33)$$

In any undominated strategy, $b_{-i}^*(1) < 1$. As r_{-i} is drawn from a continuous distribution whose upper bound equals one, $\Pr(r_{-i} > b_{-i} | V_i = 1) > 0$. Observe also that if p_i is close to one, $1 - \xi_i(1)$ is close to zero. It then follows from the equation above that if p_i is close to one, $\mu_i^-(1)$ is also close to one. As $c < 1$, there exists a $\hat{p}_1 < 1$ such that $\mu_i^-(1) \geq c$, $\forall p_i \geq \hat{p}_1$. ■

Proof of lemma 8

Observe that $\Delta_i(1; \lambda_{-i}(1) = 1, \delta = 0) > 0$. Recall from Corollary 2 that $\Delta_i(1; \lambda_{-i}(1) = 0, \delta = 1) = 0$. Recall also from Lemma 5 that her payoff from waiting is increasing in the other player's drilling probability $\lambda_{-i}(1)$. Hence, $\Delta_i(1; \lambda_{-i}(1) = \delta = 1) < 0$. As $\Delta_i(s_i)$ is decreasing in δ , there exists a $\hat{\delta}_i < 1$ such that $\Delta_i(1; \lambda_{-i}(1) = 1, \delta = \hat{\delta}_i) = 0$.

Observe that $\hat{\delta}_i$ depends on her beliefs about his bidding strategy. To see this, suppose she believes that $b_{-i}(0) = b_{-i}(1)$. In that case $\Pr(s_{-i} = 1 | s_i, r_{-i} < b_{-i}) = \Pr(s_{-i} = 1 | s_i)$.

If, however, she believes that $b_{-i}(0) = 0 < b_{-i}(1)$, $\Pr(s_{-i} = 1 | s_i, r_{-i} < b_{-i}) = 1$. As tract values are correlated, the higher $\Pr(s_{-i} = 1 | s_i, r_{-i} < b_{-i})$, the higher $\mu_i^+(1)$. A higher value of $\Pr(s_{-i} = 1 | s_i, r_{-i} < b_{-i})$ also increases her payoff from waiting as only type-one players drill at time one. Hence, the value of $\Delta_i(s_i)$, and, by extension, the value of $\hat{\delta}_i$ depend on her beliefs about Player $-i$'s bidding strategy. To make this dependency explicit, I will write $\hat{\delta}_i(b_{-i}(0))$ instead of simply $\hat{\delta}_i$. (The critical discount factor also depends on her beliefs about $b_{-i}(1)$, but, as shown in Lemma 7, if signals are sufficiently precise or if the drilling cost c is sufficiently low, $b_{-i}(1)$ is equal to some constant.) The lemma then follows from the following definitions: let $\hat{\delta}_i \equiv \max\{\hat{\delta}_i(b_{-i}(0)) : b_{-i}(0) \in [0, \frac{1}{2}(\xi_{-i}(1) - c)]\}$, and $\hat{\delta} \equiv \max\{\hat{\delta}_1, \hat{\delta}_2\}$. ■

Proof of Proposition 3

I am left to argue that if $\delta > \max\{\hat{\delta}, \hat{\delta}\}$, there does not exist a non-coordinating equilibrium in which Player i drills with positive probability whenever she is a type-zero player and whenever both players won their tracts. Recall from Lemma 4 that in this case type-one players face a positive payoff from drilling. Hence, conditions (ci) and (cii)—stated at the start of Section 5—are satisfied. Condition (ciii) is satisfied as well as—by construction of $\hat{\delta}$ —type-one players are assumed to possess a positive option value of waiting: If Player $-i$ expects Player i to drill whenever she is a type-one player, he prefers to wait. Observe that such an equilibrium only exists if Player i weakly prefers to drill, i.e. if $\Delta_i(0) \geq 0$. Lemma 6, however, then implies that, if Player i were to possess signal one, she would strictly prefer to drill at time one. As the discount factor is high, Player $-i$ then prefers to wait. Hence, in such a candidate equilibrium, players are coordinating their drilling activities. This behavior, however, is ruled out in a non-coordinating equilibrium. ■

Proof of Proposition 6

Let $TD^{<<}(s_1, s_2; k)$ denote total expected drilling conditional on the event that both players win their tracts,³⁴ conditional on their signals and conditional on the auction design k . If $k = \mathcal{D}$,

$$\begin{aligned}
TD^{<<}(s_1, s_2; \mathcal{D}) &= \lambda_1^*(s_1, s_2; \mathcal{D})\lambda_2^*(s_1, s_2; \mathcal{D})2 + \lambda_1^*(s_1, s_2; \mathcal{D}) \left[1 - \lambda_2^*(s_1, s_2; \mathcal{D})\right] \left[1 + \Pr(V_1 = 1 | s_1, s_2)\right] \\
&+ \lambda_2^*(s_1, s_2; \mathcal{D}) \left[1 - \lambda_1^*(s_1, s_2; \mathcal{D})\right] \left[1 + \Pr(V_2 = 1 | s_1, s_2)\right] \\
&+ \left[1 - \lambda_1^*(s_1, s_2; \mathcal{D})\right] \left[1 - \lambda_2^*(s_1, s_2; \mathcal{D})\right] \left[\mathcal{I}_1^{s_1}(\mathcal{D}) + \mathcal{I}_2^{s_2}(\mathcal{D})\right], \tag{34}
\end{aligned}$$

³⁴The “<<”-superscript refers to the event that $r_i < b_i$ and that $r_{-i} < b_{-i}$.

where $\mathcal{I}_i^{s_i}(\mathcal{D})$ is defined as

$$\mathcal{I}_i^{s_i}(\mathcal{D}) = \begin{cases} 1 & \text{if } \Pr(V_i = 1 | s_i, s_{-i}) \geq c \\ 0 & \text{otherwise.} \end{cases} \quad (35)$$

$TD^{<<}(s_1, s_2; \mathcal{N}\mathcal{D})$ is computed in a similar way except for the following three main differences. First, $\lambda_i^*(s_i = 0, s_{-i}; \mathcal{N}\mathcal{D}) = 0$. Second, $\lambda_i^*(s_i = 1, s_{-i}; \mathcal{N}\mathcal{D})$ is independent of s_{-i} . Third, in case both players wait, Player i drills at time two only if $\Pr(V_i = 1 | s_i, r_{-i} < b_{-i}, a_{-i} = \text{wait}) \geq c$. Total expected drilling in auction k , $TD(k)$, is thus computed as:

$$\begin{aligned} TD(k) &= \sum_{s_1} \sum_{s_2} \Pr(s_1, s_2) \left\{ b_1^*(s_1; k) b_2^*(s_2; k) TD^{<<}(s_1, s_2; k) \right. \\ &\quad \left. + b_1^*(s_1; k) \left[1 - b_2^*(s_2; k) \right] \mathcal{I}_1^{s_1}(k) + b_2^*(s_2; k) \left[1 - b_1^*(s_1; k) \right] \mathcal{I}_2^{s_2}(k) \right\}. \end{aligned} \quad (36)$$

In the above equation, $\mathcal{I}_i^{s_i}(\mathcal{D})$ is defined in Equation 35, while

$$\mathcal{I}_i^{s_i}(\mathcal{N}\mathcal{D}) = \begin{cases} 1 & \text{if } \mu_i^-(s_i) \geq c \\ 0 & \text{otherwise.} \end{cases} \quad (37)$$

Using 36, one has

$$\begin{aligned} \frac{\partial TD(k)}{\partial b_i(s_i)} &= \sum_{s_{-i}} \Pr(s_i, s_{-i}) \left\{ b_{-i}(s_{-i}; k) TD^{<<}(s_i, s_{-i}; k) + \left(1 - b_{-i}(s_{-i}; k) \right) \mathcal{I}_i^{s_i}(k) \right. \\ &\quad \left. - b_{-i}(s_{-i}; k) \mathcal{I}_{-i}^{s_{-i}}(k) \right\} \end{aligned}$$

Observe that $TD^{<<}(s_i, s_{-i}; \mathcal{D}) \geq \mathcal{I}_{-i}^{s_{-i}}(\mathcal{D})$ and that the inequality becomes strict if $\delta < 1$. I am therefore left to show that the partial derivative is non decreasing when $k = \mathcal{N}\mathcal{D}$. Observe that the term between curly brackets is positive when $\mathcal{I}_{-i}^{s_{-i}}(\mathcal{N}\mathcal{D}) = 0$. I now show that that term is non-negative when $\mathcal{I}_{-i}^{s_{-i}}(\mathcal{N}\mathcal{D}) = 1$ and when the discount factor is sufficiently high. Recall that $\mathcal{I}_{-i}^{s_{-i}}(\mathcal{N}\mathcal{D}) = 1 \Leftrightarrow \mu_{-i}^-(s_{-i}) \geq c$. Recall from Corollary 2 that $\lambda_i^*(1; \mathcal{N}\mathcal{D}) = 0$ whenever $\delta = 1$. This implies that

$$\Pr(V_{-i} = 1 | s_{-i}, r_{-i} < b_{-i}, a_{-i} = \text{wait}; \lambda_i^*(1; \mathcal{N}\mathcal{D}) = 0) = \mu_{-i}^+(s_{-i}) > \mu_{-i}^-(s_{-i}) \geq c.$$

As $\lambda_i^*(1; \mathcal{N}\mathcal{D})$ is continuous in δ , for δ high enough the time-two posterior of Player $-i$ remains higher than c . This implies that $TD^{<<}(s_i, s_{-i}; \mathcal{N}\mathcal{D}) \geq \mathcal{I}_{-i}^{s_{-i}}(\mathcal{N}\mathcal{D})$, $\forall s_i, s_{-i}$. Hence, $\frac{\partial TD(k)}{\partial b_i(s_i)} \geq 0$, $\forall i$, $\forall s_i$, and $\forall k$.

I now prove the first statement of the proposition. Suppose that $\Pr(V_1 = 1 | s_1 = 0, s_2 = 1) < c < \Pr(V_1 = 1 | s_1 = 1, s_2 = 0)$. It follows from above and from the explanations provided in the paper that in this case

$$\begin{aligned} \lim_{\delta \rightarrow 1} TD(\mathcal{D}) &= \Pr(s_1 = 1, s_2 = 0) \left\{ b_1(1; \mathcal{D}) b_2(0; \mathcal{D}) \left[1 + \Pr(V_1 = 1 | s_1 = 1, s_2 = 0) \right] + b_1(1; \mathcal{D}) \left(1 - b_2(0; \mathcal{D}) \right) \right\} \\ &\quad + \Pr(s_1 = 0, s_2 = 1) \left\{ b_1(0; \mathcal{D}) b_2(1; \mathcal{D}) \left[1 + \Pr(V_2 = 1 | s_1 = 0, s_2 = 1) \right] + b_2(1; \mathcal{D}) \left(1 - b_1(0; \mathcal{D}) \right) \right\} \\ &\quad + \Pr(s_1 = s_2 = 1) 2. \end{aligned}$$

Recall from Corollary 2 that if $\delta = 1$, no one drills at time one. Hence, if $k = \mathcal{N}\mathcal{D}$, if $(s_i, s_{-i}) = (1, 0)$ and if both players win their tracts, only Player i drills at time two. Therefore,

$$\begin{aligned} \lim_{\delta \rightarrow 1} TD(\mathcal{N}\mathcal{D}) &= \Pr(s_1 = 1, s_2 = 0) \left\{ b_1(1; \mathcal{N}\mathcal{D})b_2(0; \mathcal{N}\mathcal{D}) + b_1(1; \mathcal{N}\mathcal{D}) \left(1 - b_2(0; \mathcal{N}\mathcal{D}) \right) \right\} \\ &+ \Pr(s_1 = 0, s_2 = 1) \left\{ b_1(0; \mathcal{N}\mathcal{D})b_2(1; \mathcal{N}\mathcal{D}) + b_2(1; \mathcal{N}\mathcal{D}) \left(1 - b_1(0; \mathcal{N}\mathcal{D}) \right) \right\} \\ &+ \Pr(s_1 = s_2 = 1)2, \end{aligned}$$

It is immediate that $\lim_{\delta \rightarrow 1} TD(\mathcal{N}\mathcal{D}) < \lim_{\delta \rightarrow 1} TD(\mathcal{D})$. This result, combined with my finding (summarized in Point 2 of Lemma 10 and Point 2 of Lemma 11) that $b_i(s_i; \mathcal{N}\mathcal{D}) \leq b_i(s_i; \mathcal{D})$, and with the insights present in my previous paragraph, proves the first statement of the proposition.

To prove the second statement, consider the following example: Suppose that $p_1 = p_2 = 0.7$, that $\nu = \frac{1}{2}$, that $c = \frac{1}{2}$, that $\delta = 0.9$, and that $\rho^1 = \rho^0 = 1$. As $\Pr(V_1 = 1 | s_1 = 1, s_2 = 0) = \frac{1}{2} = c$, $b_i^*(1; \mathcal{D}) = b_i^*(1; \mathcal{N}\mathcal{D})$. Suppose that if $(s_i, s_{-i}) = (1, 0)$, Player i does not drill in the disclosure case. Then $0 = b_i^*(0; \mathcal{D}) < b_i^*(0; \mathcal{N}\mathcal{D})$. Hence, for c sufficiently close to $\frac{1}{2}$, $R(\mathcal{D}) < R(\mathcal{N}\mathcal{D})$. Furthermore, it can be checked that total expected drilling is 73% higher in the auction without bid disclosure.

I now prove the last statement of the proposition. It follows from Point 2 of Lemma 10 and from Point 1 of Lemma 11 that—if the discount factor is sufficiently high and if c is sufficiently close to $\Pr(V_i = 1 | s_i = s_{-i} = 0)$ —disclosing bids raises revenues. I am left to show that—in this parameter range— $TD(\mathcal{D}) < TD(\mathcal{N}\mathcal{D})$. Suppose $p_1 = p_2$. Observe that if $\delta = 1$ and if $c = \Pr(V_i = 1 | s_i = s_{-i} = 0)$, $\mu_i^-(0) > c$ and $\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}) = \mu_i^+(0) > c$. By continuity, there exists a $\bar{c} > \Pr(V_i = 1 | s_i = s_{-i} = 0)$ and a $\bar{\delta} < 1$ such that $\mu_i^-(0) > c$ and $\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}) > c$ whenever $c \in [\Pr(V_i = 1 | s_i = s_{-i} = 0), \bar{c}]$ and $\delta \in [\bar{\delta}, 1]$. It follows from Equations 20, 21, 22, and 23 that, if $\delta = 1$ and if $c = \Pr(V_i = 1 | s_i = s_{-i} = 0)$, $b_i^*(s_i; \mathcal{D}) = b_i^*(s_i; \mathcal{N}\mathcal{D}) \forall i$ and $\forall s_i$. In particular, $b_i(0; \mathcal{N}\mathcal{D}) = \frac{1}{2}(\xi_i(0) - c) > 0$. Suppose that $c = \Pr(V_i = 1 | s_i = s_{-i} = 0)$ and that if $(s_i, s_{-i}) = (0, 0)$ both players refrain from drilling in the disclosure case. This is without loss of generality as if c were slightly higher, both players strictly prefer no to drill. Hence, if $c = \Pr(V_i = 1 | s_i = s_{-i} = 0)$ and if $\delta = 1$,

$$\begin{aligned} TD(\mathcal{N}\mathcal{D}) &= \Pr(s_1 = s_2 = 0) \left\{ b_1(0; \mathcal{N}\mathcal{D})b_2(0; \mathcal{N}\mathcal{D})TD^{<<}(0, 0; \mathcal{N}\mathcal{D}) + b_1(0; \mathcal{N}\mathcal{D}) \left(1 - b_2(0; \mathcal{N}\mathcal{D}) \right) \right. \\ &\quad \left. + b_2(0; \mathcal{N}\mathcal{D}) \left(1 - b_1(0; \mathcal{N}\mathcal{D}) \right) \right\} + TD(\mathcal{D}). \end{aligned}$$

As $b_i(0; \mathcal{N}\mathcal{D}) > 0$, $TD(\mathcal{D}) < TD(\mathcal{N}\mathcal{D})$ in this case. Observe that—if $c \in [\Pr(V_i = 1 | s_i = s_{-i} = 0), \bar{c}]$ and if $\delta \in [\bar{\delta}, 1]$ —bids are continuous in c , in δ and in the drilling probabilities. Furthermore, in the same parameter range, drilling probabilities are also continuous in δ . Hence, $TD(\mathcal{D}) < TD(\mathcal{N}\mathcal{D})$ whenever $c \in [\Pr(V_i = 1 | s_i = s_{-i} = 0), \bar{c}]$ and if $\delta \in [\bar{\delta}, 1]$. ■

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Appendix B (Not for Publication)

In this Appendix, I prove Propositions 2 and 3 when at least one type-one player faces a negative payoff from drilling when the other player does not win his tract. I also prove Inequality 19 and Equation 25.

Proof of Proposition 2

First, note that $\forall b_{-i}(1) \in [0, 1]$ and $\forall b_{-i}(0) \leq b_{-i}(1)$, $\Delta_i(1; \lambda_{-i}(1) = 1, \delta = 0) > 0$ and $\Delta_i(1; \lambda_{-i}(1) = 1, \delta = 1) < 0$. Let $\hat{\delta}_i(b_{-i}(0), b_{-i}(1)) \in (0, 1)$ denote the critical discount factor such that $\Delta_i(1; \lambda_{-i}(1) = 1) = 0$. In this proof, I redefine $\hat{\delta}_i \equiv \max\{\hat{\delta}_i(b_{-i}(0), b_{-i}(1)) : b_{-i}(1) \in [0, 1], b_{-i}(0) \leq b_{-i}(1)\}$. Let $\hat{\delta} \equiv \max\{\hat{\delta}_1, \hat{\delta}_2\}$. It then follows from lemma 5 that $\forall \delta \in [\hat{\delta}, 1]$ there exists a $\lambda_{-i}^*(1) \in [0, 1]$, such that $\Delta_i(1; \lambda_{-i}(1) = \lambda_{-i}^*(1)) = 0$.

Redefine $h : [0, 1]^2 \rightarrow [0, 1]$ as the probability with which Player one must drill (in case $s_1 = 1$) such that $\Delta_2(1) = 0$ given $(b_1(0), b_1(1))$. Redefine $j : [0, 1]^2 \rightarrow [0, 1]$ with the interpretation that $j(b_2(0), b_2(1))$ represents the probability with which Player two must drill (in case $s_2 = 1$) such that $\Delta_1(1) = 0$.

The equilibrium value $(b_1^*(1), b_2^*(1), b_1^*(0), b_2^*(0), \lambda_1^*(1), \lambda_2^*(1))$ is now computed such that

$$\begin{aligned} b_1^*(1) &= \frac{1}{2}u_1^<(1; b_2^*(0), b_2^*(1)), & b_2^*(1) &= \frac{1}{2}u_2^<(1; b_1^*(0), b_1^*(1)), & b_1^*(0) &= \frac{1}{2}u_1^<(0; b_2^*(0), b_2^*(1), \lambda_2^*(1)), \\ b_2^*(0) &= \frac{1}{2}u_2^<(0; b_1^*(0), b_1^*(1), \lambda_1^*(1)), & \lambda_1^*(1) &= h(b_1^*(0), b_1^*(1)), & \lambda_2^*(1) &= j(b_2^*(0), b_2^*(1)). \end{aligned} \quad (38)$$

Let $S \equiv [0, 1]^6$, and $F \equiv (\frac{1}{2}u_1^<(1), \frac{1}{2}u_2^<(1), \frac{1}{2}u_1^<(0), \frac{1}{2}u_2^<(0), h, j)$. Observe that, if $\delta \in [\hat{\delta}, 1]$, $F : S \rightarrow S$ and that F is continuous in $(b_1(1), b_2(1), b_1(0), b_2(0), \lambda_1(1), \lambda_2(1))$. Existence of a non-coordinating equilibrium thus follows from Brouwer's fixed point theorem. ■

Proof of Proposition 3

In this proof I show that if $\delta \geq \delta'$ the set of simultaneous equations given in 38 possess a unique fixed point. I first state and prove the following lemma.

Lemma 13 *For all i , one has:*

$$\begin{aligned} \frac{\partial \frac{1}{2}u_i^<(0)}{\partial b_{-i}(0)} &\in \left[-\frac{1}{2}, \frac{1}{2}\right], & \frac{\partial \frac{1}{2}u_i^<(1)}{\partial b_{-i}(0)} &\in \left[-\frac{1}{2}, 0\right], & \frac{\partial \frac{1}{2}u_i^<(0)}{\partial b_{-i}(1)} &\in \left[-\frac{1}{2}, \frac{1}{2}\right], \\ \frac{\partial \frac{1}{2}u_i^<(1)}{\partial b_{-i}(1)} &\in \left[0, \frac{1}{2}\right], & \text{and} & & \frac{\partial \frac{1}{2}u_i^<(0)}{\partial \lambda_{-i}(1)} &\in \left[0, \frac{1}{4}\right]. \end{aligned}$$

Proof: Recall that Lemma 4 established that $b_i^*(0) < b_i^*(1)$. Therefore, I define the following sets:

$\Omega_i^1 \equiv \{r_i | r_i < b_i(0)\}$, $\Omega_i^2 \equiv \{r_i | b_i(0) < r_i < b_i(1)\}$, and $\Omega_i^3 \equiv \{r_i | b_i(1) < r_i\}$. As $r_i \sim U[0, 1]$,

$$\frac{\partial \Pr(r_i \in \Omega_i^1)}{\partial b_i(0)} = 1, \quad \frac{\partial \Pr(r_i \in \Omega_i^2)}{\partial b_i(0)} = -1, \quad \text{and} \quad \frac{\partial \Pr(r_i \in \Omega_i^3)}{\partial b_i(0)} = 0, \quad (39)$$

and

$$\frac{\partial \Pr(r_i \in \Omega_i^1)}{\partial b_i(1)} = 0, \quad \frac{\partial \Pr(r_i \in \Omega_i^2)}{\partial b_i(1)} = 1, \quad \text{and} \quad \frac{\partial \Pr(r_i \in \Omega_i^3)}{\partial b_i(1)} = -1. \quad (40)$$

Let

$$\begin{aligned} x_i(0) &\equiv \Pr(V_{-i} = s_{-i} = 1 | s_i = 0) \lambda_{-i}(1) \delta \left[\Pr(V_i = 1 | s_i = 0, V_{-i} = 1) - c \right] \\ &+ \Pr(s_{-i} = 1 | s_i = 0) (1 - \lambda_{-i}(1)) \mathcal{K}_i(0) \delta \left[\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c \right] \\ &- \Pr(s_{-i} = 0 | s_i = 0) \mathcal{K}_i(0) \delta \left[c - \Pr(V_i = 1 | s_i = 0, s_{-i} = 0) \right], \end{aligned} \quad (41)$$

where

$$\mathcal{K}_i(0) = \begin{cases} 1 & \text{if } \Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}) \geq c \\ 0 & \text{otherwise.} \end{cases}$$

Let

$$\begin{aligned} y_i(0) &\equiv \Pr(V_{-i} = s_{-i} = 1 | s_i = 0) \lambda_{-i}(1) \delta \left[\Pr(V_i = 1 | s_i = 0, V_{-i} = 1) - c \right] \\ &+ \Pr(s_{-i} = 1 | s_i = 0) (1 - \lambda_{-i}(1)) \mathcal{K}_i(0) \delta \left[\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c \right] \\ &- \Pr(s_{-i} = 0 | s_i = 0) \mathcal{L}_i(0) \left[c - \Pr(V_i = 1 | s_i = 0, s_{-i} = 0) \right], \end{aligned} \quad (42)$$

where

$$\mathcal{L}_i(0) = \begin{cases} 1 & \text{if } \mu_i^-(0) \geq c \\ 0 & \text{otherwise.} \end{cases}$$

Let

$$\begin{aligned} z_i(0) &\equiv \Pr(s_{-i} = 1 | s_i = 0) \mathcal{L}_i(0) \left[\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c \right] \\ &- \Pr(s_{-i} = 0 | s_i = 0) \mathcal{L}_i(0) \left[c - \Pr(V_i = 1 | s_i = 0, s_{-i} = 0) \right]. \end{aligned} \quad (43)$$

Recall that, in equilibrium, type-zero players wait. Therefore,

$$u_i^<(0) = \Pr(r_{-i} \in \Omega_{-i}^1) x_i(0) + \Pr(r_{-i} \in \Omega_{-i}^2) y_i(0) + \Pr(r_{-i} \in \Omega_{-i}^3) z_i(0). \quad (44)$$

Observe that $b_{-i}(0)$ only influences $u_i^<(0)$ through its effect on $\Pr(r_{-i} \in \Omega_{-i}^l)$ ($l = 1, 2$). Using 39 and 44, one has

$$\begin{aligned} \frac{\partial u_i^<(0)}{\partial b_{-i}(0)} &= x_i(0) - y_i(0) \\ &= -\Pr(s_{-i} = 0 | s_i = 0) \mathcal{K}_i(0) \delta \left[c - \Pr(V_i = 1 | s_i = s_{-i} = 0) \right] \\ &+ \Pr(s_{-i} = 0 | s_i = 0) \mathcal{L}_i(0) \left[c - \Pr(V_i = 1 | s_i = s_{-i} = 0) \right]. \end{aligned}$$

which proves Inequality 19. Observe that $\frac{\partial u_i^<(0)}{\partial b_{-i}(0)} \in [-1, 1]$, which implies that $\frac{1}{2} \frac{\partial u_i^<(0)}{\partial b_{-i}(0)} \in [-\frac{1}{2}, \frac{1}{2}]$.

This proves the first claim of the lemma.

I now tackle the case in which $s_i = 1$. Let

$$\begin{aligned} x_i(1) &\equiv \Pr(s_{-i} = 1 | s_i = 1) \left[\Pr(V_i = 1 | s_i = 1, s_{-i} = 1) - c \right] \\ &\quad + \Pr(s_{-i} = 0 | s_i = 1) \left[\Pr(V_i = 1 | s_i = 1, s_{-i} = 0) - c \right], \end{aligned} \quad (45)$$

and

$$\begin{aligned} y_i(1) &\equiv \Pr(s_{-i} = 1 | s_i = 1) \left[\Pr(V_i = 1 | s_i = 1, s_{-i} = 1) - c \right] \\ &\quad + \Pr(s_{-i} = 0 | s_i = 1) \mathcal{L}_i(1) \left[\Pr(V_i = 1 | s_i = 1, s_{-i} = 0) - c \right], \end{aligned} \quad (46)$$

where

$$\mathcal{L}_i(1) = \begin{cases} 1 & \text{if } \mu_i^-(1) \geq c \\ 0 & \text{otherwise,} \end{cases}$$

As type-one players are indifferent between drilling and waiting, one has:

$$u_i^<(1) = \Pr(r_{-i} \in \Omega_{-i}^1) x_i(1) + \Pr(r_{-i} \in \Omega_{-i}^2) y_i(1) + \Pr(r_{-i} \in \Omega_{-i}^1) \mathcal{L}_i(1) x_i(1), \quad (47)$$

Using 47 and 39, one has:

$$\begin{aligned} \frac{\partial u_i^<(1)}{\partial b_{-i}(0)} &= x_i(1) - y_i(1) \\ &= -\Pr(s_{-i} = 0 | s_i = 1) \left(1 - \mathcal{L}_i(1) \right) \left[c - \Pr(V_i = 1 | s_i = 1, s_{-i} = 0) \right]. \end{aligned} \quad (48)$$

If $\mathcal{L}_i(1) = 0$, $\mu_i^-(1) < c$. As $\Pr(V_i = 1 | s_i = 1, s_{-i} = 0) \leq \mu_i^-(1)$, the partial derivative above is negative. Observe also that the partial derivative is greater than -1 . Hence, $\frac{1}{2} \frac{\partial u_i^<(1)}{\partial b_{-i}(0)} \in [-\frac{1}{2}, 0]$.

This proves the second claim of the lemma.

Using 44 and 40, one has:

$$\begin{aligned} \frac{\partial u_i^<(0)}{\partial b_{-i}(1)} &= y_i(0) - z_i(0) \\ &= \Pr(V_{-i} = s_{-i} = 1 | s_i = 0) \lambda_{-i}(1) \delta \left[\Pr(V_i = 1 | s_i = 0, V_{-i} = 1) - c \right] \\ &\quad + \Pr(s_{-i} = 1 | s_i = 0) (1 - \lambda_{-i}(1)) \mathcal{K}_i(0) \delta \left[\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c \right] \\ &\quad - \Pr(s_{-i} = 1 | s_i = 0) \mathcal{L}_i(0) \delta \left[\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c \right] \in [-1, 1] \end{aligned}$$

Hence, $\frac{1}{2} \frac{\partial u_i^<(0)}{\partial b_{-i}(1)} \in [-\frac{1}{2}, \frac{1}{2}]$. This proves the third claim of the lemma.

Using 47 and 40, one has:

$$\begin{aligned} \frac{\partial u_i^<(1)}{\partial b_{-i}(1)} &= y_i(1) - \mathcal{L}_i(1) x_i(1) \\ &= \Pr(s_{-i} = 1 | s_i = 1) \left(1 - \mathcal{L}_i(1) \right) \left[\Pr(V_i = 1 | s_i = s_{-i} = 1) - c \right] \in [0, 1]. \end{aligned}$$

Hence, $\frac{1}{2} \frac{\partial u_i^<(1)}{\partial b_{-i}(1)} \in [0, \frac{1}{2}]$. This proves the fourth claim of the lemma.

Using 44, one has

$$\frac{\partial u_i^<(0)}{\partial \lambda_{-i}(1)} = \Pr(r_{-i} \in \Omega_{-i}^1) \frac{\partial x_i(0)}{\partial \lambda_{-i}(1)} + \Pr(r_{-i} \in \Omega_{-i}^2) \frac{\partial y_i(0)}{\partial \lambda_{-i}(1)}.$$

Using 41 and 42, one has

$$\begin{aligned} \frac{\partial x_i(0)}{\partial \lambda_{-i}(1)} &= \Pr(V_{-i} = s_{-i} = 1 | s_i = 0) \delta \left[\Pr(V_i = 1 | s_i = 0, V_{-i} = 1) - c \right] \\ &\quad - \Pr(s_{-i} = 1 | s_i = 0) \mathcal{K}_i(0) \delta \left[\Pr(V_i = 1 | s_i = 0, s_{-i} = 1) - c \right] \\ &= \frac{\partial y_i(0)}{\partial \lambda_{-i}(1)} \geq 0, \end{aligned}$$

where the inequality follows from Lemma 5. As $\Pr(r_{-i} \in \Omega_{-i}^1) + \Pr(r_{-i} \in \Omega_{-i}^2) = b_{-i}(1)$, as $b_{-i}(1) \in [0, \frac{1}{2}]$, and as $\frac{\partial x_i(0)}{\partial \lambda_{-i}(1)} \in [0, 1]$, I conclude that $\frac{\partial \frac{1}{2} u_i^<(0)}{\partial \lambda_{-i}(1)} \in [0, \frac{1}{4}]$. This proves the fifth claim of the lemma. ■

Suppose $\mathbf{x}' \equiv (b_1(1)', b_2(1)', b_1(0)', b_2(0)', \lambda_1(1)', \lambda_2(1)')$ and $\mathbf{x}'' \equiv (b_1(1)'', b_2(1)'', b_1(0)'', b_2(0)'', \lambda_1(1)'', \lambda_2(1)'')$ are two different fixed points of the simultaneous equations summarized in 38.

Using the second fundamental theorem of calculus, one has:

$$\begin{aligned} b_1(1)'' &= b_1(1)' + \int_{b_2(0)'}^{b_2(0)''} \frac{\partial \frac{1}{2} u_1^<(1; b_2(1) = b_2(1)')}{\partial b_2(0)} \Big|_{b_2(0)=x} dx \\ &\quad + \int_{b_2(1)'}^{b_2(1)''} \frac{\partial \frac{1}{2} u_1^<(1; b_2(0) = b_2(0)'')}{\partial b_2(1)} \Big|_{b_2(1)=x} dx, \tag{49} \\ b_2(1)'' &= b_2(1)' + \int_{b_1(0)'}^{b_1(0)''} \frac{\partial \frac{1}{2} u_2^<(1; b_1(1) = b_1(1)')}{\partial b_1(0)} \Big|_{b_1(0)=x} dx + \int_{b_1(1)'}^{b_1(1)''} \frac{\partial \frac{1}{2} u_2^<(1; b_1(0) = b_1(0)'')}{\partial b_1(1)} \Big|_{b_1(1)=x} dx, \\ b_1(0)'' &= b_1(0)' + \int_{b_2(0)'}^{b_2(0)''} \frac{\partial \frac{1}{2} u_1^<(0; b_2(1) = b_2(1)', \lambda_2(1) = \lambda_2(1)')}{\partial b_2(0)} \Big|_{b_2(0)=x} dx \\ &\quad + \int_{b_2(1)'}^{b_2(1)''} \frac{\partial \frac{1}{2} u_1^<(0; b_2(0) = b_2(0)'', \lambda_2(1) = \lambda_2(1)')}{\partial b_2(1)} \Big|_{b_2(1)=x} dx \\ &\quad + \int_{\lambda_2(1)'}^{\lambda_2(1)''} \frac{\partial \frac{1}{2} u_1^<(0; b_2(0) = b_2(0)'', b_2(1) = b_2(1)'')}{\partial \lambda_2(1)} \Big|_{\lambda_2(1)=x} dx, \\ b_2(0)'' &= b_2(0)' + \int_{b_1(0)'}^{b_1(0)''} \frac{\partial \frac{1}{2} u_2^<(0; b_1(1) = b_1(1)', \lambda_1(1) = \lambda_1(1)')}{\partial b_1(0)} \Big|_{b_1(0)=x} dx \\ &\quad + \int_{b_1(1)'}^{b_1(1)''} \frac{\partial \frac{1}{2} u_2^<(0; b_1(0) = b_1(0)'', \lambda_1(1) = \lambda_1(1)')}{\partial b_1(1)} \Big|_{b_1(1)=x} dx \\ &\quad + \int_{\lambda_1(1)'}^{\lambda_1(1)''} \frac{\partial \frac{1}{2} u_2^<(0; b_1(0) = b_1(0)'', b_1(1) = b_1(1)'')}{\partial \lambda_1(1)} \Big|_{\lambda_1(1)=x} dx, \\ \lambda_1(1)'' &= \lambda_1(1)' + \int_{b_1(0)'}^{b_1(0)''} \frac{\partial h(b_1(1) = b_1(1)')}{\partial b_1(0)} \Big|_{b_1(0)=x} dx + \int_{b_1(1)'}^{b_1(1)''} \frac{\partial h(b_1(0) = b_1(0)'')}{\partial b_1(1)} \Big|_{b_1(1)=x} dx, \\ \lambda_2(1)'' &= \lambda_2(1)' + \int_{b_2(0)'}^{b_2(0)''} \frac{\partial j(b_2(1) = b_2(1)')}{\partial b_2(0)} \Big|_{b_2(0)=x} dx + \int_{b_2(1)'}^{b_2(1)''} \frac{\partial j(b_2(0) = b_2(0)'')}{\partial b_2(1)} \Big|_{b_2(1)=x} dx, \tag{50} \end{aligned}$$

Define κ_i ($i = 1, \dots, 14$) such that

$$\begin{aligned}
\int_{b_2(0)'}^{b_2(0)''} \frac{\partial \frac{1}{2} u_1^<(1)}{\partial b_2(0)} \Big|_{b_2(0)=x} dx &\equiv -\kappa_1 [b_2(0)'' - b_2(0)'] \\
\int_{b_2(1)'}^{b_2(1)''} \frac{\partial \frac{1}{2} u_1^<(1)}{\partial b_2(1)} \Big|_{b_2(1)=x} dx &\equiv \kappa_2 [b_2(1)'' - b_2(1)'] \\
\int_{b_1(0)'}^{b_1(0)''} \frac{\partial \frac{1}{2} u_2^<(1)}{\partial b_1(0)} \Big|_{b_1(0)=x} dx &\equiv -\kappa_3 [b_1(0)'' - b_1(0)'] \\
\int_{b_1(1)'}^{b_1(1)''} \frac{\partial \frac{1}{2} u_2^<(1)}{\partial b_1(1)} \Big|_{b_1(1)=x} dx &\equiv \kappa_4 [b_1(1)'' - b_1(1)'] \\
\int_{b_2(0)'}^{b_2(0)''} \frac{\partial \frac{1}{2} u_1^<(0)}{\partial b_2(0)} \Big|_{b_2(0)=x} dx &\equiv -\kappa_5 [b_2(0)'' - b_2(0)'] \\
\int_{b_2(1)'}^{b_2(1)''} \frac{\partial \frac{1}{2} u_1^<(0)}{\partial b_2(1)} \Big|_{b_2(1)=x} dx &\equiv \kappa_6 [b_2(1)'' - b_2(1)'] \\
\int_{\lambda_2(1)'}^{\lambda_2(1)''} \frac{\partial \frac{1}{2} u_1^<(0)}{\partial \lambda_2(1)} \Big|_{\lambda_2(1)=x} dx &\equiv \kappa_7 [\lambda_2(1)'' - \lambda_2(1)'] \\
\int_{b_1(0)'}^{b_1(0)''} \frac{\partial \frac{1}{2} u_2^<(0)}{\partial b_1(0)} \Big|_{b_1(0)=x} dx &\equiv -\kappa_8 [b_1(0)'' - b_1(0)'] \\
\int_{b_1(1)'}^{b_1(1)''} \frac{\partial \frac{1}{2} u_2^<(0)}{\partial b_1(1)} \Big|_{b_1(1)=x} dx &\equiv \kappa_9 [b_1(1)'' - b_1(1)'] \\
\int_{\lambda_1(1)'}^{\lambda_1(1)''} \frac{\partial \frac{1}{2} u_2^<(0)}{\partial \lambda_1(1)} \Big|_{\lambda_1(1)=x} dx &\equiv \kappa_{10} [\lambda_1(1)'' - \lambda_1(1)'] \\
\int_{b_1(0)'}^{b_1(0)''} \frac{\partial h}{\partial b_1(0)} \Big|_{b_1(0)=x} dx &\equiv \kappa_{11} [b_1(0)'' - b_1(0)'] \\
\int_{b_1(1)'}^{b_1(1)''} \frac{\partial h}{\partial b_1(1)} \Big|_{b_1(1)=x} dx &\equiv \kappa_{12} [b_1(1)'' - b_1(1)'] \\
\int_{b_2(0)'}^{b_2(0)''} \frac{\partial j}{\partial b_2(0)} \Big|_{b_2(0)=x} dx &\equiv \kappa_{13} [b_2(0)'' - b_2(0)'] \\
\int_{b_2(1)'}^{b_2(1)''} \frac{\partial j}{\partial b_2(1)} \Big|_{b_2(1)=x} dx &\equiv \kappa_{14} [b_2(1)'' - b_2(1)']
\end{aligned}$$

Lemma 13 allows me to conclude that:

COROLLARY 4 $\kappa_l \in [0, \frac{1}{2}]$ ($l \in \{1, 2, 3, 4\}$), $\kappa_m \in [-\frac{1}{2}, \frac{1}{2}]$ ($m \in \{5, 6, 8, 9\}$), $\kappa_n \in [0, \frac{1}{4}]$ ($n \in \{7, 10\}$).

Recall from Corollary 2 that, if $\delta = 1$, $\lambda_i(1) = 0 \forall (b_i(0), b_i(1))$. Hence, if $\delta = 1$, $\kappa_o = 0$ ($o \in \{11, 12, 13, 14\}$). By continuity, for δ close to one, κ_o is close to zero. It is important to realize that the above κ 's are no constants. In particular, some κ 's depend on the value taken by $\mathcal{L}_i(1)$,

$\mathcal{L}_i(0)$ or $\mathcal{K}_i(0)$, which, in its turn, depends on the (endogenous) bids and drilling probabilities. Lemma 13, however, implies that— independent of the value taken by those indicator functions— all κ 's are small, a crucial property which I will use below. All those κ 's allow me to rewrite Equations 49 to 50 as:

$$\Delta b_1(1) = -\kappa_1 \Delta b_2(0) + \kappa_2 \Delta b_2(1) \quad (51)$$

$$\Delta b_2(1) = -\kappa_3 \Delta b_1(0) + \kappa_4 \Delta b_1(1) \quad (52)$$

$$\Delta b_1(0) = -\kappa_5 \Delta b_2(0) + \kappa_6 \Delta b_2(1) + \kappa_7 \Delta \lambda_2(1) \quad (53)$$

$$\Delta b_2(0) = -\kappa_8 \Delta b_1(0) + \kappa_9 \Delta b_1(1) + \kappa_{10} \Delta \lambda_1(1) \quad (54)$$

$$\Delta \lambda_1(1) = \kappa_{11} \Delta b_1(0) + \kappa_{12} \Delta b_1(1) \quad (55)$$

$$\Delta \lambda_2(1) = \kappa_{13} \Delta b_2(0) + \kappa_{14} \Delta b_2(1), \quad (56)$$

where $\Delta r \equiv r'' - r'$ ($r \in \{b_1(1), b_2(1), b_1(0), b_2(0), \lambda_1(1), \lambda_2(1)\}$). Inserting 56 into 53 and 55 into 54, allows me to rewrite Equations 53 and 54 as:

$$\Delta b_1(0) = (\kappa_7 \kappa_{13} - \kappa_5) \Delta b_2(0) + (\kappa_6 + \kappa_7 \kappa_{14}) \Delta b_2(1) \quad (57)$$

$$\Delta b_2(0) = (\kappa_{10} \kappa_{11} - \kappa_8) \Delta b_1(0) + (\kappa_9 + \kappa_{10} \kappa_{12}) \Delta b_1(1) \quad (58)$$

Inserting 58 and 52 into 51, and rearranging, one has:

$$\Delta b_1(1) = -\frac{\kappa_2 \kappa_3 + \kappa_1 (\kappa_{10} \kappa_{11} - \kappa_8)}{1 + \kappa_1 (\kappa_9 + \kappa_{10} \kappa_{12}) - \kappa_2 \kappa_4} \Delta b_1(0). \quad (59)$$

Inserting 58 and 52 into 57, and rearranging, one has:

$$\Delta b_1(0) = \frac{(\kappa_7 \kappa_{13} - \kappa_5)(\kappa_9 + \kappa_{10} \kappa_{12}) + \kappa_4 (\kappa_6 + \kappa_7 \kappa_{14})}{1 + \kappa_3 (\kappa_6 + \kappa_7 \kappa_{14}) - (\kappa_7 \kappa_{13} - \kappa_5)(\kappa_{10} \kappa_{11} - \kappa_8)} \Delta b_1(1). \quad (60)$$

Finally, inserting 60 into 59, I conclude that $\Delta b_1(1) = -\alpha \beta \Delta b_1(1)$, where

$$\alpha \equiv \frac{\kappa_2 \kappa_3 + \kappa_1 (\kappa_{10} \kappa_{11} - \kappa_8)}{1 + \kappa_1 (\kappa_9 + \kappa_{10} \kappa_{12}) - \kappa_2 \kappa_4}, \text{ and}$$

$$\beta \equiv \frac{(\kappa_7 \kappa_{13} - \kappa_5)(\kappa_9 + \kappa_{10} \kappa_{12}) + \kappa_4 (\kappa_6 + \kappa_7 \kappa_{14})}{1 + \kappa_3 (\kappa_6 + \kappa_7 \kappa_{14}) - (\kappa_7 \kappa_{13} - \kappa_5)(\kappa_{10} \kappa_{11} - \kappa_8)}.$$

Suppose that the discount factor δ is close to one. As argued above, this implies that α is close to α' and β close to β' , where

$$\alpha' \equiv \frac{\kappa_2 \kappa_3 - \kappa_1 \kappa_8}{1 + \kappa_1 \kappa_9 - \kappa_2 \kappa_4}, \text{ and}$$

$$\beta' \equiv \frac{-\kappa_5 \kappa_9 + \kappa_4 \kappa_6}{1 + \kappa_3 \kappa_6 - \kappa_5 \kappa_8}.$$

Using Corollary 4, one can check that—for all values of my endogenous variables— $\alpha' \in [-\frac{1}{3}, \frac{2}{3}]$ and that $\beta' \in [-1, 1]$. Hence, I conclude that— independent of the values of my endogenous

variables— $\alpha'\beta' \neq -1$. By continuity, this implies that there exists a $\delta' < 1$ such that $\alpha\beta \neq -1$, $\forall \delta \geq \delta'$. This implies that $\Delta b_1(1)$ must be equal to zero to be a solution of the simultaneous equations 51 - 56. Using 60, I conclude that $\Delta b_1(0) = 0$. Equation 52 then implies that $\Delta b_2(1) = 0$. Equation 51 then implies that $\Delta b_2(0) = 0$. Equations 55 and 56 then imply that $\Delta \lambda_1(1) = \Delta \lambda_2(1) = 0$. This, however, contradicts my assumption that \mathbf{x}' and \mathbf{x}'' are two different fixed points of the simultaneous equations given by 38. ■

Proof of Equation 25

Let $\mathcal{L}_i(s_i) = 1$ if $\mu_i^-(s_i) \geq c$ and zero otherwise. Let $\mathcal{L}_i^+(1) = 1$ if $\mu_i^+(1) \geq c$ and zero otherwise. Recall that $\mathcal{K}_i(0) = 1$ if $\Pr(V_i = 1 | s_i = 0, r_{-i} < b_{-i}, a_{-i} = \text{wait}) \geq c$ and zero otherwise.

Define welfare in auction k as

$$W(k) \equiv \sum_{s_i} \sum_{s_{-i}} \Pr(s_i, s_{-i}) \left[u_i^0(s_i, s_{-i}; k) + u_{-i}^0(s_i, s_{-i}; k) \right],$$

where

$$u_i^0(s_i, s_{-i}; k) \equiv b_i^*(s_i; k) \left[u_i^<(s_i, s_{-i}; k) - b_i^*(s_i; k) \right],$$

and where the interim payoff of a type-one player, conditional on the other player's signal and on auction design k , is defined as:

$$\begin{aligned} u_i^<(s_i = 1, s_{-i}; \mathcal{D}) &\equiv \max \left\{ 0, \Pr(V_i = 1 | s_i = 1, s_{-i}) - c \right\}, \\ u_i^<(s_i = 1, s_{-i}; \mathcal{ND}) &\equiv \Pr(r_{-i} < b_{-i} | s_{-i}) \mathcal{L}_i^+(1) \left[\Pr(V_i = 1 | s_i = 1, s_{-i}) - c \right] \\ &\quad + \Pr(r_{-i} > b_{-i} | s_{-i}) \mathcal{L}_i(1) \left[\Pr(V_i = 1 | s_i = 1, s_{-i}) - c \right]. \end{aligned}$$

The interim payoff of a type-zero player, conditional on s_{-i} and on auction design k , is defined as:

$$\begin{aligned} u_i^<(s_i = 0, s_{-i}; \mathcal{D}) &\equiv \Pr(r_{-i} < b_{-i} | s_{-i}) \left[\mathcal{I}_i(0, s_{-i}) \delta \bar{W}_i(0, s_{-i}) \right. \\ &\quad \left. + \left(1 - \mathcal{I}_i(0, s_{-i}) \right) \max \left\{ 0, \Pr(V_i = 1 | s_i = 0, s_{-i}) - c \right\} \right] \\ &\quad + \Pr(r_{-i} > b_{-i} | s_{-i}) \max \left\{ 0, \Pr(V_i = 1 | s_i = 0, s_{-i}) - c \right\} \\ u_i^<(s_i = 0, s_{-i}; \mathcal{ND}) &\equiv \Pr(r_{-i} < b_{-i} | s_{-i}) \delta \Pr(V_{-i} = 1, a_{-i} = \text{drill} | s_i = 0, s_{-i}) \left[\Pr(V_i = 1 | s_i = 0, V_{-i} = 1) - c \right] \\ &\quad + \Pr(r_{-i} < b_{-i} | s_{-i}) \delta \Pr(a_{-i} = \text{wait} | s_{-i}) \mathcal{K}_i(0) \delta \left[\Pr(V_i = 1 | s_i = 0, s_{-i}) - c \right] \\ &\quad + \Pr(r_{-i} > b_{-i} | s_{-i}) \mathcal{L}_i(0) \left[\Pr(V_i = 1 | s_i = 0, s_{-i}) - c \right]. \end{aligned}$$

Observe that

$$\sum_{s_{-i}} \Pr(s_i, s_{-i}) u_i^0(s_i, s_{-i}; k) = \Pr(s_i) u_i^0(s_i; k).$$

Hence, $W(k) = \sum_i \sum_{s_i} \Pr(s_i) u_i^0(s_i; k)$. ■