Bidding and Drilling on Offshore Wildcat Tracts *

Nicolas Melissas†

January 25, 2014

Abstract

A simple model is presented in which firms first participate in an offshore oil and gas auction and next decide whether and when to drill. If the discount factor is close to one, there essentially exists a unique equilibrium in which a "low" bid may signal an unwillingness to drill early. This induces the other player to drill early. In turn, this induces some types to strategically bid low. If players are sufficiently patient, disclosing bids increases the quality of the drilling decision and it also increases revenues. Disclosing bids, however, may reduce total drilling.

JEL-codes: D44, D82, C72, Q49

Keywords: Information externality, signaling, auctions, oil exploration

*This paper benefited from comments by Roberto Burguet, Estelle Cantillon, Dan Kovenock, Vijay Krishna, Paul Heidhues, Sergei Izmalkov, David McAdams, Richard McLean, Claudio Mezzetti, Marshall Rose and Tom Wiseman. I am also grateful to the editor George-Marios Angeletos and to three anonymous referees for their many helpful suggestions. I gratefully acknowledge financial support from the Consejo Nacional de Ciencia y Tecnología (grant #79741) and from the Asociación Mexicana de Cultura, A.C..

†Centro de Investigación Económica, Instituto Tecnológico Autónomo de México, Camino a Santa Teresa 930, México D.F. 10700, Mexico. E-mail: nicolas.melissas@itam.mx.

1 Introduction

In the last decade the price of oil increased from 25 USD per barrel to around 100 USD. This induced many companies to look for new oil (and gas) deposits. Oil exploration programs are often initiated by private firms which acquired the right to explore and extract oil (and gas) after participating in an auction. In this paper I analyze incentives to bid and to drill on *offshore* oil and gas tracts. Offshore oil and gas auctions are interesting to study for many reasons. First, a lot of oil is extracted out of the seabed: Yergin (2011) documents that production from offshore oil wells in 2011 accounts for about 30% of total world production. Second, a lot of money is involved as a bid can easily exceed several million dollars. Third, offshore oil and gas auctions are widely used: Brazil, Colombia, Cuba, Greenland, Kenya, Libya, Nigeria, Russia, and the U.S., for example, organized such auctions in the past decade. Other countries—most notably Mexico—are likely to follow.

In a seminal paper Hendricks and Porter (HP, 1996) argued that offshore drilling suffers from a public good problem: If a firm drills and finds oil, this is costlessly observed by other firms. They next showed that the U.S. drilling data is consistent with the idea that—on average—firms do *not* coordinate their drilling decisions, i.e. firms typically play a war of attrition to determine who drills first. Comparing different auctions in this context is thus a delicate matter as the post-auction war of attrition should influence bidding behavior and vice versa. Furthermore, ranking auctions in terms of their expected revenues does not suffice. Policy makers should also care about total drilling and about efficiency in drilling. For one can either increase the discovery rate by drilling more (i.e. increasing the probability that any given tract is drilled) or by drilling better (i.e. increasing the probability that any given tract contains oil conditional on it being drilled). After all, increasing the discovery rate should reduce oil prices and a country's dependence on imported oil.

To analyze the interaction between bidding and drilling, I build a simple bidding-drilling game which enables me to answer the following questions: Does equilibrium behavior in my game shed additional light on HP's empirical findings? Can bids sometimes be used as a coordination device? If so, how? Does the auction format influence the efficiency of the drilling decision? Should bids be disclosed? To be more specific, I consider a set-up in which two players possess private information (i.e. the result of a seismic test) about the value of a tract they are interested in acquiring. Private information is either favorable or unfavorable. In the first stage, both players participate in an auction. After this first stage, both players might end up owning tracts that are relatively close to each other. In that case they play a waiting game to determine who drills first: Players can drill in two periods, if Player i drills at time one, both players observe the value of Tract i. Tract values are assumed to be "very" (though not perfectly) correlated: If Player $-i$ learns that Player i drills a dry hole, his payoff from drilling becomes negative—even if he possesses a favorable signal. Similarly, if Player i finds oil, Player $-i$'s payoff from drilling becomes positive—even if he possesses an unfavorable signal. Waiting thus yields an informational benefit but comes at the cost of discounting. I consider two different auction designs. First in Subsection 3.4 the auctioneer is assumed to follow a minimal-disclosure policy, i.e. Player i is only informed about whether she wins her tract or not. The second auction design closely matches the one used by the U.S. government: A sealed-bid first-price auction followed by bid disclosure, i.e. after the auction, but prior to the first drilling date, the auctioneer discloses both players' bids.

If the auctioneer follows a minimal-disclosure policy, there exists a unique symmetric equilibrium in which drilling decisions are very inefficient. In particular, players may drill too little (i.e. Player i may not drill despite the fact that—based on both player's signals—it yields a positive expected payoff) or too much. These inefficiencies have originally been documented by Hendricks and Kovenock (1989).

In the second auction design, which is analyzed in Subsections 3.6 and 3.7, players are restricted to using non-coordinating strategies. A strategy is said to be non-coordinating if both players—whenever possible—play a war of attrition to determine who drills first. The qualification "whenever possible" must be added for the following reason: As tract values are imperfectly correlated, at the start of the waiting game both players need not possess the same posterior probability of finding oil. Player i and Player $-i$ may thus respectively face a positive and negative payoff from drilling early. As Player i knows that Player $-i$ does not drill, she prefers to drill at time one. Successful coordination of drilling activities, however, only arises in this situation. In particular, if both players face a positive payoff from drilling, they play a (possibly asymmetric) war of attrition, i.e. Player i drills with some probability to make Player $-i$ indifferent between drilling and waiting. Two main results emerge. First, I prove that if the discount factor is sufficiently high, any equilibrium outcome lies within an epsilon-neighborhood of the one generated by the separating equilibrium. (This is summarized in Propositions 3 and 4. In the separating equilibrium players with unfavorable private information bid "low" while those with favorable private information bid "high". A player's bid thus perfectly reveals her type in this case.) Second, underbidding may occur: Players with unfavorable private information may bid less than what they would have bid had the auctioneer disclosed signals instead of bids. Intuitively, those types underbid because by doing so they signal that they face a negative payoff from drilling at time one. As explained above, this induces the other player to drill at time one. Players thus sometimes use past bids to coordinate their drilling activities. As stressed in Proposition 3, however, successful coordination of drilling activities only occurs if the drilling cost is intermediate and if one player bids "high" while the other one bids "low".

In Section 4, I argue that disclosing bids improves the quality of the drilling decision and that it also generates more revenues. This is intuitive: As players use invertible bidding strategies, Player i's drilling decision is based on two signals in the bid-disclosure case. In the minimal-disclosure case, however, her drilling decision is only based on one signal. Furthermore if Players i and $-i$ respectively face a positive and negative payoff from drilling early, disclosing bids actually implements the most efficient drilling profile: Player i drills while Player $-i$ waits. The effect of disclosing bids on revenues is ambiguous when the discount factor is not "high". On the one hand it increases revenues precisely because it improves the quality of the drilling decision. The prospect of successful coordination, however, induces players with unfavorable private information to underbid. Nevertheless, if the discount factor is sufficiently high it is optimal to disclose bids. For in the auction with minimal disclosure, players with unfavorable private information only drill in case the other player finds oil at time one—a very unlikely event when the discount factor is close to one. In the limit—i.e. as the discount factor approaches one—players with unfavorable private information actually bid zero in the minimal-disclosure case. Finally, I argue that the effect of bid disclosure on total drilling is ambiguous.

HP developed a bidding-drilling model in which different types are assumed to bid differently. I provide sufficient conditions for such a bidding behavior to arise endogenously. HP assumed that players *always* play a war of attrition in the waiting game. I show that the waiting game may be characterized by a unique equilibrium in which player use bids to coordinate their drilling activities. This sheds some additional light on some of their empirical findings. In particular, they showed that a tract is more likely to be drilled at the start than during the middle of her lease term. Part of this finding is due to the fact that firms use bids to coordinate their drilling plans.[1]

This is not the first paper to analyze an auction as part of a larger market interaction. Haile (2000), considers a game in which players can resell after the auction took place. He shows that the possibility of reselling affects bids in two opposing ways. On the one hand, some types have an incentive to overbid in order to extract seller's surplus at the reselling stage. On the other hand, some types have an incentive to underbid to extract buyer's surplus at the reselling stage. In general, either effect can dominate and an English auction (followed by resale) does not necessarily yield higher expected revenues than a second-, or a first-price one (followed by resale). Goeree (2003) and Das Varma (2003) analyze an auction followed by some downstream

[1] As mentioned in HP, Firm i may prefer to drill some tract early because she may possess a lot of other nearby-situated tracts. She may also prefer to drill early because she does not possess any "neighbor" with whom to start a war of attrition.

interaction among all players. If after the auction players compete à la Cournot, there exists a separating equilibrium in which overbidding occurs. A separating equilibrium may fail to exist if the auction-stage is followed by Bertrand competition. If it does exist, players underbid. Haile, Goeree and Das Varma restrict attention to separating equilibria. In contrast, I also check for semi-separating and pooling equilibria. Avery (1998) studies an English auction in which players "jump bid" to signal that their valuations lie above some threshold level and to select an asymmetric continuation equilibrium. I show that bidders in offshore oil and gas auctions behave similarly: A low bid signals a low valuation and may select an equilibrium in which the high bidder drills while the low bidder waits. Nonetheless, in Avery's model it are the bidders with high valuations who signal their types and jump bidding reduces revenues. In my model, the low types underbid and—as it implements an efficient drilling pattern—it actually increases revenues.

In an elegant model, Bolton and Harris (1999) analyze an N-player, continuous-time, two-armed bandit problem. At every moment in time, each player must decide whether to play the safe or the risky arm of her slot machine. Players are uncertain whether the risky arm outperforms the safe one or not. Players observe past choices and past payoffs. Because of the information externality, players experiment less with the risky arm than what would be socially optimal. Interestingly, the authors show that a player may experiment with the risky arm to induce other players to experiment with theirs. There is a subtle difference between Bolton and Harris's set-up and mine. In their model, if a player chooses the safe arm, no information is provided about the profitability of the risky arm. In my model if a player decides to wait, she partially reveals that she possesses unfavourable private information. Hence, in my model players have to choose between a less informative action (waiting) and a more informative one (drilling).

The empirical evidence on social learning mainly supports the view that people experiment too little. HP and Farrow and Rose (1992) present evidence that firms choose their offshore drilling activities non-cooperatively. Foster and Rosenzweig (1995) analyze the adoption of high-yield variety crops in India and also argue that farmers choose their adoption strategies non-cooperatively. In a study on adoption rates of high-yield variety cotton in an Indian village, Besley and Case (1994) cannot tell whether farmers choose their adoption strategies cooperatively or non-cooperatively.

2 Some institutional features

In this section, I enumerate several important institutional features of the U.S. offshore leasing program. This will help the reader to understand the game I analyze in my next section. I focus on wildcat tracts. Such a tract is situated in an offshore geographical area where no exploratory drilling has occurred in the past. Tracts that are situated next to already developed ones are called drainage tracts. Hendricks and Porter (1988) showed that firms possess an informational advantage over the value of a neighboring tract. In contrast, no firm should possess superior information about the value of a wildcat tract.

Feature 1. (Solo bidding) A bid is a dollar figure that the firm must pay if it wins her tract. Firms submit their bids simultaneously. Firms bid on a small subset of the tracts offered for sale. For example, between 1998 and 2005 (inclusive) the U.S. government organized 22 auctions. On average 3,145 tracts were offered in each one of them. On average only 305 of them received at least one bid.[2] Hence, in those auctions the number of tracts offered for sale by far exceeds total demand. As a result of this, few of the tracts offered for sale receive more than one bid. Summed over all those 22 auctions, for example, 6,705 tracts received at least one bid and 5,255 received exactly one bid. Stated differently, conditional on the event that a tract received at least one bid, there is a 78.4% probability that that tract received only one bid.[3]

Feature 2. (Random reservation price) If a tract happens to possess only one bid, the U.S. government decides whether or not to reject the bid. To do so, it estimates the "fair market value" of the tract. Henceforth, this fair-market-value estimate will be called the (government's) reservation price. A tract which received only one bid is sold if the bid exceeds the reservation price. The reservation price is computed after all bids were submitted. Hence, ex-ante bidders don't know what the realization of the reservation price will be. This insight, combined with my earlier finding that few tracts receive more than one bid, indicate that a player's bidding strategy is primarily determined by her desire to "beat" the reservation price rather than to "beat" a hypothetical competing bid. So far, only Hendricks, Porter and Spady (HPS, 1989) analyzed the government's rejection decision on offshore tracts. They focussed on drainage and development tracts that were sold during the period 1959 - 1979. Unfortunately, wildcat tracts were not included in their sample. They found that the rejection decision on drainage tracts was positively correlated with a tract's size, with the average wellhead price of offshore oil and with the identity of the highest bidder (i.e. the government was more likely to reject a given

[2]Source: own computations based on data taken from http://www.boemre.gov/econ/econHIST.htm.

[3]Solo bidding, however, has not always been the norm in OCS auctions. In particular, Hendricks, Porter and Boudreau (1987) documented that Pr(tract i receives only one bid|tract i receives at least one bid) was approximately 32% for wildcat auctions held during the period 1954-1969. Solo bidding led to a dramatic fall in the average bid. Prior to 1983 the mean winning bid was around 15 million dollars. Nowadays it is well below 1 million dollars. Haile, Hendricks and Porter (2010) provide interesting information about those auctions since they were first held in 1954 until 2002.

high bid submitted by a neighbor firm than by a non-neighbor one). The rejection decision was also negatively correlated with the value of the highest bid. The decision, however, was *not* significantly correlated with the amount of oil extracted nor with the bidding history of the neighboring tract. As the reservation price on drainage tracts did not depend on the expected quantity of oil nor on the neighbors' bids, there is no reason to assume that the contrary situation would prevail on wildcat tracts.

Feature 3. (Bid disclosure) After firms submitted their bids, but before the first drilling date, the government releases the identity of all bidders along with their bids.

Feature 4. (War of attrition) After winning her tract, a firm is given five years to initiate an exploratory drilling program. If after five years it has not drilled its tract, its lease expires and the tract is returned to the government which may decide to resell it in some future auction. The tracts are usually smaller than the size of the deposits. For example, Lin (2009) documents that the largest petroleum field in the Gulf of Mexico spans 23 tracts. Depending on water depth, 57% to 67% of all productive tracts had to share their deposits with at least one neighboring firm. Furthermore, the costs of drilling an exploratory well are not trivial. According to Forbes and Zampelli (2000) in 1996 the average exploratory well in the Gulf of Mexico had a depth of 11,203 feet (3,414 meters) and cost 3.3 million USD. This cost dramatically increases with well depth: A 15,000 feet (4,572 meters) exploratory well cost 10 million USD. Drilling in Alaskan waters is even more expensive: Yergin (1991), for example, documents that the most expensive dry hole to date was build in Mukluk in 1983 and cost over 2 billion dollars! Given those facts, one would expect firms to play a waiting game, i.e. a firm has an incentive to postpone its exploratory drilling in the hope that its neighbor drills first. This plausible strategic behavior is consistent with the available empirical evidence. HP documented that the hazard rate of drilling (i.e. the probability to drill at time t given that the tract has not been drilled before) features a U-shaped pattern. A tract is most likely to be drilled at the start or at the end of her lease term. In years 2, 3 and 4, however, the hazard rate is significantly lower. If a firm drills its tract during the final year of her lease, this indicates that it must hold sufficiently optimistic beliefs about her prospects of finding oil. The fact that it postponed its drilling decision indicates that there was a positive option value of waiting. A plausible explanation behind this option value of waiting is that the firm hoped to learn from its neighbor's drilling outcomes. Furthermore, HP also found that the probability to drill during the second and the third year of the lease term is positively influenced by the number of past successful drilling outcomes in the same area-cohort.

3 The Model

3.1 Set-up

A seller offers two offshore tracts in two simultaneously-held, first-price, sealed-bid auctions. Two risk-neutral players are interested in acquiring one of those two tracts. Both players have thus unit demand and I assume that there is only one bidder per tract. The unit-demand assumption captures the presumption that firms cannot bid on all tracts offered for sale. (Recall that in the period 1998-2005 on average 3,145 tracts were simultaneously offered for sale!) This can be due to financing constraints, to a limited refining capacity, to a bottleneck in the supply of drilling rigs or because of risk-aversion. Assuming one bidder per tract captures Feature 1. It also implies that Player i's only strategic motive at the bidding stage arises from her desire to influence the other player's incentives to drill early. (More on this below.)

$V_i \in \{0,1\}$ denotes the value of Tract i ($i = 1,2$). If $V_i = 1$, Tract i is said to posses oil (and gas). If $V_i = 0$, Tract i does not possess any oil nor gas. It is assumed that $\Pr(V_i = 1) = \nu \in (0,1)$ $\forall i$. Tract values are correlated. In particular, it is assumed that $\Pr(V_i = 0|V_{-i} = 0) = \rho^0 \in (1-\nu, 1]$ and that $\Pr(V_i = 1|V_{-i} = 1) = \rho^1 \in (\nu, 1]$. If ρ^0 is close to $1-\nu$ and if ρ^1 is close to ν, tract values are almost uncorrelated. This is most realistic when the distance between both tracts is very large. If $\rho^0 = \rho^1 = 1$, my model boils down to a common value problem. This corresponds to the case when both tracts are neighbors who share a common pool. For intermediate values of ρ^0 and ρ^1, tract values are imperfectly correlated. In general, one would expect ρ^0 and ρ^1 to decrease with the distance between both tracts.

Player i receives an informative but imperfect signal $s_i \in \{0,1\}$ concerning the value of her tract. Both players possess an equally informative signal. Formally, Player i receives the correct signal with probability p, i.e. $\Pr(s_i = 1|V_i = 1) = \Pr(s_i = 0|V_i = 0) = p \in \left(\frac{1}{2}, 1\right)$. Signals are (conditionally) independent. I denote the common drilling cost by c. ξ^{s_i} denotes Player i's posterior probability that $V_i = 1$ conditional on her[4] signal s_i, i.e. $\xi^{s_i} \equiv \Pr(V_i = 1|s_i)$. Henceforth, a player who receives signal zero (one) is called a type-zero (type-one) player. I assume that

Assumption 1 $\xi^0 \leq c \leq \xi^1$.

The first inequality implies that, *absent any additional favorable information about* V_i, type-zero players refrain from drilling. The second inequality implies that, in the absence of any additional negative information, type-one players eventually drill. Tract values are assumed to be "sufficiently" correlated. Formally, I assume that

[4]Henceforth, I assume that Player i is female while Player $-i$ is male.

Assumption 2 $\Pr(V_i = 1 | s_i = 1, V_{-i} = 0) < c < \Pr(V_i = 1 | s_i = 0, V_{-i} = 1)$.

In words, Assumption 2 ensures that my model possess a meaningful information externality: A negative drilling outcome on Tract $-i$ completely removes Player i's incentives to explore her tract, even if she is a type-one player. Conversely, a positive drilling outcome on Tract $-i$ induces Player i to explore her tract, even if she is a type-zero player.

The government's reservation price for both tracts is denoted by r. As both tracts are ex ante identical, it is reasonable to assume that both tracts possess the same reservation price. To capture Feature 2, and to simplify computations, I assume that r is drawn from a uniform distribution with support $[0, 1]$. Let b_i denote the bid of Player i. I consider the following sequencing of events:

-1 Nature draws V_i, V_{-i}, s_i, s_{-i} and the reservation price r.

0 Player One bids on Tract One, Player Two bids on Tract Two.

$\frac{1}{2}$ The auctioneer announces b_i, b_{-i} and which tracts are sold.[5]

1 If $\max\{b_i, b_{-i}\} < r$, both players get a zero payoff and the game ends. If $b_{-i} < r < b_i$, Player i drills at time one if her payoff from drilling—conditional on her signal and on his bid—is positive and the game ends. If both players win their tracts, they decide whether to drill or wait.

2 Player i observes the first-period decision of Player $-i$. If he drilled, Player i also observes V_{-i}. If Player i waited, she decides whether to drill or not. Players receive their payoffs and the game ends.

Besides her signal, Player i thus gets three additional pieces of information about the value of her tract. First, she observes the other player's bid. If, for example, the other player bids high, she may infer that probably $V_{-i} = 1$. As both tracts are correlated, this makes her more "optimistic" about her chances of finding oil. Second, she observes the other player's time-one action. If, for example, the other player waits at time one, she may infer that probably $s_{-i} = 0$. In turn, this may reduce her incentives to drill at time two. Third, if the other players drills at time one she observes the value of the other tract.

Henceforth, I refer to the continuation game that starts at time one when both players win their tracts as the *waiting game*. By waiting, Player i may learn the value of V_{-i} but, of course,

[5] In reality, the reservation price for tracts with a rejected high bid can be downloaded (after some years) from the Bureau of Ocean Energy Management, Regulation and Enforcement website (which is the division in the Department of the Interior responsible for organizing those auctions). The reservation price for tracts with an accepted high bid, however, is not made public.

any payoff she gets at time two is discounted. Let $\delta < 1$ denote the common discount factor. As documented by HP, the relevant time period in my model is about three months. (This is the time that elapses between building an exploratory well and finding out whether sufficient quantities of oil are present to warrant production.) In any reasonable calibration of my model, one should thus work with a very high discount factor. Suppose, for example, that oil firms face an annual opportunity cost of 10%. This corresponds to a three-monthly discount factor of approximately 0.976. In what follows, I will thus restrict attention to the case when the discount factor δ is close to one.

Prior to this waiting game, both players observe each other's bids. Let $F_e^{s_i}(b)$ denote the probability that Player-type[6] i^{s_i} submits a bid $b_i \leq b$ in equilibrium e. Let $f_e^{s_i}(b) \equiv \frac{\partial F_e^{s_i}}{\partial b}$. Player i's knowledge of $F_e^1(b)$ and $F_e^0(b)$ enables her to compute $\Pr_e(b_{-i} = b | s_{-i})$, the probability that Player $-i$ submits bid b given his signal s_{-i} and given that players focus on equilibrium e. Suppose b lies on the equilibrium path of e, which implies that either $\frac{\Pr_e(b_{-i}=b|s_{-i}=1)}{\Pr_e(b_{-i}=b|s_{-i}=0)} \neq \frac{0}{0}$ or that $\frac{f_e^1(b)}{f_e^0(b)} \neq \frac{0}{0}$. Let $l_{-i}(b) \in [0, \infty)$ denote the likelihood that bid b is submitted by a type-one player in equilibrium e, i.e.

$$l_{-i}(b) = \begin{cases} \frac{\Pr_e(b_{-i}=b|s_{-i}=1)}{\Pr_e(b_{-i}=b|s_{-i}=0)} & \text{if } \frac{\Pr_e(b_{-i}=b|s_{-i}=1)}{\Pr_e(b_{-i}=b|s_{-i}=0)} \neq \frac{0}{0}, \text{ and} \\ \frac{f_e^1(b)}{f_e^0(b)} & \text{otherwise.} \end{cases}$$

Thus, if $l_{-i}(b) = 0$, Player i infers that $s_{-i} = 0$ with probability one. If $l_{-i}(b) = \infty$, Player i infers that $s_{-i} = 1$ with probability one. If $l_{-i}(b) = 1$, bid b is as likely to have been submitted by Player-type $-i^1$ as by Player-type $-i^0$. In that case, $\Pr(s_{-i} = 1|s_i, b_{-i} = b) = \Pr(s_{-i} = 1|s_i)$. In what follows, and with a slight abuse of notation, I will often refer to $l_{-i}(b)$ as l_{-i}.

3.2 Equilibrium

Throughout the paper, I solve for sequential equilibria. I also restrict attention to the class of symmetric and *non-coordinating* strategies. Loosely speaking, a strategy is said to be symmetric if players with identical private information and with identical histories behave identically. Suppose both players possess signal one and that they both submitted the same bid. The symmetry restriction then implies that both players must drill at time one with the same probability. Stated differently, in this case both players play a *symmetric* war of attrition in which Player i drills with some probability to make Player $-i$ indifferent. Suppose now that both players possess signal one but that they submitted different bids. Suppose Player one believes that Player two is a type-one player with probability 50% while Player two believes that Player one is a type-one

[6] I use the term Player-type i^{s_i} to refer to Player i when her signal s_i takes a particular value. For example, Player-type 1^1 refers to player one is she possesses favorable information about V_1.

player with probability 50.001%. The symmetry restriction is then vacuous.[7] My restriction of non-coordinating strategies, however, obliges me to focus on the *asymmetric* war of attrition in which both players drill with some probability to make each other indifferent between drilling and waiting. Formally, consider a continuation equilibrium ce in my waiting game. Let Z^{ce} denote the set of player-types who are indifferent between drilling and waiting in the continuation equilibrium ce. A continuation equilibrium ce is said to be non-coordinating if there does not exist another continuation equilibrium (say ce') with the property that $|Z^{ce'}| > |Z^{ce}|$. The real world is obviously not symmetric. Yet, as stressed in Feature 4, firms engage in a war of attrition. In this light, it is reasonable to restrict attention to non-coordinating strategies. In their analysis of offshore drilling, HP also assume that players focus on the (possibly asymmetric) mixed-strategy Nash equilibrium in the waiting game. A sequential equilibrium is said to be non-coordinating if its strategies prescribe both players to focus on a non-coordinating continuation equilibrium in the waiting game.

I also restrict attention to the class of *monotone* bidding strategies. Suppose b and b' are bids that lie *on* the equilibrium path. Suppose $b < b'$. A bidding strategy is said to be monotone if a higher bid is weakly more likely to have been submitted by a type-one player, i.e. if $l_{-i}(b) \leq l_{-i}(b')$. In the auction-theoretic literature, bidding strategies are typically assumed to be strictly increasing.[8] In this two-type model, this is tantamount to restricting attention to separating equilibria in which $l_{-i}(b) = 0$ and $l_{-i}(b') = \infty$. I don't want to assume away any possible semi-separating or pooling equilibrium. Hence, I allow both $l_{-i}(b)$ and $l_{-i}(b')$ to lie in $(0, 1)$ and I allow $l_{-i}(b)$ to be equal to $l_{-i}(b')$. Suppose Player $-i$ submits a bid *off* the equilibrium path. I assume that Player i's out-of-equilibrium beliefs then satisfy Cho and Kreps's (1987) intuitive criterion.[9] Henceforth, I will simply refer to a sequential, symmetric, non-coordinating equilibrium with monotone bidding strategies and intuitive out-of-equilibrium beliefs as a *non-coordinating equilibrium*.

3.3 The waiting game

The waiting game need not be symmetric. Player i for example (after observing b_{-i}) may believe that Player $-i$ is a type-one player with probability 10%, while Player $-i$ may believe that

[7] My symmetry restriction would also be vacuous if, for example, one player were to possess a slightly more informative signal.

[8] See Krishna (2002) for an excellent introduction to this literature.

[9] In Melissas (2014), I actually prove that my specified out-of-equilibrium beliefs also satisfy Banks and Sobel's (1987) divinity criterion.

Player i is a type-one player with probability one. In Melissas (2014), I characterize the set of the non-coordinating continuation equilibria for all (l_i, l_{-i}). Fortunately, to intuitively grasp the main results of this paper, one need not understand how to compute all possible continuation equilibria for every possible history. A few results suffice.

Let $\mu^{s_i}(l_{-i}) \equiv \Pr(V_i = 1|s_i, l_{-i})$ denote the time-one posterior of Player i given her signal s_i and given the likelihood ratio l_{-i}. It is straightforward to check that $\mu^{s_i}(l_{-i})$ is increasing in l_{-i}: The higher l_{-i}, the higher the probability that Player $-i$ possesses signal one; as tract values are correlated this increases the probability that Tract i contains oil.

Let $a_i \in \{drill, wait\}$ denote the time-one action of Player i. Let $W^{s_i}\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right)$ denote Player i's undiscounted payoff from waiting given that both players win their their tracts, given her signal s_i, given l_{-i} and given that he drills with probability λ^0_{-i} if he is a type-zero player and with probability λ^1_{-i} if $s_{-i} = 1$. Formally,

$$\begin{aligned} W^{s_i}\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right) &= \Pr(V_{-i} = 1, a_{-i} = drill|s_i, l_{-i}) \Big[\Pr(V_i = 1|s_i, V_{-i} = 1) - c\Big] \\ &+ \Pr(a_{-i} = wait|s_i, l_{-i}) \max\Big\{0, \Pr(V_i = 1|s_i, l_{-i}, a_{-i} = wait) - c\Big\}. \end{aligned} \tag{1}$$

Intuitively, with probability $\Pr(V_{-i} = 1, a_{-i} = drill|\cdot)$ the other player drills at time one and finds oil. This induces Player i to drill at time two, and she then gets an undiscounted payoff equal to $\Pr(V_i = 1|s_i, V_{-i} = 1) - c$. In case the other player drills and doesn't find oil, she refrains from drilling and gets zero. If the other player waits at time one, she drills at time two only if it is profitable for her to do so.

Suppose that

$$\delta W^{s_i}(l_{-i}, 0, 0) \leq \mu^{s_i}(l_{-i}) - c \leq \delta W^{s_i}(l_{-i}, 0, 1).$$

The first inequality holds only if $0 \leq \mu^{s_i}(l_{-i}) - c$. For if the payoff from drilling were negative, Player i would strictly prefer to wait. Furthermore, if $0 \leq \mu^{s_i}(l_{-i}) - c$ and if $\left(\lambda^0_{-i}, \lambda^1_{-i}\right) = (0, 0)$, Player i does not learn anything about (V_{-i}, s_{-i}) by waiting. She just incurs a discounting cost. Unsurprisingly, she then prefers to drill. The second inequality implies that $l_{-i} > 0$ and that the discount factor δ is sufficiently high. Intuitively, waiting is costly when the discount factor δ is low and she then prefers to drill; if $l_{-i} = 0$, Player i infers that $s_{-i} = 0$ and, thus, that he waits. As waiting is costly, she then also prefers to drill. If $l_{-i} > 0$ and if δ is sufficiently high, however, waiting is "cheap" and it might enable her to learn that $V_{-i} = 0$ and, thus, that drilling is not such a good idea after all.

In Melissas (2014) I show that—under those inequalities—$W^{s_i}\left(l_{-i}, 0, \lambda^1_{-i}\right)$ is increasing in λ^1_{-i}: The higher λ^1_{-i}, the greater the probability that Player i will learn the value of Tract $-i$. (An increase in λ^1_{-i} also allows Player i to be more confident that $s_{-i} = 0$ in case he waits.) This increases her payoff from waiting as she then takes a better informed drilling decision at time two. Using the intermediate value theorem and the fact that $W^{s_i}(\cdot)$ is increasing in λ^1_{-i}, I conclude that there exists a unique $\lambda^*_{-i} \in [0,1]$ such that $\mu^{s_i}(l_{-i}) - c = \delta W^{s_i}\left(l_{-i}, 0, \lambda^*_{-i}\right)$.

The equality between her payoff from drilling and the one from waiting can also be stated differently. Formally, my last equality is equivalent to[10]

$$
\begin{aligned}
(1-\delta)\left(\mu^{s_i}(l_{-i}) - c\right) &= \delta \Pr(V_{-i}=0, s_{-i}=1|s_i, l_{-i})\lambda^*_{-i}\left[c - \Pr(V_i=1|s_i, V_{-i}=0)\right] \\
&+ \delta \Pr(a_{-i}=wait|s_i, l_{-i}) \max\left\{0, c - \Pr(V_i=1|s_i, l_{-i}, a_{-i}=wait)\right\}.
\end{aligned} \tag{2}
$$

The left-hand-side of the above equation represents Player i's opportunity cost of waiting. The right-hand side represents her discounted *informational benefit* of waiting. From my previous paragraph, we know that her informational benefit of waiting is increasing in λ^1_{-i}. The right-hand side reveals that Player i has two good reasons to wait. I already discussed the first one above: With probability $\Pr(V_{-i}=0, s_{-i}=1|s_i, l_{-i})\lambda^*_{-i}$ Player $-i$ drills a dry hole. Player i then refrains from drilling and avoids an expected loss equal to $\delta\left(c - \Pr(V_i=1|s_i, V_{-i}=0)\right)$. Second, with probability $\Pr(a_{-i}=wait|s_i, l_{-i})$ Player $-i$ waits. As type-zero players are more likely to wait, this is bad news and leads to a downward revision of Player i's posterior probability of finding oil. In case $\delta\left(c - \Pr(V_i=1|s_i, l_{-i}, a_{-i}=wait)\right)$ is positive, Player i then refrains from drilling and avoids this expected loss.

To illustrate the waiting game, suppose that $(\nu, p, c, \delta) = (0.5, 0.6, 0.5, 0.95)$ and that both tract values are almost perfectly correlated, i.e. that $\rho^0 = \rho^1 = 1 - \epsilon$ where ϵ is an arbitrarily small number. This implies that $\Pr(V_i=1|s_i=1, V_{-i}=0)$ is approximately equal to zero and that $\Pr(V_i=1|s_i=0, V_{-i}=1)$ is approximately equal to one. I will often use this example in my next subsections. Suppose also that $l_i = l_{-i} = 1$, i.e. no information about a player's type is revealed through her bid. If $s_i = 0$, she computes a time-one posterior $\mu^0(1) = \xi^0 = 1 - p < c$. As type-zero players face a negative payoff from drilling, they wait. Suppose now that $s_i = 1$. Her time-one posterior is then equal to $\mu^1(1) = \xi^1 = p = 0.6$. Her opportunity cost of waiting is then equal to $(1-0.95)(0.6-0.5)$. Suppose both players wait. Her time-two posterior probability $\Pr(V_i=1|s_i=1, l_{-i}=1, a_{-i}=wait)$ is then bounded below by $\mu^1(0)$ which is slightly above

[10] This is easy to check. Adding and subtracting $\Pr\left(V_{-i}=0, a_{-i}=drill|s_i, l_{-i}\right)\left[\Pr(V_i=1|s_i, V_{-i}=0) - c\right]$ and $\Pr\left(a_{-i}=wait|s_i, l_{-i}\right)\max\left\{0, c - \Pr(V_i=1|s_i, l_{-i}, a_{-i}=wait)\right\}$ to the right-hand side of Equation 1, and rearranging, allows me to rewrite her payoff from waiting as:

$$
\begin{aligned}
W^{s_i}(l_{-i}, 0, \lambda^*_{-i}) &= \mu^{s_i}(l_{-i}) - c + \Pr(V_{-i}=0, a_{-i}=drill|s_i, l_{-i})\left[c - \Pr(V_i=1|s_i, V_{-i}=0)\right] \\
&+ \Pr(a_{-i}=wait|s_i, l_{-i})\max\left\{0, c - \Pr(V_i=1|s_i, l_{-i}, a_{-i}=wait)\right\}.
\end{aligned}
$$

$\nu = c = \frac{1}{2}$.[11] Using (2), her informational benefit of waiting is thus approximately equal to

$$\delta \underbrace{\Pr\left(V_{-i} = 0, s_{-i} = 1 | s_i = 1, l_{-i} = 1\right)}_{=(1-p)^2} \lambda^*_{-i}[c - 0] = 0.95 \times (0.4)^2 \times \lambda^*_{-i} \times 0.5.$$

Her opportunity cost of waiting is thus equal to her informational benefit from waiting if λ^*_{-i} is approximately equal to 6.6%.

3.4 Benchmark case: minimal disclosure

Consider the following slightly different auction design: At time $\frac{1}{2}$ the auctioneer only announces whether or not Player i wins her tract. Bids are kept secret. Nor does the auctioneer reveal whether or not Player $-i$ wins his tract. The rest of the game stays the same. I now argue that there exists an equilibrium in which type-zero players submit b^0_{md} with probability one, in which type-one players bid $\frac{1}{2}\left(\xi^1 - c\right)$ $\left(> b^0_{md}\right)$ with probability one, in which type-zero players drill with probability zero at time one and in which Player-type i^1 drills with probability λ° such that $\xi^1 - c = \delta W^1\left(1, 0, \lambda^\circ\right)$ if she wins her tract.

To compute Player i's optimal bid, I must first compute her expected payoff gross of her bid and conditional on the event that she wins her tract. Henceforth, I refer to this expected payoff as her *interim payoff*. Consider Player-type i^0. Suppose she wins her tract. As she does not infer any information about Player $-i$'s type at time $\frac{1}{2}$, her time-one posterior probability of finding oil remains equal to ξ^0. Recall from Assumption 1 that $\xi^0 < c$. As she faces a negative payoff from drilling, she waits at time one, i.e. $\lambda^0_i = 0$. (As a matter of fact, she only drills at time two if the other player finds oil at time one.)

I now compute Player-type i^1's interim payoff given that $\lambda^0_{-i} = 0$. Suppose she wins her tract after having submitted a bid $b \leq \frac{1}{2}\left(\xi^1 - c\right)$. As she does not learn anything about s_{-i}, her time-one posterior is equal to ξ^1. If she drills at time one, she thus gets (gross of her bid) $\xi^1 - c$. As both tracts possess the same reservation price r, she knows that if the other player possesses signal one, he also wins his tract. Formally,

$$\Pr\left(r < \frac{1}{2}\left(\xi^1 - c\right) \middle| s_i = 1, r < b\right) = 1 \qquad \forall b \leq \frac{1}{2}\left(\xi^1 - c\right).$$

[11] As the discount factor is close to one, type-one players wait with positive probability. Thus observing $a_{-i} = wait$ is not as bad as observing $s_{-i} = 0$. Thus, $\Pr(V_i = 1 | s_i = 1, l_{-i} = 1, a_{-i} = wait) \geq \mu^1(0)$. To understand why the latter probability is above ν, suppose that both tract values are perfectly correlated, i.e. that $\rho^0 = \rho^1 = 1$. Her posterior probability $\mu^1(0)$ is then equal to ν: In this case she possesses two contradictory pieces of information about the realization of the random variable V_i. Suppose now that tracts are imperfectly correlated. As Player i's signal s_i is more informative about the realization of V_i than s_{-i}, the bad news embodied in the statistic $s_{-i} = 0$ does not completely cancel out the good news embodied in the statistic $s_i = 1$.

Note that not observing b_{-i} is equivalent to observing b_{-i} but to update one's posteriors beliefs using $l_{-i}(b_{-i}) = 1 \ \forall b_{-i}$. If she waits she thus gets a gross payoff equal to $\delta W^1(1, 0, \lambda^\circ)$. By construction of λ°, her payoff from waiting is equal to the one she gets if she drills at time one. Hence, her interim payoff if she submits a bid $b \leq \frac{1}{2}\left(\xi^1 - c\right)$ is equal to $\xi^1 - c$. Her interim payoff is also equal to $\xi^1 - c$ when she bids $b > \frac{1}{2}\left(\xi^1 - c\right)$. To see this, observe that

$$\Pr\left(r < \frac{1}{2}\left(\xi^1 - c\right) \middle| s_i = 1, r < b\right) < 1 \qquad \forall b > \frac{1}{2}\left(\xi^1 - c\right).$$

For with some probability $\frac{1}{2}\left(\xi^1 - c\right) < r < b$ in which case only Player i wins her tract. Recall that λ° is computed such that Player-type i^1 is indifferent between drilling and waiting given that she receives no additional information about his type and given that she expects Player-type $-i^1$ to also win his tract *with probability one.* Hence, Player-type i^1's payoff from waiting is lower after winning her tract with a bid $b > \frac{1}{2}\left(\xi^1 - c\right)$ than with a bid $b \leq \frac{1}{2}\left(\xi^1 - c\right)$. In turn, this implies that if she wins her tract with a bid $b > \frac{1}{2}\left(\xi^1 - c\right)$, she strictly prefers to drill.

At time zero, she thus chooses b to maximize $U^1(b) \equiv \Pr(r < b)\left(\xi^1 - c - b\right)$. As $r \sim U[0, 1]$, her objective function can be rewritten as $U^1(b) = b(\xi^1 - c - b)$. In words, $U^1(b)$ represents Player-type i^1's *unconditional and net* expected payoff if she bids b. It follows from the first-order condition that her optimal bid is $\frac{1}{2}\left(\xi^1 - c\right)$.

Recall that type-zero players wait at time one. Suppose Player-type i^0 submits a bid $b \leq \frac{1}{2}\left(\xi^1 - c\right)$ and that she wins her tract. As both tracts possess the same reservation price r, she knows that Player-type $-i^1$ also wins his tract. She thus gets an interim payoff equal to $\delta W^0(1, 0, \lambda^\circ)$. In Melissas (2014), I show that $\delta W^0(1, 0, \lambda^\circ) < \xi^1 - c$. This is intuitive: λ° is computed to make type-one players indifferent between drilling and waiting. As type-zero players are less confident that both V_{-i} and s_{-i} are equal to one, their payoff from waiting is lower. Observe that if she were to submit a bid $b > \frac{1}{2}\left(\xi^1 - c\right)$ her interim payoff would be lower for the same reason as the one I detailed two paragraphs above. Hence, Player-type i^0's optimal bid is equal to $\frac{1}{2}\delta W^0(1, 0, \lambda^\circ) \equiv b^0_{md}$. In Melissas (2014), I show that this equilibrium is unique within the class of the symmetric strategies. Summarizing:

PROPOSITION 1 *Suppose the auctioneer follows a minimal-disclosure policy. Type-one players then bid* $b^1_{md} = \frac{1}{2}\left(\xi^1 - c\right)$, *while type-zero players bid* $b^0_{md} = \frac{1}{2}\delta W^0(1, 0, \lambda^\circ) < b^1_{md}$. *Type-zero players wait while type-one players drill with probability* λ° *at time one.*

Consider the numerical example of my previous subsection, i.e. $(\nu, p, c, \delta) = (0.5, 0.6, 0.5, 0.95)$ and $\rho^0 = \rho^1 = 1 - \epsilon$. Recall that type-one players drill with a probability approximately equal to 6.6% in this example. It follows from above that type-one players bid $\frac{1}{2}(0.6 - 0.5) = 5$ cents.

Suppose $s_i = 0$. From above, we know that her interim payoff is approximately equal to

$$\begin{aligned} \delta W^0\left(1,0,\lambda_{-i}^* \simeq 6.6\%\right) &\simeq \delta \Pr(V_{-i} = s_{-i} = 1|s_i = 0) 6.6\%(1-c) \\ &= 0.95 \times 0.6 \times 0.4 \times 6.6\% \times 0.5 \simeq 0.74\%. \end{aligned}$$

Type-zero players thus only bid 0.37 cents in this example! This is intuitive: As the discount factor is very high, the opportunity cost of waiting of type-one players is very low. To be indifferent between drilling and waiting they therefore only drill with probability 6.6%. Type-zero players thus know that—most probably—the other player does not drill at time one. In turn, this gives them little incentives to purchase their tracts.

3.5 Benchmark case: disclosure of signals

In this section, I assume that the auctioneer discloses both players' *signals* instead of their bids. The auctioneer also discloses which player won her tract.

Recall that tract values are imperfectly correlated. Recall also that $\mu^0(\infty)$ represents the time-one posterior of a type-zero player who learns that Player $-i$ possesses signal one. Similarly, $\mu^1(0)$ represents the time-one posterior of a type-one player who learns that Player $-i$ possesses signal zero. If tract values were perfectly correlated, $\mu^0(\infty) = \mu^1(0)$. In this case both players receive two contradictory pieces of information about the realization of the *same* random variable and, thus, compute the same posterior. (As explained in Footnote 11, both posteriors then actually coincide with the prior ν.) If tract values are imperfectly correlated, however, $\mu^0(\infty) < \nu < \mu^1(0)$. This is also intuitive: Player i's signal s_i is more informative about the realization of V_i than s_{-i}. The drilling cost c is said to be low if $c \in \left(\xi^0, \mu^0(\infty)\right)$. The drilling cost c is said to be intermediate if $\mu^0(\infty) < c < \mu^1(0)$. The drilling cost is said to be high if $c \in \left(\mu^1(0), \xi^1\right)$.

Henceforth, I denote by $\bar{\lambda}$ the value of λ_{-i}^1 which equates $\mu^1(\infty) - c$ with $\delta W^1\left(\infty, 0, \lambda_{-i}^1\right)$. In words, $\bar{\lambda}$ represents the probability which makes Player-type i^1 indifferent between drilling and waiting after she inferred that Player $-i$ also possesses signal one. Similarly, $\underline{\lambda}$ denotes the probability which makes Player-type i^0 indifferent after inferring that Player $-i$ possesses signal one. Formally, $\underline{\lambda}$ represents the value of λ_{-i}^1 which equates $\mu^0(\infty) - c$ with $\delta W^0\left(\infty, 0, \lambda_{-i}^1\right)$ whenever $0 \leq \mu^0(\infty) - c$. Finally, $\lambda_\circ$ represents the probability which makes Player-type i^1 indifferent after inferring that Player $-i$ possesses signal zero. Formally, $\lambda_\circ$ denotes the value of λ_{-i}^0 which equates $\mu^1(0) - c$ with $\delta W^1\left(\infty, \lambda_{-i}^0, 0\right)$ whenever $0 \leq \mu^1(0) - c$.

I now explain time-one drilling behavior in case both players win their tracts. I need to consider three different possibilities: Either $(s_i, s_{-i}) = (1,1)$, or $(s_i, s_{-i}) = (1,0)$, or $(s_i, s_{-i}) = (0,0)$. In case $(s_i, s_{-i}) = (1,1)$, both players drill with probability $\bar{\lambda}$ to make each other indifferent between drilling and waiting. In case $(s_i, s_{-i}) = (0,0)$, both players face a negative payoff from drilling and no one ever drills. More interestingly, suppose that $(s_i, s_{-i}) = (1, 0)$. If, additionally, the drilling cost is high, both players face a negative payoff from drilling and no one ever drills. If the drilling cost is intermediate, Player i faces a positive payoff from drilling while Player $-i$ doesn't. Any strategy which prescribes Player $-i$ to drill with probability $\lambda^0_{-i} > 0$ is therefore dominated. Anticipating this, it is a best response for Player i to drill with probability one. If the drilling cost is low, both players face a positive payoff from drilling at time one. There exists then a unique non-coordinating continuation equilibrium in which Player i drills with probability $\underline{\lambda}$ and in which Player $-i$ drills with probability $\lambda_\circ$.

Suppose $s_i = 1$. I now compute her interim payoff u^1_{sd} and her bid b^1_{sd}.[12] Obviously, if the other player does not win his tract, she drills at time one if it is profitable for her to do so. She thus gets a gross payoff equal to $\max\left\{0, \mu^1(l_{-i}) - c\right\}$ where $l_{-i} = \infty$ if $s_{-i} = 1$ and where $l_{-i} = 0$ if $s_{-i} = 0$. Thus, suppose the other players also wins his tract. She then also gets a gross payoff equal to $\max\left\{0, \mu^1(l_{-i}) - c\right\}$. In the light of my previous paragraph, this is intuitive: If $s_{-i} = 1$ she is indifferent between drilling and waiting; If $s_{-i} = 0$ she either never drills (in case $\mu^1(0) < c$), or she strictly prefers to drill (in case $\mu^0(\infty) < c < \mu^1(0)$) or she is indifferent between drilling and waiting (in case $c < \mu^0(\infty)$). Hence,

$$u^1_{sd} = \sum_{s_{-i}} \Pr\left(s_{-i}|s_i = 1\right) \max\left\{0, \Pr(V_i = 1|s_i = 1, s_{-i}) - c\right\}. \qquad (3)$$

At time zero, she thus chooses b to maximize $b\left(u^1_{sd} - b\right)$. The solution to this problem is $b^1_{sd} = \frac{1}{2}u^1_{sd}$. Observe that if the drilling cost is not high (i.e. if $c < \mu^1(0)$) her interim payoff boils down to

$$\begin{aligned} u^1_{sd} &= \Pr\left(s_{-i} = 0|s_i = 1\right)\left[\mu^1(0) - c\right] + \Pr\left(s_{-i} = 1|s_i = 1\right)\left[\mu^1(\infty) - c\right] \\ &= \xi^1 - c, \end{aligned}$$

where the last equality follows from the fact that drilling returns are a martingale. Type-one players thus enjoy the same interim payoff in this case as the one in my previous subsection. In both benchmark cases, they weakly prefer to drill. One can thus think of them as types who always—i.e. independent of s_{-i} and independent of whether signals are disclosed or not—drill at time one. As far as they are concerned, disclosing s_{-i} does thus not provide them with any information which makes them change their drilling plans. They are therefore indifferent between the signals-disclosure and the minimal-disclosure case.

I now compute Player-type i^0's interim payoff u^0_{sd} and her bid b^0_{sd} if the drilling cost is *not* intermediate. If the other player does not win his tract, she drills only if it's profitable

[12] The subscript sd refers to the case in which signals are disclosed.

for her to do so. She then gets a gross payoff of $\max\left\{0,\mu^0(s_{-i})-c\right\}$. Observe that if the other player wins his tract, she also gets a gross payoff equal to $\max\left\{0,\mu^0(s_{-i})-c\right\}$. This is also intuitive: If the drilling cost is high or if $s_{-i}=0$, both players face a negative payoff from drilling and no one ever drills; if the drilling cost is low and if the other player possesses signal one, as explained above, she is indifferent between drilling and waiting. Hence, $u^0_{sd}=\sum_{s_{-i}}\Pr\left(s_{-i}|s_i=0\right)\max\left\{0,\mu^0(s_{-i})-c\right\}$. As in my previous paragraph, her equilibrium bid is equal to

$$b^0_{sd}=\frac{1}{2}\sum_{s_{-i}}\Pr\left(s_{-i}|s_i=0\right)\max\left\{0,\Pr(V_i=1|s_i=0,s_{-i})-c\right\}. \tag{4}$$

Observe that if the drilling cost is high, type-zero players bid zero. In this case type-zero players anticipate that the divulgence of their bad private information "kills" the other player's incentives to drill. As they cannot free-ride on the other player's drilling cost, they bid zero.

I now compute Player-type i^0's interim payoff and her optimal bid if the drilling cost is intermediate. If s_{-i} is also equal to zero, no one ever drills and both players get a gross payoff equal to zero. Suppose $(s_i,s_{-i})=(0,1)$ and that both players win their tracts. As argued above, Player i then waits while Player $-i$ drills. Player i's gross payoff is therefore equal to $\delta W^0\left(\infty,0,1\right)$. Hence, Player-type i^0's interim payoff in the intermediate-cost case is equal to

$$\Pr\left(s_{-i}=1|s_i=0\right)\Pr\left(r<b^1_{sd}\,|r<b\right)\delta W^0\left(\infty,0,1\right)\equiv\bar{u}^0_{sd}. \tag{5}$$

Observe that $\bar{u}^0_{sd}$ could be higher than u^1_{sd}. This is intuitive: If the drilling cost is intermediate and if $(s_i,s_{-i})=(0,1)$, Player i gets to free-ride on Player $-i$'s drilling cost. Because of this information externality, type-zero players may enjoy a higher interim payoff than type-one players despite them being less optimistic about their prospects of finding oil. At time zero, Player-type i^0 chooses b to maximize $\bar{U}^0\equiv b(\bar{u}^0_{sd}-b)$. Observe that $\bar{U}^0$ decreases when $b>b^1_{sd}$. This is also intuitive: Type-zero players only drill if the other player finds oil at time one. Player-type i^0 therefore only wants to win her tract whenever Player-type $-i^1$ wins his. If she were to submit a bid $b>b^1_{sd}$, she thus runs the risk that $r\in\left(b^1_{sd},b\right)$ in which case she is the only player with an accepted bid. Player i's objective function can thus, without loss of generality, be rewritten as: $\max_{b\le b^1_{sd}} b\left[\bar{u}^0_{sd}-b\right]$. As $\Pr\left(r<b^1_{sd}\,|r<b\right)=1$ whenever $b\le b^1_{sd}$, I conclude that her optimal bid is equal to

$$b^0_{sd}=\min\left\{b^1_{sd},\bar{b}^0\right\}, \tag{6}$$

where $\bar{b}^0\equiv\frac{1}{2}\Pr\left(s_{-i}=1|s_i=0\right)\delta W^0\left(\infty,0,1\right)$. My main results are summarized below.

PROPOSITION 2 *Suppose signals are disclosed. Type-one players then bid $b^1_{sd}=\frac{1}{2}u^1_{sd}$. If the drilling cost is not intermediate—i.e. if $c\notin\left(\mu^0(\infty),\mu^1(0)\right)$—type-zero players bid according to (4). If the drilling cost is intermediate, they bid according to (6). Suppose both players win their* *tracts. If* $(s_i, s_{-i}) = (1,1)$*, both players drill with probability* $\bar{\lambda}$ *at time one. If* $(s_i, s_{-i}) = (1,0)$ *and if the drilling cost is high, no one ever drills. If* $(s_i, s_{-i}) = (1,0)$ *and if the drilling cost is intermediate, Player* i *drills with probability one at time one while Player* $-i$ *waits. If* $(s_i, s_{-i}) = (1,0)$ *and if the drilling cost is low Player* i *drills at time one with probability* $\underline{\lambda}$*, while Player* $-i$ *drills with probability* $\lambda_\circ$*. If* $(s_i, s_{-i}) = (0,0)$ *no one ever drills.*

Consider the example of my previous subsections, i.e. $(\nu, p, c, \delta) = (0.5, 0.6, 0.5, 0.95)$ and $\rho^0 = \rho^1 = 1 - \epsilon$. Suppose $s_i = 1$ while $s_{-i} = 0$. Observe that both $\mu^0(\infty)$ and $\mu^1(0)$ are approximately equal to $\nu = c$. Yet, as explained in the first paragraph of this subsection, $\mu^0(\infty)$ is slightly less than the drilling cost c, while $\mu^1(0)$ is slightly above c. As summarized in the above proposition, Player i then drills at time one while Player $-i$ waits. It then follows from above that both types get an interim payoff approximately equal to:

$$
\begin{aligned}
u^1_{sd} &= \xi^1 - c = 0.6 - 0.5 = 0.1, \text{ and,} \\
\bar{u}^0_{sd} &= \Pr(s_{-i} = 1 | s_i = 0)\, \delta W^0(\infty, 0, 1) \\
&= \delta \Pr(V_{-i} = s_{-i} = 1 | s_i = 0)(1 - c) = 0.95 \times 0.6 \times 0.4 \times 0.5 = 0.114.
\end{aligned}
$$

Observe that the interim payoff of the type-zero player actually exceeds the one of the type-one players. As explained above, this implies that both types bid 5 cents in equilibrium. Recall that type-zero players bid 0.37 cents in the minimal-disclosure case. They thus bid much more aggressively in this case as they know that if $s_{-i} = 1$, he drills with probability one and not with probability 6.6%.

3.6 Strategic underbidding

I now go back to my original game as explained in Subsection 3.1. Recall from that subsection that $l_{-i}(b_{-i})$ represents the likelihood that Player $-i$'s bid is submitted by a type-one player. The higher $l_{-i}(b_{-i})$, the more likely that $s_{-i} = 1$. In what follows, Player-type i^{s_i} is said to distort her bid when she bids differently in the bid-disclosure case as opposed to the signals-disclosure case. Player-type i^{s_i} is said to strategically underbid if her bid in in the bid-disclosure case is less than the one in the signals-disclosure case and if, by underbidding, she increases her interim payoff via an increase in Player $-i$'s drilling probability λ^1_{-i}.

In this subsection, I assume that the the drilling cost c is "intermediate", i.e. that $c \in (\mu^0(\infty), \mu^1(0))$. As $\mu^0(l_{-i}) \leq \mu^0(\infty) < c$, type-zero players—independent of their beliefs about the other player's type—face a negative payoff from drilling at time one and wait.

Recall that I restrict attention to the class of the non-coordinating strategies. To compute interim payoffs, I therefore don't need to consider all possible beliefs that Player i can have about Player $-i$'s type after observing his bid b_{-i}. Instead, I only need to bother whether she is "sufficiently optimistic" about his type, i.e. whether $l_{-i}(b_{-i})$ lies above or below some critical threshold level. To see this, recall from Subsection 3.3 that if $c < \mu^1(0)$ and if $\delta < 1$,

$$\mu^1(0) - c > \delta W^1(0, 0, 1).$$

(In this case Player-type i^1 knows that $s_{-i} = 0$ and that he waits. To avoid the discounting cost, she prefers to drill.) Recall from Subsection 3.3 that if the discount factor is high enough,

$$\mu^1(1) - c < \delta W^1(1, 0, 1).$$

In Melissas (2014), I prove that—if $c < \mu^1(0)$—the payoff from waiting $W^1(l_{-i}, 0, 1)$ increases faster in l_{-i} than the payoff from drilling. This is intuitive: As $\left(\lambda^0_{-i}, \lambda^1_{-i}\right) = (0, 1)$, any increase in l_{-i}—and thus in the the probability that $s_{-i} = 1$—has a big positive impact on the probability that Player $-i$ drills at time one. In turn, this increases her payoff from waiting by a "big" amount. Hence, there exists a unique $\tilde{l} \in (0, 1)$ such that $\mu^1\left(\tilde{l}\right) - c = \delta W^1\left(\tilde{l}, 0, 1\right)$. This insight implies that if Player i updates her beliefs using a likelihood $l_{-i} < \tilde{l}$, and if Player $-i$ updates his beliefs using a likelihood $l_i \geq \tilde{l}$, there exits a unique continuation equilibrium in which Player-type i^1 drills while Player $-i$ waits. Suppose both players update their beliefs using likelihoods l_i and l_{-i} no less than $\tilde{l}$. It then follows from my discussion in Subsection 3.3 that there exists a unique non-coordinating continuation equilibrium in which Player-types i^1 and $-i^1$ are indifferent between drilling and waiting.

Consider a candidate separating equilibrium in which type-zero players bid b^0 and type-one players b^1.[13] Consider Player-type i^1. Suppose she bids b and that this bid induces Player $-i$ to update his beliefs using a likelihood $l_i(b) < \tilde{l}$. As $c < \mu^1(0) \leq \mu^1(l_{-i})$, type-one players—independent of their beliefs about the other player's type—face a positive payoff from drilling at time one. Thus, if Player $-i$ does not win his tract, she gets a gross payoff equal to $\mu^1(l_{-i}) - c$, where $l_{-i} = 0$ when $b_{-i} = b^0$ and $l_{-i} = \infty$ when $b_{-i} = b^1$. Suppose now that both players win their tracts. If $b_{-i} = b^0$, Player i updates her beliefs using a likelihood $l_{-i} = 0 < \tilde{l}$. It then follows from my previous paragraph that she drills and gets a gross payoff equal to $\mu^1(0) - c$. If $b_{-i} = b^1$, she updates her beliefs using a likelihood $l_{-i} = \infty > \tilde{l}$. As $l_i < \tilde{l} < l_{-i}$, from my previous paragraph we know that she then gets a gross payoff of $\delta W^1(\infty, 0, 1)$. Hence, Player-type i^1's interim payoff in this case is equal to

$$\bar{u}^1 \quad \equiv \quad \Pr\left(b_{-i} = b^0 \middle| s_i = 1, r < b\right)\left(\mu^1(0) - c\right) + \Pr\left(b_{-i} = b^1 < r \middle| s_i = 1, r < b\right)\left(\mu^1(\infty) - c\right)$$

[13]In Melissas (2014), I show that it is without loss of generality to assume that both types only submit one bid. To be more precise, there does not exist a separating equilibrium in which some type randomizes between b and b' and in which $\Pr(s_i = 1|s_{-i}, b_i = b) = \Pr(s_i = 1|s_{-i}, b_i = b') \in \{0, 1\}$.

$$+ \quad \Pr\left(b_{-i} = b^1 > r \middle| s_i = 1, r < b\right) \delta W^1(\infty, 0, 1). \tag{7}$$

Let $\bar{U}^1(b) \equiv b(\bar{u}^1 - b)$. In words, $\bar{U}^1(b)$ represents Player-type i^1's unconditional and net expected payoff if she bids b and if this bid induces Player $-i$ to update his beliefs using a likelihood $l_i(b) < \tilde{l}$. Observe that $\bar{U}^1(b)$ is unimodal: It initially increases in b and then decreases.

Suppose now that Player i bids b and that this bid induces Player $-i$ to update his beliefs using a likelihood $l_i(b) \geq \tilde{l}$. Player-type i^1's unconditional and net expected payoff is then equal to $\underline{U}^1(b) \equiv b(\underline{u}^1 - b)$, where

$$\begin{aligned} \underline{u}^1 \quad &\equiv \quad \Pr\left(b_{-i} = b^0 \middle| s_i = 1, r < b\right)\left(\mu^1(0) - c\right) + \Pr\left(b_{-i} = b^1 < r \middle| s_i = 1, r < b\right)\left(\mu^1(\infty) - c\right) \\ &+ \quad \Pr\left(b_{-i} = b^1 > r \middle| s_i = 1, r < b\right) \delta W^1\left(\infty, 0, \bar{\lambda}\right). \end{aligned} \tag{8}$$

The intuition behind the first two terms of (8) is identical to the one of the first two terms of (7). If Player $-i$ bids b^1 and if he also wins his tract both l_i and l_{-i} are no less than $\tilde{l}$. It then follows from Subsection 3.3 that Player-type $-i^1$ drills with probability $\bar{\lambda}$ to make Player-type i^1 indifferent between drilling and waiting.[14] This explains the third term of (8). Observe that $\underline{U}^1$ is also unimodal. As $\underline{u}^1 < \bar{u}^1$, $\underline{U}^1$ lies below $\bar{U}^1$ as illustrated in Figure 1. As Player-type i^1 is indifferent, the payoff $\delta W^1\left(\infty, 0, \bar{\lambda}\right)$ can be replaced by $\mu^1(\infty) - c$. As drilling returns are a martingale, her interim payoff can be rewritten as $\underline{u}^1 = \xi^1 - c$.

Observe that $\underline{U}^1(b)$ is maximized when $b = \frac{1}{2}\left(\xi^1 - c\right)$. Suppose there exists a separating equilibrium in which $b^1 \neq \frac{1}{2}\left(\xi^1 - c\right)$. As this bid reveals their types, Player $-i$ updates his beliefs using a likelihood ratio $l_i\left(b^1\right) = \infty > \tilde{l}$. It then follows from above that Player-type i^1 gets $\underline{U}^1\left(b^1\right)$. This, however, implies that she has a profitable deviation: She could bid $\frac{1}{2}\left(\xi^1 - c\right)$ and—depending on the specified out-of-equilibrium beliefs—either achieve $\underline{U}^1\left(\frac{1}{2}\left(\xi^1 - c\right)\right) > \underline{U}^1\left(b^1\right)$ or $\bar{U}^1\left(\frac{1}{2}\left(\xi^1 - c\right)\right) > \underline{U}^1\left(b^1\right)$. Thus, in any candidate separating equilibrium $b^1 = \frac{1}{2}\left(\xi^1 - c\right)$. Observe that type-one players do not distort their bids in this case, i.e. $b^1 = b^1_{sd}$.

Consider now Player-type i^0. Suppose she submits b and that this bid induces Player $-i$ to update his beliefs using a likelihood $l_i(b) < \tilde{l}$. Her unconditional and net expected payoff is then equal to $\bar{U}^0(b) \equiv b\left(\bar{u}^0 - b\right)$, where

$$\bar{u}^0 \equiv \Pr\left(b_{-i} = b^1 \middle| s_i = 0\right) \Pr\left(r < b^1 \middle| r < b\right) \delta W^0(\infty, 0, 1). \tag{9}$$

This is intuitive: Type-zero players only drill if the other player finds oil at time one. Thus if $b_{-i} = b^0$ or if the other player does not win his tract, she gets zero. As argued above, if

[14] Recall that $\bar{\lambda}$ is computed to equate $\mu^1(\infty) - c$ with $\delta W^1\left(\infty, 0, \bar{\lambda}\right)$. Recall also that I assume that the discount factor δ is close to one. This implies that $\mu^1(\infty) - c < \delta W^1(\infty, 0, 1)$. Recall also that $W^1\left(\infty, 0, \lambda_{-i}\right)$ is increasing in λ_{-i}. Hence, $\bar{\lambda} < 1$, and $\underline{u}^1 < \bar{u}^1$.

$b_{-i} = b^1$ and if he also wins his tract, he drills at time one with probability one. She then gets a gross payoff of $\delta W^0(\infty, 0, 1)$. As above, $\bar{U}^0(b)$ is unimodal. Recall that $b^1 = b^1_{sd}$ and that $b_{-i} = b^1 \Leftrightarrow s_{-i} = 1$. A careful comparison between Equations 5 and 9 then reveals that $\bar{u}^0 = \bar{u}^0_{sd}$.

Suppose Player-type i^0 submits b and that this bid induces Player $-i$ to update his beliefs about her type using a likelihood $l_i(b) \geq \tilde{l}$. She then gets an unconditional and net expected payoff of

$$b\Big(\Pr\left(b_{-i} = b^1 \middle| s_i = 0\right) \Pr\left(r < b^1 \middle| r < b\right) \delta W^0\left(\infty, 0, \bar{\lambda}\right) - b\Big) \equiv \underline{U}^0(b).$$

The intuition is similar to the one behind (9) except that in case $b_{-i} = b^1$ both players hold sufficiently optimistic beliefs about each other's types. As argued above, this implements a unique non-coordinating equilibrium in which Player-type $-i^1$ drills with probability $\bar{\lambda}$ to make Player-type i^1 indifferent between drilling and waiting. Observe that $\underline{U}^0$ is unimodal and that, as $\bar{\lambda} < 1$, it lies below $\bar{U}^0$ as illustrated in Figure 1.

Call $\underline{b}$ the lowest bid such that $\bar{U}^1\left(\underline{b}\right) = \underline{U}^1\left(b^1\right)$. Observe that if the discount factor δ is close to one, $\underline{U}^0(b) < \bar{U}^0\left(\underline{b}\right)\ \forall b$ as shown in Figure 1. This is intuitive: Recall from above that $\underline{U}^0(b)$ is primarily determined by the probability $\bar{\lambda}$ with which Player-type $-i^1$ drills at time one. Recall also from 2 that $\bar{\lambda}$ is determined to equate Player-type i^1's opportunity cost with her informational benefit of waiting. If the discount factor δ is high, Player-type i^1's opportunity cost is low. The drilling probability $\bar{\lambda}$ is then also low. In turn, this implies that $\underline{U}^0(b)$ is low. (In the limit, i.e. as δ is close to one, $\bar{\lambda}$ is close to zero and the maximal value of $\underline{U}^0(b)$ is then also close to zero.) Recall from Subsection 3.5 that $\bar{b}^0$ represents the bid which maximizes $\bar{U}^0(b)$. In Figure 1, I assume that the values of my exogenous parameters are chosen such that such that $\underline{b} < \bar{b}^0$ and such that $\bar{U}^0$ lies below $\underline{U}^1$. More on the role of those assumptions below.

Recall from Subsection 3.5 that if signals instead of bids were disclosed, type-zero players would bid $\bar{b}^0$. Consider a candidate separating equilibrium in which $b^0 = \bar{b}^0$. Using Figure 1, it is easy to see that Player-type i^1 then has a profitable deviation: She can bid $\bar{b}^0$ and achieve a payoff $\bar{U}^1\left(\bar{b}^0\right)$ which is higher than $\underline{U}^1\left(b^1\right)$.

I now argue that there exists a separating equilibrium in which $b^0 = \underline{b}$ and in which type-one players bid b^1. This separating equilibrium is supported by the following beliefs: If $b_i \leq \underline{b}$, Player i is supposed to be a type-zero player with probability one; if $b_i > \underline{b}$, she is supposed to be a type-one player with probability one. To see this, observe that the specified out-of-equilibrium beliefs satisfy the intuitive criterion.[15] Observe also that, given the specified beliefs, no player-type can gain from deviating: Submitting a bid $b < \underline{b}$ is dominated for both types, submitting

[15] Independent of the specified out-of-equilibrium beliefs, *both* player-types strictly lose from submitting a bid lower than $\underline{b}$. The intuitive criterion then puts no restrictions on the specified out-of-equilibrium beliefs. Further-

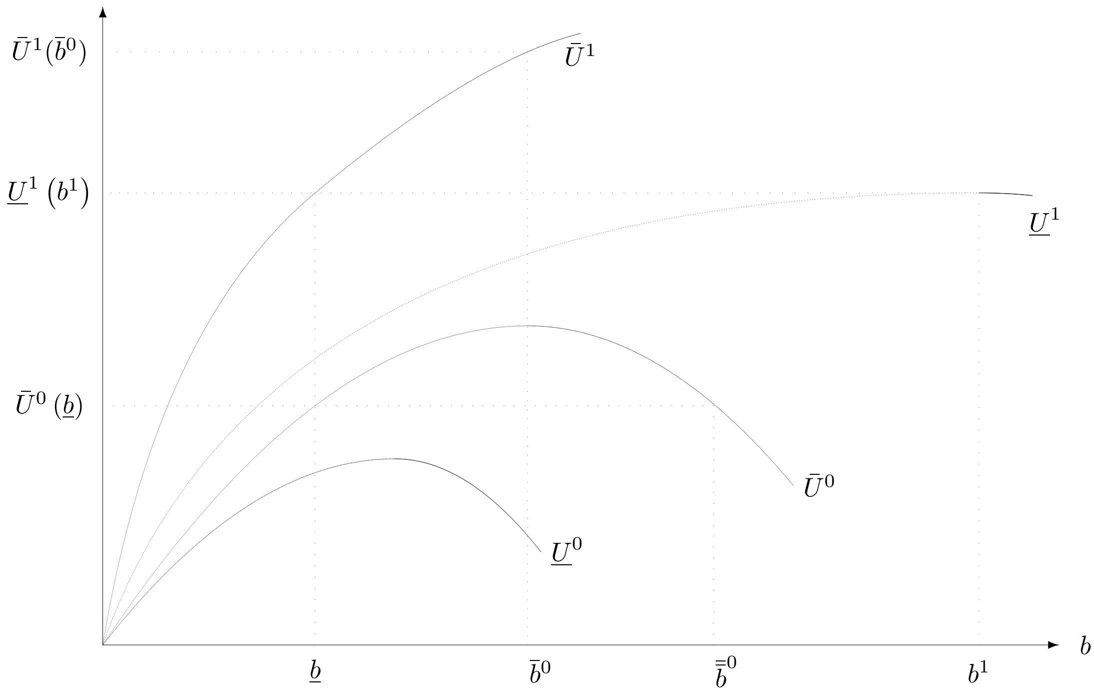

Figure 1: Strategic underbidding by type-zero players. If signals were disclosed, type-zero players would bid $\bar{b}^0$. If bids are disclosed, there exists a separating equilibrium in which they bid $\underline{b}$.

$b > \underline{b}$ is not a good idea either as for all $b > \underline{b}$, $\underline{U}^0(b) < \bar{U}^0\left(\underline{b}\right)$ and $\underline{U}^1(b) \leq \underline{U}^1\left(b^1\right)$.

Underbidding thus allows type-zero players to credibly signal that they are not going to drill at time one. This, in turn, induces the other player to drill in case he bid "high". Numerically, the underbidding problem need not be small. Suppose, for example, that $(\nu, p, c, \rho^0, \rho^1) = (0.5, 0.6, 0.5, 0.8, 08)$ and that $\delta = 1 - \epsilon$ where ϵ is an arbitrarily small, yet strictly positive number.[16] Type-zero players would then approximately bid 3 cents if signals instead of bids were disclosed. In the separating equilibrium, they approximately bid 2.1 cents, a 30% decrease!

In Figure 1, I illustrated a separating equilibrium when $\underline{b} < \bar{b}^0$ and when $\bar{U}^0$ lies below $\underline{U}^1$. I now elaborate on the role of those assumptions.

Suppose that $\bar{b}^0 < \underline{b}$ and that there exists a non-coordinating equilibrium in which type-zero players bid $b^0 \neq \bar{b}^0$. Observe that—independent of the specified out-of-equilibrium beliefs—type-one players strictly lose from submitting a bid $b < \underline{b}$. Hence, if Player i deviates and submits

more, both Player-type i^0 and Player-type i^1 win from submitting $b \in \left(b^0, \bar{\bar{b}}^0\right]$ if it induces Player $-i$ to drill with probability one in case he wins his tract. The intuitive criterion thus also puts no restrictions on the specified out-of-equilibrium beliefs. Finally, Submitting a bid higher than $\bar{\bar{b}}^0$ is strictly dominated for type-zero players. Hence, the out-of equilibrium belief $\Pr\left(s_i = 1 \middle| b_i > \bar{\bar{b}}^0, b_i \neq b^1\right) = 1$ is intuitive too.

[16]It can be checked that in this numerical example $\underline{b} < \bar{b}^0$ and that $\bar{U}^0$ lies below $\underline{U}^1$.

$\bar{b}^0$ the intuitive criterion prescribes Player $-i$ to believe that $s_i = 0$ with probability one. This, in turn, induces Player-type i^0 to deviate from the hypothesized equilibrium. Thus, if $\bar{b}^0 < \underline{b}$ there exists a unique equilibrium outcome in which no types distort their bids. I now argue that $\bar{b}^0 < \underline{b}$ if signals are sufficiently precise. Suppose p is close to one. Recall that Assumption 2 states that ρ^0 must be chosen such that $\Pr(V_i = 1 | s_i = 1, V_{-i} = 0) < c$. If p is close to one, my last inequality only holds if tract values are almost perfectly correlated. This is intuitive: If p is equal to, say, 0.9 and if ρ^0 is equal to, say, 0.99, then the "bad news" that is embodied in the statistic $V_{-i} = 0$ overcompensates the "good news" that is embodied in the statistic $s_i = 1$. Hence, if p is close to one, Player-type i^0 is "very confident" that V_i, V_{-i} and s_{-i} are all equal to zero. This implies that $\Pr\left(b_{-i} = b^1 > r \middle| s_i = 0, r < b\right) \delta W^0(\infty, 0, 1)$ is close to zero and, thus, that $\bar{b}^0 < \underline{b}$.

In Figure 1, I assumed that $\bar{U}^0$ lies below $\underline{U}^1$. It follows from my previous paragraph that this assumption is satisfied when signals are sufficiently precise. When signals are imprecise, however, this need not be the case. (The intuition behind this result is explained in the paragraph below Equation 5.) Observe, however, that $\bar{U}^0$ always lies below $\bar{U}^1$ as in both cases player-type $-i^1$ drills with probability one and as Player-type i^1 is slightly more confident that s_{-i}, V_{-i} and V_i are equal to one. Thus, suppose that $\bar{U}^0$ lies between $\underline{U}^1$ and $\bar{U}^1$. Recall from Equation 6 that—in the signals-disclosure case—type-zero players would bid b^1_{sd}. Drawing a figure similar to Figure 1, it is easy to check that there still exists a separating equilibrium in which type-zero players bid $\underline{b} < b^1 = b^1_{sd}$. Underbidding does thus not rest on my assumption that $\bar{U}^0$ lies below $\underline{U}^1$. Again the underbidding problem need not be small. Consider the numerical example I detailed in my previous subsections, i.e. assume that $(\nu, p, c, \delta) = (0.5, 0.6, 0.5, 0.95)$ and that $\rho^0 = \rho^1 = 1 - \epsilon$. Recall from my previous subsection that if signals were disclosed, both types would bid 5 cents. In the separating equilibrium, however, type-zero players approximately bid 1.6 cents, a 68% decrease! Summarizing: A separating equilibrium without bid distortion exists if $\bar{b}^0 < \underline{b}$ and a separating equilibrium with bid distortion exists if $\underline{b} < \bar{b}^0$.

What about other non-coordinating equilibria? In Melissas (2014) I show that—depending on the values of my exogenous parameters—only one other non-coordinating equilibrium may exist. In this alternative equilibrium, type-one players strategically underbid. To be more specific, they bid $b^1 = \frac{1}{2}\left(\xi^1 - c\right)$ with probability $1 - x$ and b_{ss} with probability x $\left(b_{ss} < b^1\right)$. Type-zero players bid b_{ss} with probability one. (Henceforth, the subscript ss refers to this semi-separating equilibrium.) This equilibrium is supported by the following continuation strategies: If $(b_i, b_{-i}) = \left(b^1, b^1\right)$, both players drill with probability $\bar{\lambda}$; if $(b_i, b_{-i}) = \left(b_{ss}, b^1\right)$ Player i waits, while Player $-i$ drills; if $(b_i, b_{-i}) = (b_{ss}, b_{ss})$ type-one players drill at time one with probability one while type-zero players wait. By bidding b_{ss} instead of b^1 Player-type i^1 thus increases her probability of free-riding on the other player's drilling cots. Bidding b_{ss}, however, also comes with a cost as it becomes then less likely that Player i's bid exceeds the government's reservation price r. In equilibrium, x and b_{ss} are chosen such that type-one players are indifferent between submitting both bids. In Melissas (2014) I show that the difference between the equilibrium outcome of the semi-separating and the separating equilibrium vanishes as the discount factor δ tends to one.[17] This is made explicit in the Proposition below. (In the proposition, $\Pr_e(a_i = \text{drill}|s_i)$ denotes the expected drilling probability of Player-type i^{s_i} in equilibrium $e \in \{ss, sep\}$. The subscript *sep* refers to the separating equilibrium.)

PROPOSITION 3 *Suppose that* $\mu^0(\infty) < c < \mu^1(0)$.

There exists then a separating equilibrium in which type-one players bid $b^1 = \frac{1}{2}\left(\xi^1 - c\right)$ *and in which type-zero players bid* $b^0 = \min\left\{\bar{b}^0, \underline{b}\right\}$ $\left(< b^1\right)$. *Type-zero players strategically underbid if and only if* $\underline{b} < \bar{b}^0$. *If signals are sufficiently precise,* $\bar{b}^0 < \underline{b}$. *Type-one players do not distort their bids. If* $(b_i, b_{-i}) = \left(b^1, b^1\right)$, *Player i drills with probability* $\bar{\lambda}$. *If* $(b_i, b_{-i}) = \left(b^1, b^0\right)$, *Player i drills while Player $-i$ waits. If* $(b_i, b_{-i}) = \left(b^0, b^0\right)$, *no one drills.*

There may also exist a semi-separating equilibrium in which type-one players bid b^1 with probability $1-x$ and b_{ss} with probability x, and in which type-zero players bid b_{ss} with probability one. Suppose there exists a $\underline{\delta}$ such that $\forall\delta \in (\underline{\delta}, 1)$ *the vector of exogenous parameters* $\left(p, c, \delta, \nu, \rho^1, \rho^0\right)$ *supports both a separating and a semi-separating equilibrium. Then,* $\forall\epsilon > 0$, $\exists\ \tilde{\delta}(\epsilon) < 1$ *such that* $\forall\delta > \tilde{\delta}(\epsilon)$, $x < \epsilon$, $\left|b_{ss} - b^0\right| < \epsilon$, *and* $|\Pr_{sep}(a_i = \text{drill}|s_i) - \Pr_{ss}(a_i = \text{drill}|s_i)| < \epsilon\ \forall s_i$.

No other non-coordinating equilibrium exists.

It is important to realize that if signals were drawn from a continuous distribution, underbidding may then occur even if the drilling costs c is not "intermediate". Suppose, for example, that the drilling cost is "low" (i.e. lower than "intermediate") and that Player i possesses a "very low" signal. As her signal is "very low", Player i expects Player $-i$'s signal–conditional on the event that she wins her tract—to be "moderately high" in which case she still faces a negative payoff from drilling at time one. This induces many of Player $-i$'s types to drill at time one. In turn, this may warrant the existence of an equilibrium in which "very low" types underbid. Finally, observe that in equilibrium both players coordinate their drilling plans *despite* the fact that I restrict them to use non-coordinating strategies in the waiting game.

[17] For the sake of expositional clarity, I decided not to put the intuition behind this result in the paper. In Melissas (2014) I also provide necessary and sufficient conditions for existence of the semi-separating equilibrium and I argue that it rests on divine out-of-equilibrium beliefs.

3.7 No bid distortion

In this subsection I analyze bidding and drilling behavior when the drilling cost is not intermediate, i.e. either $c \in \left(\xi^0, \mu^0(\infty)\right)$, or $c \in \left(\mu^1(0), \xi^1\right)$.

Suppose that the drilling cost is "high", i.e. that $c \in \left(\mu^1(0), \xi^1\right)$. As the drilling cost is high, type-zero players only drill if the other player finds oil at time one and type-one players refrain from drilling if they find out that the other player possesses signal zero. As $\mu^1(0) < c < \xi^1 < \mu^1(\infty)$ and as $\mu^1(l_{-i})$ is increasing in l_{-i}, there exists a unique $l' \in (0,1)$ such that drilling yields a zero expected profit, i.e. such that $\mu^1(l') = c$. Thus, if Player i updates her beliefs using a likelihood $l_{-i} < l'$, she waits at time one. Suppose Player i updates her beliefs using a likelihood $l_{-i} \geq l'$. As Player i is then "confident enough" that Player $-i$ possesses signal one, it can be shown that she then prefers to wait if she anticipates Player-type $-i^1$ to drill with probability one. Formally, $0 \leq \mu^1(l_{-i}) - c < \delta W^1(l_{-i}, 0, 1)\ \forall l_{-i} \geq l'$. (See Melissas (2014) for the proof.) It then follows from Subsection 3.3 that there exists a unique λ^*_{-i} which makes Player-type i^1 indifferent between drilling and waiting. Hence, if both players update their beliefs using likelihoods l_i and l_{-i} no less than l', there exists a unique non-coordinating continuation equilibrium in which Player-types i^1 and $-i^1$ are indifferent between drilling and waiting.

Consider a candidate separating equilibrium in which type-zero players bid b^0 and type-one players bid b^1. Player-type i^0 knows that Player $-i$ does not drill if he observes that $b_i = b^0$. Hence, such an equilibrium only exists if $b^0 = b^0_{sd} = 0$. Consider Player-type i^1 and suppose she wins her tract. If Player $-i$ bids zero, she refrains from drilling and gets a gross payoff equal to zero. If $b_{-i} = b^1$ and if either he does not win his tract or if he updates his beliefs using a likelihood $l_i < l'$, she drills at time one and gets a gross payoff equal to $\mu^1(\infty) - c$. If $b_{-i} = b^1$, if he wins his tract and if he updates his beliefs using a likelihood $l_i \geq l'$, both players choose their drilling probabilities to make each other indifferent between drilling and waiting. In this case, she thus also gets $\mu^1(\infty) - c$. Player-type i^1's interim payoff in a candidate separating equilibrium is thus equal to $\Pr\left(s_{-i} = 1 \middle| s_i = 1\right)\left[\mu^1(\infty) - c\right]$, *independent of Player $-i$'s beliefs about her type.* At time zero, she thus faces the following problem: $\max_b b\left(\Pr\left(s_{-i} = 1 \middle| s_i = 1\right)\left[\mu^1(\infty) - c\right] - b\right)$. This is a strictly concave maximization problem which has b^1_{sd} as unique solution. Hence, such an equilibrium only exists if $b^1 = b^1_{sd}$. Observe that in this candidate separating equilibrium, no type distorts her bid.

Let $u^0(b)$ denote Player-type i^0's interim payoff if she bids b. Recall that in the intermediate-cost case Player-type i^0 wants to signal her type because Player-type $-i^1$ then infers that Player i faces a negative payoff from drilling. In the unique non-coordinating continuation equilibrium, Player $-i$ then drills. If the drilling cost is high, however, Player-type $-i^1$ faces a negative payoff from drilling if she observes that $b_i = b^0$. Hence, type-zero players want to "hide" their types in this case! Thus, suppose Player-type i^0 hides her type by bidding b^1 instead of b^0. Player $-i$ then believes that she is a type-one player. Her interim payoff is then equal to

$$u^0\left(b^1\right) = \Pr\left(b_{-i} = b^1 \middle|\, s_i = 0\right) \delta W^0\left(\infty, 0, \bar{\lambda}\right).$$

The intuition is almost identical to the one behind $\underline{U}^0(b)$: Type-zero players wait at time one. As she bids b^1 she knows that if she wins her tract, so does Player-type $-i^1$ in which case she gets a gross payoff of $\delta W^0\left(\infty, 0, \bar{\lambda}\right)$. A separating equilibrium only exists if type-zero players cannot profitably deviate by bidding b^1 instead of zero, i.e. only if

$$U^0\left(b^1\right) = b^1\left(u^0\left(b^1\right) - b^1\right) \leq 0. \tag{10}$$

Observe that Inequality 10 holds when the discount factor is sufficiently high. For, in that case the opportunity cost of waiting is close to zero. To make Player-type i^1 indifferent, $\bar{\lambda}$ must be close to zero. In turn, this implies that $u^0\left(b^1\right)$ is close to zero. Suppose inequality 10 holds and that out-of equilibrium beliefs are updated under the assumption that $\Pr(s_{-i} = 0|s_i, b_{-i} \neq b^1) = 1$ $\forall s_i$. Observe that those out-of-equilibrium beliefs are intuitive as type-one players strictly lose by submitting a bid different from b^1. (As argued above, Player-type i^1's interim payoff is independent of Player $-i$'s beliefs about his type.) Those out-of-equilibrium beliefs ensure that Player-type i^0 cannot gain by submitting a bid different from zero as any bid $b \neq b^1$ kills Player-type $-i^1$ incentives to drill. In Melissas (2014), I also show that if Inequality 10 holds, there does not exist any other non-coordinating equilibrium.[18]

Suppose now that drilling is "cheap", i.e. that $c \in \left(\xi^0, \mu^0(\infty)\right)$, and that the discount factor δ is close to one. Consider a candidate separating equilibrium in which type-zero players bid b^0 and in which type-one players bid b^1. Type-zero players also want to hide their types in this case for two different reasons. The first one is identical to the one detailed above. If $(b_i, b_{-i}) = \left(b^0, b^0\right)$, Player $-i$ infers that she is a type-zero player. He then faces a negative payoff from drilling and waits. If, however, $(b_i, b_{-i}) = \left(b^1, b^0\right)$ Player $-i$ (wrongly) infers that $s_i = 1$. This makes him sufficiently optimistic about his prospects of finding oil. He then drills with some probability to make Player-type i^1 indifferent under the (false) belief that she will drill with some probability to make him indifferent. Stated differently, type-zero players want to hide their types because they fear that if $b_{-i} = b^0$, they will "kill" his incentives to drill by bidding b^0. To understand the second reason, suppose that $b_{-i} = b^1$. Recall from Equation 2 that Player $-i$ chooses his

[18] To be more specific, a semi-separating or a pooling equilibrium do not exist when the discount factor is sufficiently high. Nor does there exist an equilibrium in which some type submits more than one bid along the equilibrium path.

drilling probability λ_{-i}^1 to equate her opportunity cost of waiting with her benefit of waiting. Thus if she bids b^1 instead of b^0, Player $-i$ believes that her opportunity cost of waiting is equal to $(1-\delta)\big(\mu^1(\infty)-c\big)$ instead of $(1-\delta)\big(\mu^0(\infty)-c\big)$. As $\mu^1(\infty) > \mu^0(\infty)$, this induces him to increase his time-one drilling probability. (Formally, he increases his drilling probability from $\underline{\lambda}$ to $\bar{\lambda}$.) This, in turn, increases her payoff from waiting. Observe, however, that both reasons become unimportant when the discount factor δ is close to one. For a player's opportunity cost of waiting is then close to zero—independent of her signal! Type-zero players can then not increase λ_{-i} by much by bidding b^1 instead of b^0. Therefore, in this case there exists a non-coordinating equilibrium without bid distortion. In Melissas (2014) I rule out any other candidate non-coordinating equilibrium in the low-cost case when the discount factor is close to one. My main results are summarized below:

PROPOSITION 4 *Suppose that* $c \in \Big(\mu^1(0), \xi^1\Big)$ *or that* $c \in \Big(\xi^0, \mu^0(\infty)\Big)$. *There exists then a unique non-coordinating equilibrium in which no player-type distorts her bid. Drilling probabilities are identical to the one detailed in Proposition 2.*

4 Some normative and positive implications

I first argue that disclosing bids, as opposed to following a minimal-disclosure policy, raises expected welfare and revenues. Let $k \in \{\mathcal{MD}, \mathcal{BD}\}$ denote the auction format. If $k = \mathcal{MD}$, the auctioneer follows a minimal-disclosure policy. If $k = \mathcal{BD}$, the auctioneer discloses bids. Recall from Propositions 3 and 4 that if δ is close to one, I can without loss of generality restrict attention to a separating equilibrium and that Player i distorts her bid only if $s_i = 0$ and only if the drilling cost is intermediate. In what follows $b^0(k)$ and $b^1(k)$ respectively denote the equilibrium bid of type-zero and type-one players in Auction k. Similarly, $R(k)$, $Wel(k)$ and $u^{s_i}(k)$ respectively denote expected revenues, expected welfare and Player-type i^{s_i}'s interim payoff in Auction k. Player i's equilibrium bid $b^{s_i}(\mathcal{BD})$ can be broken down in several components. Formally,

$$b^{s_i}(\mathcal{BD}) = b^{s_i}(\mathcal{MD}) + \underbrace{\Big(b_{sd}^{s_i} - b^{s_i}(\mathcal{MD})\Big)}_{\frac{1}{2}\mathrm{IVSD}^{s_i}} + \underbrace{\Big(b^{s_i}(\mathcal{BD}) - b_{sd}^{s_i}\Big)}_{\mathrm{distortion}^{s_i}}. \tag{11}$$

The term IVSD^{s_i} represents the informational value of observing signals on top of drilling outcomes. Formally, IVSD^{s_i} is defined as the difference between Player-type i^{s_i}'s interim payoff when signals are disclosed and when the auctioneer follows a minimal-disclosure policy. Recall from Subsections 3.4 and 3.5 that players bid one half their interim values in both cases. This explains why $b_{sd}^{s_i} - b^{s_i}(\mathcal{MD}) = \frac{1}{2}\mathrm{IVSD}^{s_i}$. The final term captures any bid distortion that arises due to the fact that the auctioneer discloses bids instead of signals.

Expected Revenues and expected welfare are respectively defined as

$$\begin{aligned} R(k) &\equiv 2\sum_{s_i} \Pr(s_i)\Pr(r < b_i|s_i) b^{s_i}(k) \text{ and,} \\ Wel(k) &\equiv 2\sum_{s_i} \Pr(s_i)\Pr(r < b_i|s_i) u^{s_i}(k) \, . \end{aligned}$$

Recall from Subsections 3.4 and 3.7 that if c is not intermediate, $b^{s_i}(k) = \frac{1}{2}u^{s_i}(k)$. Hence, if c is not intermediate and if Auction $\mathcal{BD}$ generates more revenues than Auction $\mathcal{MD}$, then Auction $\mathcal{BD}$ also generates a higher welfare. (At the risk of stating the obvious, the "2" before the summation signs comes from the fact that there are two players in my model. Bids do not affect welfare as they represent a transfer of money between two risk-neutral parties.)

Suppose first that the drilling cost is low and and that $s_i = 0$. Propositions 1, 2 and 4—along with Equation 11—allow me to rewrite $b^0(\mathcal{BD})$ in this case as:

$$b^0(\mathcal{BD}) = \frac{1}{2}\delta W^0(1,0,\lambda^\circ) + \frac{1}{2}\left[\Pr(s_{-i}=1|s_i=0)\left(\mu^0(\infty) - c\right) - \delta W^0(1,0,\lambda^\circ)\right] + \ 0.$$

Observe that the term between square brackets is positive when the discount factor is close to one. This is intuitive: As argued in Subsection 3.3 in equilibrium λ° balances Player-type i^1's opportunity cost of waiting with her informational benefit of waiting. If the discount factor δ is close to one, both the opportunity cost of waiting and the equilibrium probability λ° are close to zero. In turn, this implies that $W^0(1,0,\lambda^\circ)$ is close to zero. Recall from the discussion after Equation 3 that IVSD^1 and distortion^1 are both equal to zero when the drilling cost is low.

When the drilling cost is low, disclosing bids—as opposed to following a minimal-disclosure policy—thus increases revenues and welfare. For bid disclosure provides useful information to type-zero players about the profitability of drilling. In case $s_{-i} = 1$, they then face a positive payoff from drilling. If bids are not disclosed, they only drill at time two if the other player finds oil at time one—an unlikely event when the discount factor is close to one.

Suppose now that the drilling cost is high. Recall from Proposition 4 that no type distorts her bid and that type-zero players bid zero in the bid-disclosure case. Hence,

$$b^0(\mathcal{BD}) = \frac{1}{2}\delta W^0(1,0,\lambda^\circ) + \left[0 - \frac{1}{2}\delta W^0(1,0,\lambda^\circ)\right] + \ 0. \tag{12}$$

The equality above reveals that disclosing bids may actually reduce welfare and revenues. For, some types fear that the divulgence of their bad private information will "kill" the other player's incentive to drill. This induces them to bid less aggressively. Recall from Propositions 2 and 4 that type-one players bid $\frac{1}{2}\Pr(s_{-i}=1|s_i=1)\left(\mu^1(\infty) - c\right)$ in the bid-disclosure case. Thus

$$b^1(\mathcal{BD}) = \frac{1}{2}\left(\xi^1 - c\right) + \frac{1}{2}\left[\Pr(s_{-i}=1|s_i=1)\left(\mu^1(\infty) - c\right) - \left(\xi^1 - c\right)\right] + \ 0. \tag{13}$$

Recall from Subsection 3.5 that

$$\xi^1 - c = \Pr\left(s_{-i} = 0 | s_i = 1\right)\left[\mu^1(0) - c\right] + \Pr\left(s_{-i} = 1 | s_i = 1\right)\left[\mu^1(\infty) - c\right].$$

As $\mu^1(0) - c < 0$, I conclude that the term between square brackets of (13) is positive. This highlights a second reason why disclosing bids increases revenues and welfare: It reveals useful information to some types by informing them that *not* drilling is the right course of action. In the high-cost case disclosing bids is thus not an ambiguously good thing: Some types bid less aggressively (the type-zero players) while others more aggressively. Observe, however, that the term between square brackets of (12) is close to zero when the discount factor δ is close to one. Hence, whenever $\mu^1(0) < c$ there exists a $\tilde{\delta} < 1$ such that for all $\delta > \tilde{\delta}$,[19]

$$\begin{aligned}
\frac{1}{2}R(\mathcal{MD}) &= \Pr(s_i = 0)\Pr\left(r < b^0(\mathcal{MD})\right)\left[\frac{1}{2}\delta W^0\left(1, 0, \lambda^\circ\right)\right] \\
&+ \Pr(s_i = 1)\Pr\left(r < b^1(\mathcal{MD})\right)\left[\frac{1}{2}\left(\xi^1 - c\right)\right] \\
&< \Pr(s_i = 1)\Pr\left(r < b^1(\mathcal{BD})\right)\left[\Pr\left(s_{-i} = 1 | s_i = 1\right)\left(\mu^1(\infty) - c\right)\right] \\
&= \frac{1}{2}R(\mathcal{BD}).
\end{aligned}$$

Finally, suppose that the drilling cost is intermediate and that $s_i = 0$. Propositions 1, 2 and 3—along with Equation 11—allow me to rewrite $b^0(\mathcal{BD})$ in this case as:

$$b^0(\mathcal{BD}) = \frac{1}{2}\delta W^0\left(1, 0, \lambda^\circ\right) + \underbrace{\underbrace{\left[\min\left\{b^1_{sd}, \bar{b}^0\right\} - \frac{1}{2}\delta W^0\left(1, 0, \lambda^\circ\right)\right]}_{>0 \text{ if } \delta \text{ is close to one}} + \underbrace{\left[\min\left\{\underline{b}, \bar{b}^0\right\} - \min\left\{b^1_{sd}, \bar{b}^0\right\}\right]}_{\leq 0}}_{>0 \text{ if } \delta \text{ is close to one}}.$$

The first term between square brackets is positive as $W^0\left(1, 0, \lambda^\circ\right)$ is close to zero when δ is close to one. As explained in Subsection 3.6, the second term between square brackets is non-positive. Recall from my discussion after equation 3 that type-one players bid $\frac{1}{2}(\xi^1 - c)$ in both auction formats. Disclosing bids is thus a double-edged sword in this case. On the one hand, it allows both players to coordinate their drilling activities. As type-zero players then get to free-ride on the drilling activity of the type-one players, this increases their interim payoffs and thus their bids. This is captured by the first term between brackets. To benefit from the information externality, however, type-zero players may have to signal their types through strategic underbidding. Nonetheless, my model predicts that the positive effects dominates the negative one. For the sum of the last two terms is equal to $\min\left\{\underline{b}, \bar{b}^0\right\} - \frac{1}{2}\delta W^0\left(1, 0, \lambda^\circ\right)$, which

[19] At the risk of stating the obvious, the value of the critical discount factor $\tilde{\delta}$ depends on the values of the other exogenous parameters.

is also positive when the discount factor is close to one. Finally, observe that in this case

$$\begin{aligned}
\frac{1}{2}Wel(\mathcal{MD}) &= \Pr(s_i=0)\Pr\left(r<b^0(\mathcal{MD})\right)\delta W^0\left(1,0,\lambda^\circ\right)\\
&+ \Pr(s_i=1)\Pr\left(r<b^1(\mathcal{MD})\right)\left(\xi^1-c\right)\\
&< \Pr(s_i=0)\Pr\left(r<b^0(\mathcal{BD})\right)\Pr(s_{-i}=1|s_i=0)\delta W^0\left(\infty,0,1\right)\\
&+ \Pr(s_i=1)\Pr\left(r<b^1(\mathcal{BD})\right)\left(\xi^1-c\right)\\
&= \frac{1}{2}Wel(\mathcal{BD}),
\end{aligned}$$

where the inequality follows from the fact that $W^0\left(1,0,\lambda^\circ\right) < W^0\left(\infty,0,1\right)$. (Type-zero players enjoy a higher interim payoff if the auctioneer discloses bids as λ^1_{-i} then increases from λ°—which is close to zero when the discount factor is close to one—to one.) I therefore conclude that disclosing bids also increases expected revenues and expected welfare in this case.

Intuitively, Auction $\mathcal{MD}$ generates a low welfare because it implements a very inefficient profile of drilling strategies: If δ is close to one, the drilling probability λ° is close to zero and type-one players almost surely drill at time two while type-zero players almost surely never drill. Auction $\mathcal{BD}$, however, implements a more efficient profile of drilling strategies: If the drilling cost is low and if $(s_i, s_{-i}) = (0, 1)$, Player i almost surely drills at time two (which is more efficient than not drilling as $c < \mu^0(1)$); If the drilling cost is high and if $(s_i, s_{-i}) = (1, 0)$, Player i does not drill (which is also more efficient than drilling at time two as $\mu^1(0) < c$); If the drilling cost is intermediate and if $(s_i, s_{-i}) = (0, 1)$, Player i learns from Player $-i$'s drilling experience. It is precisely because the disclosure of bids implements a more efficient profile of drilling strategies that players bid more aggressively in this auction format.[20]

I now compare both auctions in terms of total drilling when δ is close to one. Recall that if $k = \mathcal{MD}$ type-zero players almost surely never drill while type-one players almost surely drill at time two. Hence,

$$\Pr\left(\text{Tract } i \text{ is drilled}|\mathcal{MD}\right) \simeq \Pr(s_i=1)\Pr\left(r<b^1(\mathcal{MD})\right). \tag{14}$$

If the auctioneer discloses bids and if c is low, one has:

$$\Pr\left(\text{Tract } i \text{ is drilled}|\mathcal{BD}\right) \simeq \Pr(s_i=0, s_{-i}=1)\Pr\left(r<b^0(\mathcal{BD})\right) + \Pr(s_i=1)\Pr\left(r<b^1(\mathcal{BD})\right). \tag{15}$$

[20] Of course, disclosing bids does not implement the most efficient profile of ex post drilling strategies. Actually, if the discount factor is close to one, the most efficient profile typically prescribes one player to drill at time one and the other one to wait. Observe, however, that in the bid-disclosure case this profile of drilling strategies only happens if the drilling cost is intermediate and if $(s_i, s_{-i}) = (1, 0)$...

(If $s_i = 0$ she will drill almost surely at time two if the other player possesses signal one. If $s_i = 1$, she even drills when $s_{-i} = 0$ as the drilling cost is low.) If c is intermediate, one also has:

$$\begin{aligned} \Pr(\text{Tract } i \text{ is drilled}|\mathcal{BD}) &\simeq \Pr(s_i = 0)\Pr\left(r < b^0(\mathcal{BD})\right)\Pr(V_{-i} = s_{-i} = 1|s_i = 0) \\ &+ \Pr(s_i = 1)\Pr\left(r < b^1(\mathcal{BD})\right). \end{aligned} \tag{16}$$

(If $s_i = 0$ and if $s_{-i} = 1$, Player i waits while Player $-i$ drills. She thus only drills if she wins her tract and if the other player finds oil at time one. A type-one player either drills almost surely at time two (in case $s_{-i} = 1$) or at time one (if $s_{-i} = 0$)). Finally if c is high,

$$\Pr(\text{Tract } i \text{ is drilled}|\mathcal{BD}) \simeq \Pr(s_i = s_{-i} = 1)\Pr\left(r < b^1\right).$$

Recall that in this case no one ever drills if Player i bids zero. Tract i is thus only drilled if she wins her tract and if both players bid "high" (i.e. if $s_i = s_{-i} = 1$). Comparing total expected drilling is not straightforward as the auction design affects both a player's bid and her drilling probability.

If c is high the effect of bid disclosure on total drilling is ambiguous. On the one hand, type-one players bid more aggressively if $k = \mathcal{BD}$. On the other hand, if $k = \mathcal{BD}$ Player i only drills if both players possess signal one. It is straightforward to show that either effect can dominate.[21] Suppose now that c is either intermediate or low. I already argued above that in this case both type-one and type-zero players bid more aggressively if $k = \mathcal{BD}$. Disclosing bids does thus also promote drilling if—conditional on the event that Player i wins her tract—she is more likely to drill it when $k = \mathcal{BD}$. Thus, suppose that Player i wins her tract. Observe that if $s_i = 1$, she will eventually drill—independent of the auction design. Observe also that if $s_i = 0$ and if $k = \mathcal{MD}$, Player i does not drill. If $k = \mathcal{BD}$, however, she would drill her tract if $V_{-i} = s_{-i} = 1$ and if c is intermediate or if $s_{-i} = 1$ and if c is low. Hence, in these cases disclosing bids increases both revenues and total expected drilling.[22] The proposition below summarizes my most important results:

Proposition 5 *Suppose the discount factor δ is close to one. Disclosing bids then raises expected revenues and expected welfare. The effect of bid disclosure on total expected drilling, however, is ambiguous.*

The size of both tracts should also play an important role in the design of auctions. The larger the tracts up for sale, the lower the correlation between both tract values, and the higher the

[21] If c is close to $\mu^1(0)$, for example, it can be checked that disclosing bids increases revenues but decreases total expected drilling. If c is close to ξ^1, however, disclosing bids raises both revenues and total expected drilling.

[22] At the risk of stating the obvious, total expected drilling in Auction k is computed as $2 \times \Pr(\text{Tract } i \text{ is drilled}|k)$.

probability that Players i and $-i$ respectively face a positive and a negative payoff from drilling at the start of the waiting game. Hence, auctioning larger tracts may increase the probability of sequential drilling and, thus, lead to a substantial increase in revenues. This is an interesting avenue for future research.

As mentioned in Section 2, the hazard rate of drilling features a U-shaped pattern. The equilibria analyzed in this paper are consistent with this finding. For at the first drilling date a posterior profile may occur with the characteristic that Player i faces a negative payoff from drilling while Player $-i$'s payoff is positive. Player i then waits, while Player $-i$ drills. In this case both players thus succeed to coordinate their drilling activities. This explains a high probability of drilling in year one. In case both players face a positive payoff from drilling, they play a standard war of attrition, which explains why in years 2, 3, and 4 the hazard rate of drilling is "low". In year 5 the probability of drilling is "high" because of the end-game effect.

References

Avery, Christopher, "Strategic Jump Bidding in English Auctions," *Review of Economic Studies* 65 (1998), 185-210.

Banks, Jeffrey S. and Sobel, Joel, "Equilibrium Selection in Signaling Games," *Econometrica* 55 (1987), 647-661.

Besley, Timothy and Case, Anne, "Diffusion as a Learning Process: Evidence from HYV Cotton," *Princeton University Discussion Paper* # 174, May 1994.

Bolton, Patrick and Harris, Christopher, "Strategic Experimentation," *Econometrica* 67 (1999), 349-374.

Chamley, C. and Gale, D., "Information Revelation and Strategic Delay in a Model of Investment," *Econometrica* 62 (1994), 1065-1085.

Chamley, C., "Rational Herds, Economic Models of Social Learning." Cambridge University Press, 2004b, 402p.

Das Varma, G., "Bidding for a Process Innovation under Alternative Modes of Competition," *International Journal of Industrial Organization* 21 (2003), 15-37.

Farrow, Scott and Rose, Marshall, "Public Information Externalities: Estimates for Offshore Energy Exploration," *Marine Resource Economics*, 7 (1992), 67-82.

Forbes, K.F. and Zampelli, E., "Technology and the Exploratory Success Rate in the U.S. Offshore," *The Energy Journal*, 21 (2000), 109-120.

Foster, Andrew D. and Rosenzweig, Mark R., "Learning by Doing and Learning from Others: Human Capital and Technical Change in Agriculture," *Journal of Political Economy*, 103 (1995), 1176-1209.

Goeree, J., "Bidding for the Future: Signaling in Auctions with an Aftermarket," *Journal of Economic Theory* 108 (2003), 345-364.

Haile, P., "Auctions with Private Uncertainty and Resale Opportunities," *Journal of Economic Theory* 108 (2003), 72-110.

Haile, P., Hendricks K. and Porter, R. "Recent U.S. Offshore Oil and Gas Lease Bidding: A Progress Report" *International Journal of industrial organization*, July 2010, 28(4), 390-396.

Hendricks, K. and Kovenock, D. "Asymmetric Information, Information Externalities, and Efficiency: the Case of Oil Exploration" *Rand Journal of Economics*, 20 (1989), 164-182.

Hendricks, K. and Porter, R. "An Empirical Study of an Auction with Asymmetric Information" *American Economic Review*, December 1988, 78(5), 865-883

Hendricks, K. and Porter, R. "Joint Bidding in Federal OCS Auctions" *American Economic Review*, May 1992, 82(2), 506-511

Hendricks, K. and Porter, R. "The Timing and Incidence of Exploratory Drilling on Offshore Wildcat Tracts" *American Economic Review*, June 1996, 86(3), 388-407

Hendricks, K., Porter, R. and Boudreau, B. "Information, Returns and Bidding Behavior in OCS Auctions" *The Journal of Industrial Economics*, June 1987, 35(4), 512-542

Hendricks, K., Porter, R. and Spady, R. "Random Reservation Prices and Bidding Behavior in OCS Drainage Auctions" *Journal of Law & Economics*, October 1989, 3, S83-S106

Krishna, V., "Auction Theory." Academic Press, 2002, 303p.

Lin, C., "Estimating Strategic Interactions in Petroleum Exploration," *Energy Economics* 31 (2009), 586-594.

Melissas, N., "Bidding and Drilling on Offshore Wildcat Tracts: Technical Appendix," *mimeo* (2014).

Yergin, D., "The Prize: The Epic Quest for Oil, Money & Power." New York: Simon & Schuster, 1991, 887p.

Yergin, D., "The Quest: Energy, Security, and the Remaking of the Modern World" New York: The Penguin Press, 2011, 788p.

Appendix (Not for publication)

Preliminary definitions and results

Let

$$\Delta^{s_i}\left(l_{-i},\lambda^0_{-i},\lambda^1_{-i}\right)\equiv\mu^{s_i}(l_{-i})-c-\delta W^{s_i}\left(l_{-i},\lambda^0_{-i},\lambda^1_{-i}\right). \tag{17}$$

I henceforth call Player i's expected payoff in the waiting game (excluding her bidding costs) as her *continuation* payoff. Stated differently, it represents her expected payoff conditional on the event that *both* players win their tracts and it excludes her bid. Let u^{s_i} denote her *interim* payoff: It represents her expected payoff conditional on the event that she wins her tract and after excluding her bidding cost. Let $U^{s_i}(b)\equiv b(u^{s_i}-b)$ denote the *unconditional and net expected payoff* of Player i.

Let $F^{s_i}_e(b)$ denote the probability that Player-type i^{s_i} submits a bid $b_i\leq b$ in candidate equilibrium e. Let $H^{s_i}(b_{-i})\equiv\Pr(s_{-i}=1|s_i)F^1_e(b_{-i})+\Pr(s_{-i}=0|s_i)F^0_e(b_{-i})$ denote Player i's expected distribution of b_{-i} conditional on her signal s_i in candidate equilibrium e. Let $B^{s_i}_e$ denote the set of bids that lie on the equilibrium path of some candidate equilibrium e and that may be submitted by Player-type i^{s_i}. Formally, $B^1_e\equiv\left\{b:\Pr_e(b_{-i}=b|s_{-i}=1)>0\text{ or }\frac{\partial F^1_e(b)}{\partial b}>0\right\}$. Similarly, $B^0_e\equiv\left\{b:\Pr_e(b_{-i}=b|s_{-i}=0)>0\text{ or }\frac{\partial F^0_e(b)}{\partial b}>0\right\}$.

Suppose Player i submits an out-of-equilibrium bid b. Using Bayes's rule, Player $-i$'s out-of-equilibrium belief can then be rewritten as

$$\Pr(s_i=1|s_{-i},b_i=b)=\frac{1}{1+\frac{\Pr(s_i=0|s_{-i})}{\Pr(s_i=1|s_{-i})}\frac{1}{l_i(b)}}.$$

Fixing s_{-i}, there is thus an increasing relationship between his out-of-equilibrium belief $\Pr(s_i=1|s_{-i},b_i=b)$ and the likelihood ratio $l_i(b)$. In what follows, I will therefore sometimes refer to $l_i(b)$ as Player $-i$'s out-of-equilibrium belief.

Consider a candidate equilibrium in which Player-type i^{s_i} gets an unconditional and net expected payoff denoted by $(U^{s_i})^*$. Suppose Player-type i^{s_i} deviates from this equilibrium and bids b. Suppose out-of-equilibrium beliefs specify that Player $-i$ updates his beliefs using some $l_i(b)$. This out-of-equilibrium belief, together with her beliefs about his type, determine some non-coordination continuation equilibrium. Integrating over all possible bids of Player $-i$, one obtains Player i's interim payoff if she bids b. This knowledge, in turn, allows us to compute $U^{s_i}(l_i,b)$, the unconditional and net expected payoff of Player-type i^{s_i} if she bids b and if Player $-i$ updates his beliefs using l_i. Call $T^{s_i}(b)$ the set of l_i's which ensure that Player-type i^{s_i} does not lose from submitting b. Formally, $T^{s_i}(b)\equiv\{l_i\in[0,\infty)\cup\{\infty\}:U^{s_i}(l_i,b)\geq(U^{s_i})^*\}$. Let $s'_i\in\{0,1\}$ and $s'_i\neq s_i$. An out-of-equilibrium belief is said to be intuitive if it satisfies the following restriction:

$$\text{If }\{\emptyset\}=T^{s_i}(b)\neq T^{s'_i}(b),\Pr(\text{Player }i\text{'s signal}=s_i|s_{-i},b_i=b)=0\ \forall s_{-i}. \tag{18}$$

Banks and Sobel's (1987) divinity criterion puts more restrictions on out-of-equilibrium beliefs. In particular, an out-of-equilibrium belief is said to be divine if it satisfies the following three restrictions:

$$\begin{aligned}&\text{If }\{\emptyset\}=T^{s_i}(b)\neq T^{s'_i}(b),\Pr(\text{Player }i\text{'s signal}=s_i|s_{-i},b_i=b)=0\ \forall s_{-i},\\&\text{if }\{\emptyset\}\neq T^{s_i}(b)\subset T^{s'_i}(b),\Pr(\text{Player }i\text{'s signal}=s_i|s_{-i},b_i=b)\leq\Pr(\text{Player }i\text{'s signal}=s_i|s_{-i})\ \forall s_{-i},\text{ and}\\&\text{if }\{\emptyset\}\neq T^{s_i}(b)=T^{s'_i}(b),\Pr(\text{Player }i\text{'s signal}=s_i|s_{-i},b_i=b)=\Pr(\text{Player }i\text{'s signal}=s_i|s_{-i})\ \forall s_{-i}.\end{aligned} \tag{19}$$

In this Appendix I assume that the discount factor δ is sufficiently high such that

Assumption 3 $\mu^1(1)-c<\delta W^1(1,0,1)$ *and* $\mu^1(\infty)-c<\delta W^1(\infty,0,1)$.

The first inequality of Assumption 3 states that Player-type i^1 prefers to wait if she expects Player-type $-i^1$ to drill with probability one and if she did not learn anything about his type through his bid (i.e. when $l_{-i}=1$). The assumption implies that—in the absence of the auction stage—there exists an equilibrium in the waiting game in which type-one players drill with some probability to make each other indifferent between drilling and waiting. The second inequality states that Player-type i^1 also prefers to wait if—after the auction stage—she infers that $s_{-i} = 1$, and if she expects him to drill with probability one.

Lemma 1 *Suppose $l_{-i} \in (0, \infty)$. Player i's payoff from waiting $W^{s_i}(\cdot)$ is then strictly increasing in $\left(\lambda^0_{-i}, \lambda^1_{-i}\right)$. If $l_{-i} = 0$, $W^{s_i}(\cdot)$ is strictly increasing in λ^0_{-i} and independent of λ^1_{-i}. If $l_{-i} = \infty$, $W^{s_i}(\cdot)$ is strictly increasing in λ^1_{-i} and independent of λ^0_{-i}.*

Proof: Observe that Equation 1 can be rewritten as:

$$\begin{aligned} W^{s_i}(l_{-i}, \lambda^0_{-i}, \lambda^0_{-i}) &= \Pr(V_i = 1|s_i, l_{-i}) - c + (1 - \mathcal{I}) \Pr(V_{-i} = 0|s_i, l_{-i}) \Big[c - \Pr(V_i = 1|s_i, V_{-i} = 0)\Big] \\ &+ \mathcal{I} \Pr(V_{-i} = 0, a_{-i} = drill|s_i, l_{-i}) \Big[c - \Pr(V_i = 1|s_i, V_{-i} = 0)\Big] \\ &- (1 - \mathcal{I}) \Pr(V_{-i} = 1, a_{-i} = wait|s_i, l_{-i}) \Big[\Pr(V_i = 1|s_i, V_{-i} = 1) - c\Big], \end{aligned}$$

where $\mathcal{I} = 1$ if $\Pr(V_i = 1|s_i, l_{-i}, a_{-i} = wait) \geq c$ and $\mathcal{I} = 0$ otherwise. By Assumption 2, the terms between square brackets are positive. The Lemma then follows from the fact that $\Pr(V_{-i} = 0, a_{-i} = drill|s_i, l_{-i})$ and $\Pr(V_{-i} = 1, a_{-i} = wait|s_i, l_{-i})$ are respectively increasing and decreasing in $\left(\lambda^0_{-i}, \lambda^1_{-i}\right)$ whenever $l_{-i} \in (0, \infty)$.■

Recall from Subsection 3.1 that $\Pr(V_i = 1) = \nu \ \forall i$. This assumption implies that $\Pr(V_i = 1, V_{-i} = 0) = \Pr(V_i = 0, V_{-i} = 1)$. Rewriting this equality yields $(1 - \nu)(1 - \rho^0) = \nu(1 - \rho^1)$.

Lemma 2 *Suppose p is close to one and that Assumption 2 holds. The parameters ρ^0 and ρ^1 must then also be close to one.*

Proof: Recall from Assumption 2 that $\Pr(V_i = 1|s_i = 1, V_{-i} = 0) < c$. Using Bayes's rule this inequality is equivalent to $\frac{1-c}{c} < \frac{1-p}{p} \frac{\rho^0}{1-\rho^0}$. If p is close to one, the inequality is only satisfied if ρ^0 is close to one. As $\nu(1 - \rho^1) = (1 - \nu)(1 - \rho^0)$, ρ^1 must then also be close to one.■

Lemma 3 $\Delta^1(0, 1, 1) < \Delta^1(\infty, 1, 1)$.

Proof: Using Equation 17, $\Delta^1(0, 1, 1) < \Delta^1(\infty, 1, 1)$ is equivalent to:

$$\begin{aligned} & (1 - \delta) \Big[\Pr(V_i = 1|s_i = 1, s_{-i} = 0) - c\Big] - \delta \Pr(V_{-i} = 0|s_i = 1, s_{-i} = 0) \Big[c - \Pr(V_i = 1|s_i = 1, V_{-i} = 0)\Big] \\ < \ & (1 - \delta) \Big[\Pr(V_i = 1|s_i = 1, s_{-i} = 1) - c\Big] - \delta \Pr(V_{-i} = 0|s_i = 1, s_{-i} = 1) \Big[c - \Pr(V_i = 1|s_i = 1, V_{-i} = 0)\Big], \end{aligned}$$

which under Assumption 2 is satisfied.■

Lemma 4 *Suppose $c < \mu^1(0)$. Then $\Delta^1(l_{-i}, 0, 1)$ is strictly decreasing in l_{-i}, $\Delta^1(l_{-i}, 0, 1) < 0 \Leftrightarrow l_{-i} > \tilde{l}$ ($\tilde{l} \in (0, 1)$), and $\Delta^1(l_{-i}, 1, 1) < 0$, $\forall l_{-i}$. Suppose $\mu^1(0) < c$. Then $\Delta^1(l_{-i}, 0, 1) < 0$ and $\Delta^1(l_{-i}, 1, 1) < 0 \ \forall l_{-i}$.*

Proof: Observe that $\Delta^1(l_{-i}, 0, 1) = (1 - q)\Delta^1(0, 0, 1) + q\Delta^1(\infty, 0, 1)$, where $q \equiv \Pr(s_{-i} = 1|s_i = 1, l_{-i})$. One has $\frac{\partial \Delta^1(l_{-i}, 0, 1)}{\partial l_{-i}} = \frac{\partial q}{\partial l_{-i}}[\Delta^1(\infty, 0, 1) - \Delta^1(0, 0, 1)]$. As $\frac{\partial q}{\partial l_{-i}} > 0$, and as the term between square brackets is independent of l_{-i}, I conclude that—generically—$\Delta^1(l_{-i}, 0, 1)$ either strictly increases or strictly decreases in l_{-i}.

It is easy to check that $\mu^1(0) > c \Leftrightarrow \Delta^1(0, 0, 1) > 0$. Hence, if the latter inequality is satisfied, $\Delta^1(l_{-i}, 0, 1)$ is strictly decreasing in l_{-i} as $\Delta^1(1, 0, 1) < 0$. This also implies the existence of a unique $\tilde{l} \in (0, 1)$ such that $\Delta^1(\tilde{l}, 0, 1) = 0$. Observe that $\Delta^1(l_{-i}, 1, 1) = (1 - q)\Delta^1(0, 1, 1) + q\Delta^1(\infty, 1, 1)$. Observe also that $\Delta^1(0, 1, 1) <$

$\Delta^1(\infty,1,1) = \Delta^1(\infty,0,1) < 0$, where the first inequality was proven in Lemma 3, where the equality follows from the fact that a player's payoff from waiting is independent of λ^0_{-i} when she knows that he is a type-one player, and where the second inequality was proven above. I conclude that $\Delta^1(l_{-i},1,1) < 0$.

Recall from above that if $\mu^1(0) < c$, $\Delta^1(0,0,1) < 0$. By Assumption 3, $\Delta^1(\infty,0,1) < 0$. Hence $\Delta^1(l_{-i},0,1)$ is always negative as $\Delta^1(l_{-i},0,1)$ is monotone in l_{-i}. From Lemma 1 we know that $\Delta^1(l_{-i},1,1) \leq \Delta^1(l_{-i},0,1)$. Hence, for all l_{-i}, $\Delta^1(l_{-i},1,1) < 0$.■

Lemma 5 *$\Delta^0(l_{-i},0,1) < 0$ and $\Delta^0(l_{-i},1,1) < 0$, $\forall l_{-i}$.*

Proof: Note that

$$\begin{aligned}
\Delta^1(1,1,1) &= \Pr(s_{-i}=1|s_i=1)\Delta^1(\infty,1,1) + \Pr(s_{-i}=0|s_i=1)\Delta^1(0,1,1) \\
&< \Delta^1(1,0,1) < 0,
\end{aligned}$$

where the first inequality follows from the fact that if λ^0_{-i} increases from zero to one, this increases a player's payoff from waiting and, thus, decreases $\Delta^{s_i}(\cdot)$. The second inequality holds by Assumption 3. Recall from Lemma 3 that $\Delta^1(0,1,1) < \Delta^1(\infty,1,1)$. As $\Delta^1(1,1,1) < 0$, I conclude that $\Delta^1(0,1,1) < 0$.

Using Equation 17, $\Delta^0(\infty,1,1) < \Delta^1(0,1,1)$ is equivalent to

$$\begin{aligned}
&\Pr(V_i = V_{-i} = 1|s_i = 0, s_{-i} = 1)[1-\delta(1-c)] + \Pr(V_i = 1, V_{-i} = 0|s_i = 0, s_{-i} = 1) \\
+ \; &\delta \Pr(V_i = 0, V_{-i} = 1|s_i = 0, s_{-i} = 1)c < \Pr(V_i = V_{-i} = 1|s_i = 1, s_{-i} = 0)[1-\delta(1-c)] \\
+ \; &\Pr(V_i = 1, V_{-i} = 0|s_i = 1, s_{-i} = 0) + \delta \Pr(V_i = 0, V_{-i} = 1|s_i = 1, s_{-i} = 0)c,
\end{aligned}$$

which is satisfied as $p > \frac{1}{2}$. As $\Delta^1(0,1,1) < 0$, I conclude that $\Delta^0(\infty,1,1) < 0$.

Observe that $\Delta^0(l_{-i},0,1) = (1-q)\Delta^0(0,0,1) + q\Delta^0(\infty,0,1)$, where $q \equiv \Pr(s_{-i} = 1|s_i = 0, l_{-i})$. Observe that $\Delta^0(0,0,1) < 0$ under Assumption 1. Observe also that $\Delta^0(\infty,0,1) = \Delta^0(\infty,1,1) < 0$, where the equality rests on the fact that λ^0_{-i} does not affect Player i's payoff from waiting if the other player possesses signal one, and where the inequality is proven in my previous paragraph. Hence, $\Delta^0(l_{-i},0,1)$ is a weighted average between two negative numbers. This proves the first inequality of the lemma.

Point 2 of the lemma then follows from the fact that a player's payoff from waiting cannot decrease when λ^0_{-i} goes from zero to one.■

Lemma 6 *A type-zero player has more incentives to wait than a type-one player. Formally, $\forall \left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right)$, $\Delta^0\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right) < \Delta^1\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right)$.*

Proof: Observe that, $\Delta^{s_i}(\cdot)$ can be rewritten as:

$$\begin{aligned}
\Delta^{s_i}(\cdot) &= (1-\delta)\Big[\mu^{s_i}(l_{-i}) - c\Big] - \delta(1-\mathcal{I})\Pr(V_{-i}=0|s_i,l_{-i})\Big[c - \Pr(V_i=1|s_i,V_{-i}=0)\Big] \\
&- \delta\mathcal{I}\Pr(V_{-i}=0|s_i,l_{-i})\Pr(a_{-i}=drill|V_{-i}=0,l_{-i})\Big[c - \Pr(V_i=1|s_i,V_{-i}=0)\Big] \\
&+ \delta(1-\mathcal{I})\Pr(V_{-i}=1|s_i,l_{-i})\Pr(a_{-i}=wait|V_{-i}=1,l_{-i})\Big[\Pr(V_i=1|s_i,V_{-i}=1) - c\Big],
\end{aligned}$$

where $\mathcal{I} = 1$ if $\Pr(V_i = 1|s_i, l_{-i}, a_{-i} = wait) \geq c$ and $\mathcal{I} = 0$ otherwise. Recall from Assumption 2 that the last three terms between square brackets are positive. Observe also that $\mu^0(l_{-i}) < \mu^1(l_{-i})$, that $\Pr(V_{-i} = 0|0, l_{-i}) > \Pr(V_{-i} = 0|1, l_{-i})$, that $\Pr(V_i = 1|1, V_{-i} = 0) \geq \Pr(V_i = 1|0, V_{-i} = 0)$, that $\Pr(V_{-i} = 1|1, l_{-i}) > \Pr(V_{-i} = 1|0, l_{-i})$, and that $\Pr(V_i = 1|1, V_{-i} = 1) \geq \Pr(V_i = 1|0, V_{-i} = 0)$. Those observations, combined with the fact that all the other probabilities are independent of s_i, prove the lemma.■

Lemma 7 *A type-one player's payoff from waiting cannot be lower than the one of a type-zero player. Formally, $W^0\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right) \leq W^1\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right)$.*

Proof: Observe that $W^{s_i}\left(l_{-i}, \lambda^0_{-i}, \lambda^1_{-i}\right)$ can be rewritten as

$$\begin{aligned} W^{s_i}(\cdot) &= \mu^{s_i}(l_{-i})\left[\Pr(V_{-i}=1, a_{-i}=\text{drill}|V_i=1, l_{-i})(1-c) + \Pr(a_{-i}=\text{wait}|V_i=1, l_{-i})\mathcal{I}(1-c)\right] \\ &- \left(1-\mu^{s_i}(l_{-i})\right)\left[\Pr(V_{-i}=1, a_{-i}=\text{drill}|V_i=0, l_{-i})c + \Pr(a_{-i}=\text{wait}|V_i=0, l_{-i})\mathcal{I}c\right], \end{aligned}$$

where $\mathcal{I}=1$ if $\Pr(V_i=1|s_i, l_{-i}, a_{-i}=\text{wait}) \geq c$ and $\mathcal{I}=0$ otherwise. The lemma then follows from the fact that $\mu^0(l_{-i}) < \mu^1(l_{-i})$.■

Proof of Proposition 1

Consider a candidate equilibrium in which type-one players randomize their bids in B^1 according to an arbitrary c.d.f. F. Let $t(b_i)$ denote Player-type i^1's drilling probability given that she bid b_i. Let $\lambda^1_i \equiv \int_{B^1} t(b_i)dF$ and $q \equiv \Pr(s_{-i}=1, r<b_{-i}|s_i=1, r<b_i)$. Let $B^d \equiv \left\{b_i \in B^1 : \xi^1 - c \geq \delta W^1\left(q(b_i), 0, \lambda^1_{-i}\right)\right\}$ and $B^w \equiv \left\{b_i \in B^1 : \xi^1 - c < \delta W^1\left(q(b_i), 0, \lambda^1_{-i}\right)\right\}$. An equilibrium in which $B^d = \{\emptyset\}$ does not exist as type-one players cannot strictly prefer to wait if no one drills. Observe that $\forall b_i \in B^d$, Player-type i^1's interim payoff is equal to $\xi^1 - c$. As $\frac{1}{2}\left(\xi^1 - c\right) \in \arg\max_b b\left(\xi^1 - c - b\right)$, and as Player-type i^1's interim payoff is bounded below by $\xi^1 - c$, in any candidate equilibrium $B^d = \left\{\frac{1}{2}\left(\xi^1 - c\right)\right\}$. Suppose that B^w is non-empty. Observe that $\forall b' \in B^w$, $q(b') \leq q\left(\frac{1}{2}\left(\xi^1 - c\right)\right)$ which implies that

$$W^1\left(q(b'), 0, \lambda^1_{-i}\right) \leq W^1\left(q\left(\frac{1}{2}\left(\xi^1 - c\right)\right), 0, \lambda^1_{-i}\right).$$

This, however, contradicts my finding that if Player-type i^1 bids $\frac{1}{2}\left(\xi^1 - c\right)$ she must weakly prefer to drill.■

Characterization of all the non-coordinating continuation equilibria with their associated payoffs.

I now compute equilibrium strategies and continuation payoffs (i.e. excluding the bidding costs) in the continuation game when both players win their tracts taking (l_i, l_{-i}) as given. With a slight abuse of notation, in the Lemma below λ^1_i denotes the probability with which Player-type i^1 must drill such that $\Delta^1(l_i, 0, \lambda^1_i) = 0$. Similarly, λ^0_i denotes the probability with which Player-type i^0 must drill such that $\Delta^1(l_i, \lambda^0_i, 1) = 0$. I denote a non-coordinating continuation equilibrium in which Player-type i^1 chooses λ^1_i such that Player-type $-i^0$ is indifferent and in which Player-type $-i^0$ chooses λ^0_{-i} such that Player-type i^1 is indifferent by the symbol II.

Lemma 8 *Suppose both players win their tracts. Let $a \in [0,1]$. In any non-coordinating continuation equilibrium, Player-type i^1 gets:*

$\delta W^1(l_{-i}, 0, a)$	*if*	$\Delta^1(l_{-i}, 0, 0) < 0$ *and* $0 = \Delta^1(l_i, 0, 0)$, *or*
$\delta W^1(l_{-i}, 0, 1)$	*if*	$\Delta^1(l_{-i}, 0, 0) < 0$ *and* $0 < \Delta^1(l_i, 0, 0)$,
	or if	$\Delta^1(l_{-i}, 0, 1) < 0 \leq \Delta^1(l_{-i}, 0, 0)$ *and* $\Delta^1(l_i, 1, 1) < 0 < \Delta^1(l_i, 0, 1)$
		and $\max_i\{\Delta^0(l_i, 0, 0)\} < 0$, *or*
$\max\{0, \mu^1(l_{-i}) - c\}$	*if*	*none of the above conditions are satisfied.*

Suppose both players win their tracts. In any non-coordinating continuation equilibrium, Player-type i^0 gets:

$$
\begin{array}{lll}
\delta W^0\left(l_{-i},0,\lambda^1_{-i}\right) & \textit{if} & \Delta^1(l_{-i},0,1)\le 0\le \Delta^1(l_{-i},0,0) \textit{ and } \Delta^1(l_i,0,1)\le 0\le \Delta^1(l_i,0,0),\\
 & \textit{unless} & \Delta^1(l_{-i},0,1)=0 \textit{ and } \Delta^1(l_i,0,1)<0<\Delta^1(l_i,0,0)\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}\ge 0 \textit{ and Players focus on cont. eq. } II, \textit{ or}\\
\delta W^0(l_{-i},\lambda^0_{-i},1) & \textit{if} & \Delta^1(l_{-i},1,1)<0<\Delta^1(l_{-i},0,1) \textit{ and } \Delta^1(l_i,0,1)\le 0<\Delta^1(l_i,0,0)\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}\ge 0, \textit{ or}\\
\delta W^0(l_{-i},0,a) & \textit{if} & \Delta^1(l_{-i},1,1)<0<\Delta^1(l_{-i},0,1) \textit{ and } \Delta^1(l_i,0,1)=0<\Delta^1(l_i,0,0)\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}<0,\\
 & \textit{or if} & \Delta^1(l_{-i},0,0)<0 \textit{ and } 0=\Delta^1(l_i,0,0), \textit{ or}\\
\delta W^0(l_{-i},0,1) & \textit{if} & 0\le \Delta^1(l_{-i},0,0) \textit{ and } \Delta^1(l_i,1,1)<0<\Delta^1(l_i,0,1)\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}<0,\\
 & \textit{or if} & \Delta^1(l_{-i},0,0)<0 \textit{ and } 0<\Delta^1(l_i,0,0),\\
 & \textit{or if} & \Delta^1(l_{-i},0,1)=0 \textit{ and } \Delta^1(l_i,0,1)<0<\Delta^1(l_i,0,0)\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}\ge 0 \textit{ and Players focus on cont. eq. } II, \textit{ or}\\
\mu^0(l_{-i})-c & \textit{if} & \Delta^1(l_{-i},0,1)\le 0<\Delta^1(l_{-i},0,0) \textit{ and } \Delta^1(l_i,1,1)<0<\Delta^1(l_i,0,1)\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}\ge 0,\\
 & \textit{or if} & \Delta^1(l_{-i},0,1)<0<\Delta^1(l_{-i},0,0) \textit{ and } \Delta^1(l_i,0,1)=0\\
 & & \textit{and } \max_i\{\Delta^0(l_i,0,0)\}\ge 0 \textit{ and Players focus on cont. eq. } II, \textit{ or}\\
0 & \textit{if} & \textit{none of the above conditions are satisfied.}
\end{array}
$$

Proof: I first analyze the case in which $\Delta^1(l_2,0,1)\le 0\le \Delta^1(l_2,0,0)$ and $\Delta^1(l_1,0,1)\le 0\le \Delta^1(l_1,0,0)$. This case can be broken down in the following subcases:

$$
\begin{array}{llll}
(i) & 0=\Delta^1(l_2,0,0) & \text{and} & 0=\Delta^1(l_1,0,0),\\
(ii) & \Delta^1(l_2,0,1)<0<\Delta^1(l_2,0,0) & \text{and} & 0=\Delta^1(l_1,0,0),\\
(iii) & \Delta^1(l_2,0,1)=0 & \text{and} & 0=\Delta^1(l_1,0,0),\\
(iv) & \Delta^1(l_2,0,1)<0<\Delta^1(l_2,0,0) & \text{and} & \Delta^1(l_1,0,1)<0<\Delta^1(l_1,0,0),\\
(v) & \Delta^1(l_2,0,1)=0 & \text{and} & \Delta^1(l_1,0,1)<0<\Delta^1(l_1,0,0),\\
(vi) & \Delta^1(l_2,0,1)=0 & \text{and} & \Delta^1(l_1,0,1)=0.
\end{array}
$$

Consider the first four subcases. Observe that $\Delta^1(0,0,0)=0\Leftrightarrow \mu^1(l_i=0)=c$. Hence, generically, $\Delta^1(l_i,0,0)=0$ only if $l_i>0$. Using Lemma 1 this implies that, generically, $\Delta^1(l_i,0,1)<0$ whenever $\Delta^1(l_i,0,0)=0$. Observe also that there exists a continuation equilibrium in which Player-type i^1 chooses λ^1_i such that $\Delta^1(l_i,0,\lambda^1_i)=0$. (In the first three subcases, for example, $\lambda^1_1=0$. In the fourth subcase, it follows from Lemma 1 that there exists a unique $\lambda^1_1\in(0,1)$ such that Player-type 2^1 is indifferent, i.e. such that $\Delta^1(l_1,0,\lambda^1_1)=0$.) From Lemma 6 we know that $\Delta^0(l_i,0,\lambda^1_i)<\Delta^1(l_i,0,\lambda^1_i)=0$. Hence, in this continuation equilibrium type-zero players strictly prefer to wait. As type-one players are indifferent between drilling and waiting, Player-type i^1 gets $\mu^1(l_{-i})-c$. As type-zero players wait, Player-type i^0 gets $\delta W^0\left(l_{-i},0,\lambda^1_{-i}\right)$. I now argue that this is the unique non-coordinating continuation equilibrium in these subcases. It follows from lemma 6 that a continuation equilibrium in which player-types i^0 and i^1 are indifferent does not exist. Suppose there exists a continuation equilibrium in which player-types i^0 and $-i^1$ are indifferent. Using Lemma 6, $\Delta^0\left(l_{-i},\lambda^0_{-i},\lambda^1_{-i}\right)=0<\Delta^1\left(l_{-i},\lambda^0_{-i},\lambda^1_{-i}\right)$. Hence, in this continuation equilibrium $\lambda^1_i=1$. This, however, contradicts my assumption that Player-type $-i^1$ is indifferent as $\Delta^1(l_i,\lambda^0_i,1)\le\Delta^1(l_i,0,1)<0$. Similarly, suppose there exists a continuation equilibrium in which Player-types i^0 and $-i^0$ are indifferent. As type-zero players are indifferent, it follows from Lemma 6 that both type-one players strictly prefer to drill. We then run, however, into the following contradiction $0=\Delta^0(l_{-i},\lambda^0_{-i},1)<\Delta^1(l_{-i},\lambda^0_{-i},1)\le\Delta^1(l_{-i},0,1)\le 0$.

Consider subcase (v). As above there exists a continuation equilibrium in which Player-type i^1 chooses λ_i^1 such that $\Delta^1(l_i, 0, \lambda_i^1) = 0$. This continuation equilibrium gives rise to the same payoffs as the ones I computed for my first four subcases. It follows from Lemma 4 that $\Delta^1(l_2, 0, 1) = 0$ only if $l_2 = \tilde{l} < 1$. This implies that $\Delta^0(l_2, 0, 0) < 0$. Hence, in any continuation equilibrium Player-type 1^0 strictly prefers to wait. If, additionally, $\Delta^0(l_1, 0, 0) < 0$ continuation equilibrium I is the unique non-coordinating one. Thus, suppose that $\Delta^0(l_1, 0, 0) \geq 0$. Recall from Lemma 5 that $\Delta^0(l_1, 0, 1) < 0$. It then follows from Lemma 1 that there exists a λ_1^1 such that $\Delta^0(l_1, 0, \lambda_1^1) = 0$. Recall from Lemma 3 that if Player-type 2^0 is indifferent, Player-type 2^1 strictly prefers to drill. Furthermore, if Player-type 2^0 drills with probability zero, Player-type 1^1 is indifferent as $\Delta^1(l_2, 0, 1) = 0$. Hence, if $\Delta^0(l_1, 0, 0) \geq 0$, there exists another non-coordinating equilibrium. I denote this continuation equilibrium with the symbol II. Observe that in this continuation equilibrium Player-type 1^1 gets $\mu^1(l_2) - c$, Player-type 2^1 gets $\mu^1(l_1) - c$, Player-type 2^0 gets $\mu^0(l_1) - c$, while Player-type 1^0 gets $\delta W^0(l_2, 0, 1)$.

Consider subcase (vi). It follows from Lemma 4 that in this case $l_1 = l_2 = \tilde{l} < 1$. As $\tilde{l} < 1$, $\Delta^0(l_i, 0, 0) < 0$. Hence, type-zero players strictly prefer to wait. In the unique non-coordinating continuation equilibrium Player-type i^1 thus chooses λ_i^1 such that $\Delta^1(l_i, 0, \lambda_i^1) = 0$. This continuation equilibrium gives rise to the same payoffs as the ones I detailed in my first four subcases.

I am left to analyze the following subcases:

$$
\begin{array}{llll}
(vii) & \Delta^1(l_2,0,0) < 0 & \text{and} & \Delta^1(l_1,0,0) < 0, \\
(viii) & \Delta^1(l_2,0,0) = 0 & \text{and} & \Delta^1(l_1,0,0) < 0, \\
(ix) & 0 < \Delta^1(l_2,0,0) & \text{and} & \Delta^1(l_1,0,0) < 0, \\
(x) & \Delta^1(l_2,1,1) < 0 < \Delta^1(l_2,0,1) & \text{and} & \Delta^1(l_1,0,1) < 0 = \Delta^1(l_1,0,0), \\
(xi) & \Delta^1(l_2,1,1) < 0 < \Delta^1(l_2,0,1) & \text{and} & \Delta^1(l_1,0,1) < 0 < \Delta^1(l_1,0,0), \\
(xii) & \Delta^1(l_2,1,1) < 0 < \Delta^1(l_2,0,1) & \text{and} & \Delta^1(l_1,0,1) = 0 < \Delta^1(l_1,0,0), \\
(xiii) & \Delta^1(l_2,1,1) < 0 < \Delta^1(l_2,0,1) & \text{and} & \Delta^1(l_1,1,1) < 0 < \Delta^1(l_1,0,1).
\end{array}
$$

Recall that $\Delta^1(l_i, 0, 0) < 0$ only if $c > \mu^1(0)$. As $\mu^0(\infty) \leq \mu^1(0)$, in subcases ($vii$), ($viii$) and ($ix$) Player-types 2^1, 1^0 and 2^0 strictly prefer to wait. Furthermore, in subcase (vii), $\Delta^1(l_2, 0, 0) < 0$. Hence, in this subcase Player-type 1^1 never invests and everyone gets zero. In subcase ($viii$), $\Delta^1(l_2, 0, 0) = 0$. Hence, it is best response for Player-type 1^1 to drill with probability $a \in [0, 1]$. In this subcase Player-type 1^1 gets $\mu^1(l_2) - c$, Player-type 1^0 gets zero, Player-type 2^1 gets $\delta W^1(l_1, 0, a \in [0, 1])$, and Player-type 2^0 gets $\delta W^0(l_1, 0, a \in [0, 1])$. In subcase ($ix$), $0 < \Delta^1(l_2, 0, 0)$. Hence, it is a best response for Player-type 1^1 to drill with probability one. Payoffs are the same as the ones in my previous subcase except that a should be replaced by one.

Consider subcases (x) to ($xiii$). It follows from Lemma 4 that $0 < \Delta^1(l_2, 0, 1)$ only if $l_2 < \tilde{l} < 1$. Hence, $\Delta^0(l_2, 0, 0) < 0$. Suppose, additionally, that $\Delta^0(l_1, 0, 0) < 0$. Then any strategy which prescribes a type-zero player to drill with a positive probability is dominated. Hence, $\lambda_1^0 = \lambda_2^0 = 0$. As $0 < \Delta^1(l_2, 0, 1)$ (and as $\lambda_2^0 = 0$) any strategy which prescribes Player-type 1^1 to wait is dominated. Hence, $\lambda_1^1 = 1$. In subcases (x) and (xi), $\Delta^1(l_1, 0, 1) < 0$. As $\lambda_1^0 = 0$ and as $\lambda_1^1 = 1$ any strategy which prescribes Player-type 2^1 to drill with positive probability is dominated. Hence, in the unique continuation equilibrium Player-type 1^1 gets $\mu^1(l_2) - c$, Player-type 2^1 gets $\delta W^1(l_1, 0, 1)$, Player-type 1^0 gets zero, and Player-type 2^0 gets $\delta W^0(l_1, 0, 1)$. In subcase (xii), $\Delta^1(l_1, 0, 1) = 0$. Hence, it is a best response for Player-type 2^1 to drill with probability $a \in [0, 1]$. In this non-coordinating equilibrium, Player-type 1^1 gets $\mu^1(l_2) - c$, Player-type 2^1 gets $\mu^1(l_1) - c$, Player-type 1^0 gets $\delta W^0(l_2, 0, a)$, and Player-type 2^0 gets $\delta W^0(l_1, 0, 1)$. In subcase ($xiii$), $\Delta^1(l_1, 0, 1) > 0$. Hence, it is a best response for Player-type 2^1 to drill with probability one. In this unique non-coordinating equilibrium, Player-type 1^1 gets $\mu^1(l_2) - c$, Player-type 2^1 gets $\mu^1(l_1) - c$, Player-type 1^0 gets $\delta W^0(l_2, 0, 1)$, and Player-type 2^0 gets $\delta W^0(l_1, 0, 1)$.

I now analyze subcases (x) to (xii) when $\Delta^0(l_1, 0, 0) \geq 0$. (Recall from above that $\Delta^0(l_2, 0, 0) < 0$ and thus that $\lambda_1^0 = 0$.) In subcase (x), $0 = \Delta^1(l_1, 0, 0)$ which only happens if $c > \mu^1(0)$. As $\mu^0(\infty) \leq \mu^1(0)$, $\Delta^0(l_1, 0, 0) < 0$.

Hence, inequalities $0 = \Delta^1(l_1, 0, 0)$ and $\Delta^0(l_1, 0, 0) \geq 0$ cannot be simultaneously satisfied. Recall from Lemma 5 that $\Delta^0(l_1, 0, 1) < 0$. Using Lemma 1, there exists then a unique λ_1^1 such that $\Delta^0(l_1, 0, \lambda_1^1) = 0$. It then follows from Lemma 6 that $\lambda_2^1 = 1$. Using Lemma 1, there exists a unique λ_2^0 such that $\Delta^1(l_2, \lambda_2^0, 1) = 0$. I now argue that this constitutes the unique non-coordinating continuation equilibrium. Recall that Lemma 6 precludes the existence of a continuation equilibrium in which Player-types 2^0 and 2^1 are indifferent. Suppose there exists a continuation equilibrium in which Player-types 1^1 and 2^1 are indifferent between drilling and waiting. From Lemma 6, this implies that Player-type 2^0 prefers to wait. As $0 < \Delta^1(l_2, 0, 1) \leq \Delta^1(l_2, 0, \lambda_2^1)$, Player-type 2^1 can then not make Player-type 1^1 indifferent, a contradiction. Finally, Observe that in the unique non-coordinating equilibrium, Player-type 1^1 gets $\mu^1(l_2) - c$, Player-type 2^1 gets $\mu^1(l_1) - c$, Player-type 1^0 gets $\delta W^0(l_2, \lambda_2^0, 1)$, and Player-type 2^0 gets $\mu^0(l_1) - c$.

Finally, observe that in subcase (*xiii*) both l_1 and l_2 are less than one. This implies that type-zero players wait. As $0 < \Delta^1(l_2, 0, 1)$ and as $0 < \Delta^1(l_1, 0, 1)$, in any continuation equilibrium $\lambda_1^1 = \lambda_2^1 = 1$. In the unique continuation equilibrium, Player-type i^1 gets $\mu^1(l_{-i}) - c$, while Player-type i^0 gets $\delta W^0(l_{-i}, 0, 1)$.

Summarizing all those subcases, one obtains Lemma 8.■

Equilibrium outcomes when $\mu^1(0) < c < \xi^1$

I first argue that, if $\mu^1(0) < c$, in any non-coordinating equilibrium $\Delta^1(l_{-i}, 0, 0) \geq 0$ when $b_{-i} \in B_e^1$. Recall that $\Delta^1(l_{-i}, 0, 0) \geq 0 \Leftrightarrow \mu^1(l_{-i}) \geq c$. If $b_{-i} \in B_e^1 \setminus B_e^0$, the inequality obviously holds as $c < \xi^1 < \mu^1(\infty) = \mu^1\left(l_{-i}\left(b_{-i} \in B_e^1 \setminus B_e^0\right)\right)$. Thus, suppose that $b_{-i} \in B_e^0 \cap B_e^1$. Suppose Player-type i^1 wins her tract. Suppose she always (i.e. for all b_{-i}) drills at time one. This continuation strategy yields her an interim payoff equal to $\xi^1 - c > 0$. This interim payoff represents a lower bound: For example, if she were to drill at time one if and only if her time-one posterior $\mu^1(l_{-i})$ exceeds the drilling cost c, she would get an even higher interim payoff. Hence, any strategy which prescribes a type-one player to bid zero is dominated. Next, suppose there exists a non-coordinating equilibrium in which $\Delta^1(l_{-i}, 0, 0) < 0$ for some bid $b \in B_e^0 \cap B_e^1$. As $\mu^0(l_{-i}) < \mu^1(l_{-i})$, it follows that in such a candidate non-coordinating equilibrium $\lambda_i^1 = \lambda_i^0 = 0$. As $\mu^0(\infty) \leq \mu^1(0) < c$, Player-type $-i^0$'s interim payoff is then equal to zero. Hence, such a candidate non-coordinating equilibrium only exists if $0 \in B_e^0 \cap B_e^1$. This, however, contradicts my earlier observation that type-one players never bid zero. Finally, observe that as $\mu^1(0) < c$, $\Delta^1(l_{-i}, 0, 0) < 0$ when $b_{-i} \in B_e^0 \setminus B_e^1$. Those insights, combined with Lemma 4, allow me to conclude that if $\mu^1(0) < c$,

$$\begin{aligned} \Delta^1(l_{-i}, 0, 1) < 0 \leq \Delta^1(l_{-i}, 0, 0) \quad &\text{if} \quad b_{-i} \in B_e^1, \text{ and} \\ \Delta^1(l_{-i}, 0, 0) < 0 \quad &\text{if} \quad b_{-i} \in B_e^0 \setminus B_e^1. \end{aligned} \tag{20}$$

I now argue that, if $\mu^1(0) < c$, in any non-coordinating equilibrium Player-type i^1's interim payoff is equal to

$$\int \max\{0, \mu^1(l_{-i}(b_{-i})) - c\} dH^1(b_{-i}), \tag{21}$$

independent of Player $-i$'s beliefs about her type. Obviously, if Player $-i$ did not win his tract, she gets $\max\{0, \mu^1(l_{-i}) - c\}$. I am thus left to show that—independent of his beliefs about her type—she gets the same payoff if he also wins his tract. Suppose first that Player-type i^1 bids b and that Player $-i$ computes a posterior $\mu^1(l_i(b)) > c$. Using Lemma 4, I conclude that in this case $\Delta^1(l_i, 0, 1) < 0 < \Delta^1(l_i, 0, 0)$. On the basis of (20) and of Lemma 8, I conclude that her continuation payoff is then equal to

$$\begin{aligned} &\max\{0, \mu^1(l_{-i}) - c\} \quad &&\text{if} \quad b_{-i} \in B_e^1, \text{ and} \\ &\delta W^1(l_{-i}, 0, 1) &&\text{if} \quad b_{-i} \in B_e^0 \setminus B_e^1. \end{aligned}$$

Observe that if $b_{-i} \in B_e^0 \setminus B_e^1$, Player i infers that $s_{-i} = 0$ with probability one. Therefore, in this case $\delta W^1(l_{-i}, 0, 1) = \max\{0, \mu^1(l_{-i}) - c\} = 0$. Next, suppose that Player-type i^1 bids b and that Player $-i$ computes a posterior $\mu_{-i}(l_i(b)) = c$. Lemma 4 then allows me to conclude that $\Delta^1(l_i, 0, 1) < 0 = \Delta^1(l_i, 0, 0)$ in this case. Using (20) and Lemma 8, her continuation payoff is then equal to

$$\begin{array}{lll} \max\{0, \mu^1(l_{-i}) - c\} & \text{if} & b_{-i} \in B_e^1, \text{ and} \\ \delta W^1(l_{-i}, 0, a) & \text{if} & b_{-i} \in B_e^0 \setminus B_e^1, \end{array}$$

where $a \in [0, 1]$. As $b_{-i} \in B_e^0 \setminus B_e^1$, in this case also $\delta W^1(l_{-i}, 0, a) = \max\{0, \mu^1(l_{-i}) - c\} = 0$. Finally, suppose that Player-type i^1 bids b and that Player $-i$ computes a posterior $\mu_{-i}(l_i(b)) < c$. Then $\Delta^1(l_i, 0, 0) < 0$ and it follows from Lemma 8 that she also gets $\max\{0, \mu^1(l_{-i}) - c\}$ in case both players win their tracts.

As Player-type i^1's interim payoff is independent of his beliefs, at time zero she faces the following problem: $\max_b b\left(\int \max\{0, \mu^1(l_{-i}) - c\} dH^1(b_{-i}) - b\right)$. This is a strictly concave maximization problem which has as unique solution:

$$\left(b^1\right)^* = \frac{1}{2} \int \max\{0, \mu^1(l_{-i}(b_{-i})) - c\} dH^1(b_{-i}). \tag{22}$$

As the solution is unique, independent of Player $-i$'s biding strategy, it is a best response for Player-type i^1 to submit only one bid with probability one. Hence, if $\mu^1(0) < c$, there does not exist a non-coordinating equilibrium in which type-one players submit more than one bid.

As $\mu^0(\infty) \leq \mu^1(0) < c$, Player-type i^1 only drills if $b_{-i} \in B_e^1$ and Player-type i^0 only drills if Player $-i$ finds oil at time one. Hence, if Player-type i^0 bids $b \in B_e^0 \setminus B_e^1$, her interim payoff is zero. Therefore, in any non-coordinating equilibrium $B_e^0 \setminus B_e^1 = \{0\}$ or $B_e^0 \setminus B_e^1 = \{\emptyset\}$.

It follows from my two previous paragraphs that I can, without loss of generality, restrict attention to the following three candidate equilibria: A separating equilibrium in which type-zero players bid zero and type-one players bid $\left(b^1\right)^*$, a semi-separating equilibrium in which type one players bid $\left(b^1\right)^*$ and type-zero players randomize between bidding zero and bidding $\left(b^1\right)^*$, and a pooling equilibrium in which both types submit the same bid $\left(b^1\right)^*$.

Let

$$D(x) \equiv U^0\left(b_i = 0; x\right) - U^0\left(b_i = \left(b^1\right)^*; x\right).$$

Intuitively, $D(x)$ measures Player-type i^0's incentives to bid zero (as opposed to bidding as if she had signal one) given that player $-i$ computes his posterior under the assumption that $\Pr\left(b_i = \left(b^1\right)^* | s_i = 0\right) = x$. If $D(x) \geq 0$, Player i prefers to bid zero. Hence, a separating equilibrium only exists[23] if $D(0) \geq 0$. A pooling equilibrium only exists if $D(1) \leq 0$, while a semi-separating equilibrium only exists if $D(x^*) = 0$ for some $x^* \in (0, 1)$. Observe that $U^0(b_i = 0; x) = 0 \forall x$. Furthermore, as type-zero players only drill if the other player finds oil at time one:

$$U^0\left(b_i = \left(b^1\right)^*; x\right) = \left(b^1\right)^* \left\{\delta \Pr(V_{-i} = s_{-i} = 1 | s_i = 0) \lambda_{-i}^1 \left[\Pr(V_i = 1 | s_i = 0, V_{-i} = 1) - c\right] - \left(b^1\right)^*\right\},$$

where λ_{-i}^1 represents the probability with which Player-type $-i^1$ drills if $(b_i, b_{-i}) = \left(\left(b^1\right)^*, \left(b^1\right)^*\right)$. Observe that $\left(b^1\right)^*$ and all the probabilities in the equation above (including λ_{-i}^1) are continuous in x. Hence, $D(x)$ is continuous in x, and, thus, either $D(0) \geq 0$, or $D(1) \leq 0$, or $D(x^*) = 0$ for some $x^* \in (0, 1)$.

I now compute λ_{-i}^1 in case $(b_i, b_{-i}) = \left(\left(b^1\right)^*, \left(b^1\right)^*\right)$. First, observe that, as $\mu^1(0) < c$, $\Delta^1\left(\frac{1}{x}, 0, 1\right) < 0 \leq \Delta^1\left(\frac{1}{x}, 0, 0\right)$, where the first inequality is proven in Lemma 4 and where the second inequality is proven in the first paragraph of this subsection. It then follows from Lemma 1 that there exists a unique $\lambda_{-i}^1 \in [0, 1)$ such that $\Delta^1\left(\frac{1}{x}, 0, \lambda_{-i}^1\right) = 0$. The probability λ_{-i}^1 is computed such that

$$\mu^1\left(\frac{1}{x}\right) - c \quad = \quad \delta \Pr\left(V_{-i} = 1, a_{-i} = \text{drill} \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*\right) \left[\Pr(V_i = 1 | s_i = V_{-i} = 1) - c\right]$$

[23]At the risk of stating the obvious, this is a necessary condition as I still have to check that no player-type can gain by submitting a bid that does not lie on the equilibrium path.

$$+ \quad \delta \Pr\left(a_{-i} = \text{wait} \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*\right) \tag{23}$$
$$\times \quad \max\left\{0, \Pr\left(V_i = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*, a_{-i} = \text{wait}\right) - c\right\}.$$

If $\Pr\left(V_i = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*, a_{-i} = \text{wait}\right) \geq c$, the right-hand side of 23 can be rewritten as

$$\delta\left(\mu^1\left(\frac{1}{x}\right) - c\right) + \delta \Pr\left(V_{-i} = 0, s_{-i} = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*\right)\lambda^1_{-i}[c - \Pr(V_i = 1|s_i = 1, V_{-i} = 0)] \equiv RHS_1.$$

Denote by λ_1 the value of λ^1_{-i} which equates the left-hand side of (23) with RHS_1. Using Bayes's rule, one has

$$\lambda_1 \equiv \frac{(1-\delta)\left[\Pr\left(V_i = 1, b_{-i} = \left(b^1\right)^* \middle| s_i = 1\right)(1-c) - \Pr\left(V_i = 0, b_{-i} = \left(b^1\right)^* \middle| s_i = 1\right)c\right]}{\delta \Pr(V_{-i} = 0, s_{-i} = 1|s_i = 1)\left[c - \Pr(V_i = 1|s_i = 1, V_{-i} = 0)\right]}. \tag{24}$$

If $\Pr\left(V_i = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*, a_{-i} = \text{wait}\right) < c$, the right-hand side of 23 can be rewritten as

$$\delta \Pr\left(V_{-i} = s_{-i} = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*\right)\lambda^1_{-i}\left[\Pr(V_i = 1|s_i = V_{-i} = 1) - c\right] \equiv RHS_2.$$

Denote by λ_2 the value of λ^1_{-i} which equates the left-hand-side of (23) with RHS_2. Using Bayes's rule, one has

$$\lambda_2 \equiv \frac{\Pr\left(V_i = 1, b_{-i} = \left(b^1\right)^* \middle| s_i = 1\right)(1-c) - \Pr\left(V_i = 0, b_{-i} = \left(b^1\right)^* \middle| s_i = 1\right)}{\delta \Pr(V_{-i} = s_{-i} = 1|s_i = 1)\left[\Pr(V_i = 1|s_i = 1, V_{-i} = 1) - c\right]}. \tag{25}$$

Observe that $\Pr\left(V_i = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*, a_{-i} = \text{wait}\right)$ is decreasing in λ^1_{-i}. Call λ^c the value of λ^1_{-i} such that $\Pr\left(V_i = 1 \,\middle|\, s_i = 1, b_{-i} = \left(b^1\right)^*, a_{-i} = \text{wait}\right) = c$. Observe that $\lambda^c < 1 \Leftrightarrow \mu^1(0) < c$. If $\lambda^1_{-i} < \lambda^c$, RHS_1 is the relevant right-hand side of equation 23. If $\lambda^1_{-i} > \lambda^c$, RHS_2 is the relevant right-hand side of equation 23. If $\lambda^1_{-i} = \lambda^c$, $RHS_1 = RHS_2$. Observe also that

$$\frac{\partial RHS_1}{\partial \lambda^1_{-i}} < \frac{\partial RHS_2}{\partial \lambda^1_{-i}} \Leftrightarrow c < \mu^1(\infty),$$

which is obviously satisfied. Suppose $\lambda^1_{-i} = \lambda_2$ and that $\lambda_1 < \lambda_2$. λ^1_{-i} is only equal to λ_2 if

$$\lambda_2 > \lambda^c. \tag{26}$$

As $\lambda_1 < \lambda_2$, and as $0 < \frac{\partial RHS_1}{\partial \lambda^1_{-i}} < \frac{\partial RHS_2}{\partial \lambda^1_{-i}}$, RHS_1 will only be equal to RHS_2 at a value $\lambda^c > \lambda_2$, which contradicts Inequality 26. Using a similar reasoning, one can check that λ^1_{-i} cannot be equal to λ_1 when $\lambda_2 < \lambda_1$. This result, combined with my earlier insight that there exists a unique $\lambda^1_{-i} < 1$ such that $\Delta^1\left(\frac{1}{x}, 0, \lambda^1_{-i}\right) = 0$, allows me to conclude that in equilibrium $\lambda^1_{-i} = \min\{\lambda_1, \lambda_2\}$. Using (24) and (25), the inequality $\lambda_1 < \lambda_2 \Leftrightarrow \Delta^1(\infty, 0, 1) < 0$, which, by Assumption 3, holds. I conclude that $\lambda^1_{-i} = \lambda_1$.

I now use my knowledge of λ^1_{-i} to prove that, generically, a semi-separating equilibrium does not exist. Recall that a semi separating equilibrium only exists if $D(x^*) = 0$ for some $x^* \in (0,1)$. Call b_{ss} the value of $\left(b^1\right)^*$ when $x \in (0,1)$. Recall that Player-type i^1 faces a positive payoff from drilling if and only if $b_{-i} = b_{ss}$. As $b_{-i} = b_{ss}$ with probability one if $s_{-i} = 1$ and with probability x if $s_{-i} = 0$,

$$b_{ss} = \frac{1}{2}\left[\Pr(s_{-i} = 1|s_i = 1) + \Pr(s_{-i} = 0|s_i = 1)x\right]\left(\mu^1\left(\frac{1}{x}\right) - c\right),$$

which can be rewritten as

$$b_{ss} = \frac{1}{2}\left[\Pr\left(V_i = 1, b_{-i} = b_{ss}\middle| s_i = 1\right)(1-c) - \Pr\left(V_i = 0, b_{-i} = b_{ss}\middle| s_i = 1\right)c\right]. \tag{27}$$

Recall that $U^0(b_i = 0; x) = 0 \forall x$. Hence, $D(x) = 0$ is equivalent to

$$\delta \Pr(V_{-i} = s_{-i} = 1|s_i = 0)\lambda^1_{-i}\left[\Pr(V_i = 1|s_i = 0, V_{-i} = 1) - c\right] = b_{ss}. \tag{28}$$

Substituting (24) in the equation above and using (27), Equality 28 can be rewritten as

$$\frac{1}{2}\Pr(V_{-i}=0,s_{-i}=1|s_i=1)\left[c-\Pr(V_i=1|s_i=1,V_{-i}=0)\right] = (1-\delta)\Pr(V_{-i}=s_{-i}=1|s_i=0) \times \left[\Pr(V_i=1|s_i=0,V_{-i}=1)-c\right],$$

which, generically, is never satisfied.

Suppose that $D(0)>0$. I now argue that there exist then a separating equilibrium. To prove this, I am left to show that the separating equilibrium is supported by divine out-of-equilibrium beliefs. Recall that—independent of the specified beliefs—a type-one player strictly loses by submitting a bid $b\neq\left(b^1\right)_{sep}$. Hence, $\forall b\neq\left(b^1\right)_{sep}$, $T^1(b)=\{\emptyset\}$. Suppose Player-type i^0 deviates and bids $b\notin\left\{0,\left(b^1\right)_{sep}\right\}$. As $T^1(b)=\{\emptyset\}$, any out-of-equilibrium belief that specifies that this bid is submitted by a type-zero player satisfies the divinity criterion. This out-of-equilibrium belief ensures that she loses from this deviation. This is intuitive: As $\mu^1(0)<c$, Player $-i$ does not drill at time one if he infers that she possesses signal zero. As she doesn't learn anything about V_{-i}, Player-type i^0's interim payoff is therefore zero. It can also be shown that $\forall b>\left(b^1\right)_{sep}$, $T^0(b)=T^1(b)=\{\emptyset\}$. Hence, the out-of-equilibrium belief $\Pr\left(s_i=1\,\middle|\,s_{-i},b_i>\left(b^1\right)_{sep}\right)=1\ \forall s_{-i}$ is also divine.

Suppose that $D(0)<0$. I now argue that there exist then a pooling equilibrium. Recall that there does not exist a $x^*\in(0,1)$ such that $D(x^*)=0$. Thus, if $D(0)<0$, $D(1)<0$. Furthermore, as $T^1(b)=\{\emptyset\}\ \forall\ b\neq b_{pool}$, the out-of-equilibrium belief $\Pr\left(s_i=1\,\middle|\,s_{-i},b_i\neq b_{pool}\right)=0\ \forall s_{-i}$ is divine. As above, this out-of-equilibrium belief ensures that Player-type i^0 cannot gain from this deviation. As above, it can be shown that $\forall b>b_{pool}$, $T^0(b)=T^1(b)=\{\emptyset\}$. Hence, the out-of-equilibrium belief $\Pr\left(s_i=1\,\middle|\,s_{-i},b_i>b_{pool}\right)=1\ \forall s_{-i}$ is also divine.

On the basis of my preceding paragraph, I conclude that my model is characterized by a unique pooling equilibrium if $(\nu,p,c,\delta,\rho^0,\rho^1)\in\Omega^1$, where

$$\Omega^1\equiv\Big\{(\nu,p,c,\delta,\rho^0,\rho^1):\quad \text{Assumptions 1 to 3 hold, } \mu^1(0)<c, (1-\nu)(1-\rho^0)=\nu(1-\rho^1), \text{ and } D(0)<0\Big\}.$$

Ω^1 is non-empty: $\left(\frac{1}{2},0.6,0.51,0.4,1,1\right)$, for example, $\in\Omega^1$. I now argue that if the discount rate δ is sufficiently high, the separating equilibrium prevails. Recall that, as $\mu^1(0)<c$, type-zero players only drill if the other player finds oil at time one. Hence,

$$u^0\left(b_i=\left(b^1\right)_{sep}\right)=\delta\Pr(V_{-i}=s_{-i}=1|s_i=0)\lambda^1_{-i}\left[\Pr(V_i=1|s_i=0,V_{-i}=1)-c\right],$$

where λ^1_{-i} is computed using (24). Observe also that $D(0)>0\Leftrightarrow u^0\left(b_i=\left(b^1\right)_{sep}\right)<\left(b^1\right)_{sep}$. Observe that $\forall\delta$, $\left(b^1\right)_{sep}>0$. It follows from (24) that $u^0\left(b_i=\left(b^1\right)_{sep}\right)=0$ when $\delta=1$. By continuity, there exists a $\tilde{\delta}(\nu,p,c,\rho^0)<1$, such that $u^0\left(b_i=\left(b^1\right)_{sep}\right)<\left(b^1\right)_{sep}\ \forall\delta>\tilde{\delta}(\nu,p,c,\rho^0)$.

For future reference, I summarize the main insights of this subsection below:

Proposition 6 *Suppose that $\mu^1(0)<c$. There exists then a unique equilibrium outcome. Furthermore:*

1. *If $D(0)>0$, there exists a separating equilibrium with no bid distortion, i.e. type-zero players bid zero and type-one players bid*
$$\left(b^1\right)_{sep}=\frac{1}{2}\Pr(s_{-i}=1|s_i=1)\left[\mu^1(\infty)-c\right].$$
2. *If $D(0)<0$, both types bid*
$$\left(b^1\right)_{pool}=\frac{1}{2}\left[\xi^1-c\right]\in\left(0,\left(b^1\right)_{sep}\right).$$
3. *If $\delta>\tilde{\delta}$, $D(0)>0$.*

I will prove below that $D(0)$ is also positive when signals are sufficiently precise, i.e. when p lies above some critical value.

Equilibrium outcomes when $\mu^0(\infty) < c < \mu^1(0)$

I first state and prove the following lemma. Recall from Lemma 4 that there exists a unique $\tilde{l} \in (0,1)$ such that $\Delta^1(\tilde{l},0,1) = 0$ whenever $c < \mu^1(0)$.

Lemma 9 *Suppose that $c < \mu^1(0)$.*

1. *If, additionally, Player i bids b and if Player $-i$ updates his beliefs using $l_i(b) \geq \tilde{l}$, Player-type i^1's interim payoff equals $\xi^1 - c$.*
2. *In any candidate equilibrium e, $\frac{1}{2}(\xi^1 - c) \in B_e^1$, $l_i\left(\frac{1}{2}(\xi^1 - c)\right) > \tilde{l}$, and $\forall b \in \left(B_e^1 \cup B_e^0\right) \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$, $l_i(b) < \tilde{l}$.*

Proof: I first prove the first claim. As $l_i(b) \geq \tilde{l}$, it follows from Lemma 4 that $\Delta^1(l_i,0,1) \leq 0$. As $c < \mu^1(0)$, $\Delta^1(l_i,0,0) > 0 \ \forall l_i$. It then follows from Lemma 8 that, if both players win their tracts, Player-type i^1 gets $\max\{0, \mu^1(l_{-i}) - c\}$. If only Player i wins her tract, she obviously gets the same payoff. Hence, conditional on winning her tract, Player-type i^1 gets $\int \max\{0, \mu^1(l_{-i}) - c\} dH^1(b_{-i})$. As $c < \mu^1(0) \leq \mu^1(l_{-i})$, I can rewrite her interim payoff as $\xi^1 - c$.

I now prove the second claim. As $\tilde{l} < 1$, in any candidate equilibrium there exists a bid b such that $l_i(b) > \tilde{l}$. Suppose there exists an equilibrium in which $b \neq \frac{1}{2}(\xi^1 - c)$. From my previous paragraph we know that if Player-type i^1 bids b, her unconditional and net expected payoff equals $b(\xi^1 - c - b)$. Suppose Player-type i^1 deviates and bids $\frac{1}{2}(\xi^1 - c)$. Player $-i$ then updates his beliefs using some specified out-of-equilibrium belief. This belief—along with her beliefs about his type—implements some non-coordinating continuation equilibrium. With a slight abuse of notation, denote Player-type i^1's payoff from waiting in this continuation equilibrium as $\delta W^1\left(l_{-i}, \lambda_{-i}^0, \lambda_{-i}^1\right)$. Her unconditional and net expected payoff then equals $\frac{1}{2}(\xi^1 - c)(u^1 - \frac{1}{2}(\xi^1 - c))$, where

$$
\begin{aligned}
u^1 \quad \equiv \quad & \int \Big[\Pr\left(r > b_{-i} | r < \frac{1}{2}(\xi^1 - c)\right) \left[\mu^1(l_{-i}) - c\right] \\
& + \Pr\left(r < b_{-i} | r < \frac{1}{2}(\xi^1 - c)\right) \max\left\{\mu^1(l_{-i}) - c, \delta W^1\left(l_{-i}, \lambda_{-i}^0, \lambda_{-i}^1\right)\right\}\Big] dH^1(b_{-i}).
\end{aligned}
$$

As $b \neq \frac{1}{2}(\xi^1 - c)$ and as $u^1 \geq \frac{1}{2}(\xi^1 - c)$, this is a profitable deviation. Hence, in any candidate equilibrium, $\frac{1}{2}(\xi^1 - c) \in B_e^1$ and $l_i\left(\frac{1}{2}(\xi^1 - c)\right) > \tilde{l}$.

Consider a bid $b \in \left(B_e^1 \cup B_e^0\right) \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$. If $b \in B_e^0 \setminus B_e^1$, $l_i(b) = 0 < \tilde{l}$. Thus, suppose that $b \in B_e^1 \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$. As Player-type i^1 must be indifferent between submitting b and submitting $\frac{1}{2}(\xi^1 - c)$, such an equilibrium only exists if Player-type i^1's interim payoff is different from $\xi^1 - c$ whenever she bids b. From my first claim, this implies that $l_i(b) < \tilde{l}$.■

First, observe that if $\mu^0(\infty) < c < \mu^1(0)$,

$$
\Delta^0(l_i,0,0) < 0 \ \forall l_i, \ \ 0 < \Delta^1(l_{-i},0,0) \ \forall l_{-i}, \ \text{ and } \Delta^1(l_i,1,1) < 0 \ \forall l_i. \tag{29}
$$

The first inequality rests on the fact that type-zero players always (i.e. $\forall l_i$) face a negative payoff from drilling at time one. The second inequality rests on the fact that type-one players always face a positive payoff from drilling. The third inequality is proven in Lemma 4.

Consider Player-type i^1. Suppose she submits some bid b and that Player $-i$ updates his beliefs using a likelihood $l_i(b) < \tilde{l}$. On the basis of Lemma 4 we know that the following inequality then holds:

$$
0 < \Delta^1(l_i,0,1). \tag{30}
$$

Obviously, if she does not win her tract, she gets zero. Thus, suppose she wins her tract. If Player $-i$ does not win his tract, she drills at time one and gets $\mu^1(l_{-i}) - c$. Thus, suppose Player $-i$ also wins his tract. Suppose Player $-i$ bid b and that $l_{-i}(b) \leq \tilde{l}$. Using Lemma 4, we know that the following inequality then holds $0 \leq \Delta^1\left(l_{-i},0,1\right)$.

This inequality, combined with (29), (30), and Lemma 8, allows me to conclude that she then gets a continuation payoff equal to $\mu^1(l_{-i}) - c$. Suppose now that $l_{-i}(b) > \tilde{l}$. Using Lemma 4, the following inequality then holds $\Delta^1(l_{-i}, 0, 1) < 0$. This inequality, combined with (29), (30), and Lemma 8, allows me to conclude that she then gets a continuation payoff equal to $\delta W^1(\infty, 0, 1)$. Observe that $\delta W^1(\infty, 0, 1) = \mu^1(\infty) - c + \left|\Delta^1(\infty, 0, 1)\right|$. It then follows from Lemma 9 that Player-type i^1's unconditional and net expected payoff if Player $-i$ updates his beliefs using a likelihood $l_i < \tilde{l}$ is equal to:

$$b\left(\xi^1 - c + \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) > r \middle| s_i = 1, r < b\right)\left|\Delta^1(\infty, 0, 1)\right| - b\right) \equiv \bar{U}^1(b). \tag{31}$$

Observe that $\bar{U}^1$ is unimodal: It initially increases until $b = \frac{1}{2}(\xi^1 - c)$, after which it decreases.

It follows from Lemma 9 that her unconditional and net expected payoff when Player $-i$ updates his beliefs using a $l_i(b) \geq \tilde{l}$ is equal to

$$b\left(\xi^1 - c - b\right) \equiv \underline{U}^1(b).$$

Consider now Player-type i^0. As $\mu^0(\infty) < c$, she only drills if the other player finds oil at time one. Thus suppose both players win their tracts. Suppose she submits some bid b and that Player $-i$ updates his beliefs using a likelihood $l_i < \tilde{l}$. From Lemma 4, this implies that $\Delta^1(l_i, 1, 1) < 0 < \Delta^1(l_i, 0, 1)$. It then follows from Lemma 8 and from the inequalities summarized in (29) that her time-zero utility is equal to

$$b\left(\int \Pr(r < b_{-i}|r < b)\delta W^0(l_{-i}, 0, 1)\, dH^0(b_{-i}) - b\right) \equiv \bar{U}^0(b). \tag{32}$$

Observe that $\overline{U}^0$ is also unimodal.

Suppose now that both players win their tracts and that Player $-i$ updates his beliefs using a likelihood $l_i = \tilde{l}$. From Lemma 4, this implies that $\Delta^1(l_i, 1, 1) < 0 = \Delta^1(l_i, 0, 1)$. Suppose that $b_{-i} \in \left(B_e^1 \cup B_e^0\right) \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$. Recall from above that l_{-i} is then less than $\tilde{l}$. From Lemma 4, this implies that $\Delta^1(l_{-i}, 1, 1) < 0 < \Delta^1(l_{-i}, 0, 1)$. Using Lemma 8, I conclude that Player-type i^0 then gets $\delta W^0(l_{-i}, 0, a)$, where $a \in [0, 1]$. Suppose that $b_{-i} = \frac{1}{2}(\xi^1 - c)$. Recall from above that l_{-i} is then greater than $\tilde{l}$. From Lemma 4, this implies that $\Delta^1(l_{-i}, 0, 1) < 0$. This inequality, combined with my previous insights and with Lemma 8, implies that she then gets $\delta W^0(l_{-i}, 0\lambda^1_{-i})$ where λ^1_{-i} is computed such that $\Delta^1\left(l_{-i}, 0, \lambda^1_{-i}\right) = 0$. Hence, if Player $-i$ updates his beliefs using $l_i = \tilde{l}$, Player-type i^0's unconditional and net expected payoff is equal to $b(\underline{u}^0 - b) \equiv \underline{U}^0(b)$, where

$$\begin{aligned} \underline{u}^0 &= \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) > r \middle| s_i = 0, r < b_i\right)\delta W^0\left(l_{-i}, 0, \lambda^1_{-i}\right) \\ &+ \int_{\left(B_e^1 \cup B_e^0\right)\setminus\left\{\frac{1}{2}(\xi^1 - c)\right\}} \Pr(r < b_{-i}|r < b)\delta W^0(l_{-i}, 0, a)dH^0(b_{-i}). \end{aligned} \tag{33}$$

Observe that $\underline{U}^0(b)$ is unimodal.

Finally, suppose that both players win their tracts and that Player $-i$ updates his beliefs using a likelihood $l_i > \tilde{l}$. It then follows from Lemma 4 that

$$\Delta^1(l_i, 0, 1) < 0. \tag{34}$$

If $b_{-i} \in \left(B_e^1 \cup B_e^0\right) \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$, $l_{-i} < \tilde{l}$. From Lemma 4, this implies that $0 < \Delta^1(l_{-i}, 0, 1)$. This inequality combined with (29), (34), and Lemma 8 allows me to conclude that she then gets a continuation payoff equal to zero. If $b_{-i} = \frac{1}{2}(\xi^1 - c)$, $l_{-i} > \tilde{l}$, and $\Delta^1(l_{-i}, 0, 1) < 0$. This inequality, combined with (29), (34), and Lemma 8 allows me to conclude that she then gets a continuation payoff equal to $\delta W^0(\infty, 0, \lambda^1_{-i})$ where λ^1_{-i} is computed such that $\Delta^1(\infty, 0, \lambda^1_{-i}) = 0$. Hence if Player $-i$ updates his beliefs using a $l_i(b) > \tilde{l}$, Player-type i^0's unconditional and net expected payoff is

$$b\left(\Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) > r \middle| s_i = 0, r < b\right)\delta W^0(\infty, 0, \lambda^1_{-i}) - b\right) \equiv \underline{\underline{U}}^0(b), \tag{35}$$

which is also unimodal. Observe also that $\forall b \geq 0$, $\underline{\underline{U}}^0(b) \leq \underline{U}^0(b) \leq \bar{U}^0(b)$.

We now know enough to characterize the set of equilibrium outcomes when the drilling cost is intermediate. I need to consider five candidate non-coordinating equilibria in which:

$$\begin{array}{llll} (i) & B_e^0 \setminus B_e^1 \neq \{\emptyset\}, & B_e^0 \cap B_e^1 \neq \{\emptyset\}, & B_e^1 \setminus B_e^0 \neq \{\emptyset\}, \\ (ii) & B_e^0 \setminus B_e^1 \neq \{\emptyset\}, & B_e^0 \cap B_e^1 = \{\emptyset\}, & B_e^1 \setminus B_e^0 \neq \{\emptyset\}, \\ (iii) & B_e^0 \setminus B_e^1 = \{\emptyset\}, & B_e^0 \cap B_e^1 \neq \{\emptyset\}, & B_e^1 \setminus B_e^0 \neq \{\emptyset\}, \\ (iv) & B_e^0 \setminus B_e^1 \neq \{\emptyset\}, & B_e^0 \cap B_e^1 \neq \{\emptyset\}, & B_e^1 \setminus B_e^0 = \{\emptyset\}, \\ (v) & B_e^0 \setminus B_e^1 = \{\emptyset\}, & B_e^0 \cap B_e^1 \neq \{\emptyset\}, & B_e^1 \setminus B_e^0 = \{\emptyset\}. \end{array}$$

I first rule out the existence of a non-coordinating equilibrium in which (i) holds. Recall from Lemma 9 that such an equilibrium only exists if $B_e^1 \setminus B_e^0 = \left\{\frac{1}{2}(\xi^1 - c)\right\}$. Lemma 9 also states that $l_i < \tilde{l}$ both when $b_i \in B_e^0 \setminus B_e^1$ and when $b_i \in B_e^0 \cap B_e^1$. Player-type i^0 thus gets $\overline{U}^0(b)\ \forall b \in B_e^0$. As $\overline{U}^0$ is unimodal, and as type-zero players must be indifferent between submitting $b \in B_e^0 \setminus B_e^1$ and $b \in B_e^0 \cap B_e^1$, I conclude that $B_e^0 \setminus B_e^1$ and $B_e^0 \cap B_e^1$ are singletons as well. Call b^0 the bid in $B_e^0 \setminus B_e^1$. Call $b^{0\cap 1}$ the bid in $B_e^0 \cap B_e^1$. Let $b^1 \equiv \frac{1}{2}(\xi^1 - c)$. As I restrict attention to monotone bidding strategies, $b^0 < b^{0\cap 1} < b^1$. As $\overline{U}^0$ is unimodal, such an equilibrium only exists if $0 < \left.\frac{\partial \overline{U}^0}{\partial b}\right|_{b=b^0}$ and if $\left.\frac{\partial \overline{U}^0}{\partial b}\right|_{b=b^{0\cap 1}} < 0$. Thus $T^0(b) \neq \{\emptyset\}$ for b close to (but less than) $b^{0\cap 1}$. As $\overline{U}^1\left(b^{0\cap 1}\right) = \underline{U}^1\left(b^1\right)$ and as $\underline{U}^1(b)$ and $\overline{U}^1(b)$ are increasing when $b < b^1$, $T^1(b) = \{\emptyset\}\ \forall b < b^{0\cap 1}$. Player-type i^0 thus has a profitable deviation: She can bid b (where b is slightly less than $b^{0\cap 1}$), the intuitive criterion then prescribes Player $-i$ to update his beliefs using $l_i = 0$, and she then gets a payoff equal to $\overline{U}^0(b) > \overline{U}^0\left(b^{0\cap 1}\right)$.

Case (ii) corresponds to a separating equilibrium. Recall from the body of the paper that $b^1 \left(= \frac{1}{2}(\xi^1 - c)\right)$ is defined as the bid which maximizes $\underline{U}^1(b)$, that b^0 is defined as the lowest bid such that $\bar{U}^1(b^0) = \underline{U}^1(b^1)$ and that $\bar{b}^0$ is defined as the bid which maximizes $\bar{U}^0(b)$. Recall from Lemma 9 that such an equilibrium only exists if $B_e^1 = \left\{\frac{1}{2}(\xi^1 - c)\right\}$. I now argue that B_e^0 is also a singleton. Suppose that B_e^0 contains more than one bid. As Player-type i^0 must be indifferent between submitting any bid $b \in B_e^0$, and as $\overline{U}^0$ is unimodal, B_e^0 contains exactly two bids. Call b^{min} the lowest and b^{max} the highest bid in B_e^0. Observe that $b^{min} < \bar{b}^0 < b^{max}$. Recall that the intuitive criterion prescribes Player $-i$ to believe that any bid below b^0 must have been submitted by a type-zero player. Thus, if $\bar{b}^0 < b^0$, Player-type i^0 can profitable deviate by bidding $\bar{b}^0$ instead of randomizing between b^{min} and b^{max}. Thus, suppose that $b^0 < \bar{b}^0$. Observe that such an equilibrium only exists if $b^{min} = b^0$. As I restrict attention to monotone bidding strategies, $b^{max} < \frac{1}{2}(\xi^1 - c)$. It is, however, then clear from Figure 1 that Player-type i^1 can profitably deviate by bidding b^{max} instead of b^1.

I am left to argue that if $\underline{U}^1$ lies below $\bar{U}^0$ and if $\forall b\ \underline{U}^0(b) < \bar{U}^0(b^0)$, there still exists a separating equilibrium. As B_e^1 is a singleton, I can rewrite Equation 32 as

$$b\left(\Pr\left(b_{-i} = b^1 > r \middle| s_i = 0, r < b\right)\delta W^0(\infty, 0, 1) - b\right) = \bar{U}^0(b).$$

As B_e^0 is also a singleton, I can rewrite Equation 31 as

$$\begin{aligned} \bar{U}^1(b) \quad &\equiv \quad b\Big(\Pr(b_{-i} = b^0|s_i = 1)(\mu^1(0) - c) + \Pr(b_{-i} = b^1 < r|s_i = 1, r < b)(\mu^1(\infty) - c) \\ &+ \quad \Pr(b_{-i} = b^1 > r|s_i = 1, r < b)\delta W^1(\infty, 0, 1) - b\Big). \end{aligned} \tag{36}$$

Observe that for any given b, $\Pr\left(b_{-i} = b^1 > r \middle| s_i = 0, r < b\right) \leq \Pr\left(b_{-i} = b^1 > r \middle| s_i = 1, r < b\right)$. Recall from Lemma 7 that $W^0(\infty, 0, 1) \leq W^1(\infty, 0, 1)$. It then follows from my two last equations that $\forall b\ \bar{U}^0(b) \leq \bar{U}^1(b)$. Let $\bar{\bar{b}}^0 \equiv \max\{b : \bar{U}^0(b) = \bar{U}^0(b^0)\}$. As $\bar{U}^0$ lies below $\bar{U}^1$, the following out-of-equilibrium beliefs are divine:

$$\Pr(s_i = 1|s_{-i}, b_i) = \begin{cases} 1 & \text{if } b_i \geq \bar{\bar{b}}^0, \\ \Pr(s_i = 1|s_{-i}) & \text{if } b_i \in \left(b^0, \bar{\bar{b}}^0\right) \setminus b^1, \text{ and} \\ 0 & \text{if } b_i < b^0 \end{cases}$$

Those out-of-equilibrium beliefs support a separating equilibrium in which type-zero players bid b^0 and in which type-one players bid b^1. To see this, observe that both player types lose from submitting a bid below b^0. If Player i deviates and submits a bid $b \in \left(b^0, \bar{\bar{b}}^0\right) \setminus b^1$, Player $-i$ updates his beliefs about her type using $l_i = 1$. As $\tilde{l} < 1$, Player i then gets $\underline{U}^{s_i}(b)$. As $\forall b$ $\underline{U}^0(b) < \bar{U}^0(b^0)$ and as $\underline{U}^1(b) \leq \bar{U}^0(b^1)$ she cannot gain from deviating.

I now analyze existence of a non-coordinating equilibrium in which (*iii*) holds. Recall that in this candidate equilibrium $B_e^0 \setminus B_e^1 = \{\emptyset\}$, $B_e^0 \cap B_e^1 \neq \{\emptyset\}$ and $B_e^1 \setminus B_e^0 \neq \{\emptyset\}$. It follows from Lemma 9 that such an equilibrium only exists if $\Delta^1(l_i, 0, 1) > 0$ when $b_i \in B_e^0 \cap B_e^1$ and if $B_e^1 \setminus B_e^0 = \left\{\frac{1}{2}(\xi^1 - c)\right\}$.

Observe that, in this candidate semi-separating equilibrium, if Player-type i^1 submits $b = \frac{1}{2}(\xi^1 - c)$, she gets $\underline{U}^1\left(\frac{1}{2}(\xi^1 - c)\right)$, while if she submits $b \in B_e^0 \cap B_e^1$, she gets $\bar{U}^1(b)$. Observe also that $\forall b$, $\underline{U}^1(b) \leq \bar{U}^1(b)$. Let $b_{ss} \equiv \min\left\{b : \bar{U}^1(b) = \underline{U}^1\left(\frac{1}{2}(\xi^1 - c)\right)\right\}$. Observe that $b_{ss} < \frac{1}{2}(\xi^1 - c)$. As type-one players must be indifferent between submitting $b \in B_e^0 \cap B_e^1$ and $b = \frac{1}{2}(\xi^1 - c)$, and as I restrict attention to monotone bidding strategies, such an equilibrium only exists if $B_e^0 \cap B_e^1 = \{b_{ss}\}$.

Those insights allow me to rewrite Equations 31 and 32 respectively as

$$\begin{aligned}\bar{U}^1(b) &= b\Bigg(\Pr(b_{-i} = b_{ss} < r|s_i = 1, r < b)\left[\mu^1(l_{-i}) - c\right] + \Pr(b_{-i} = b_{ss} > r|s_i = 1, r < b)\left[\mu^1(l_{-i}) - c\right] \\ &+ \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) < r|s_i = 1, r < b\right)\left[\mu^1(l_{-i}) - c\right] \\ &+ \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) > r|s_i = 1, r < b\right)\delta W^1(\infty, 0, 1) - b\Bigg), \qquad (37)\end{aligned}$$

and

$$\begin{aligned}\bar{U}^0(b) &= b\Bigg(\Pr(b_{-i} = b_{ss} > r|s_i = 0, r < b)\delta W^0\left(l_{-i}, 0, 1\right) \\ &+ \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) > r|s_i = 0, r < b\right)\delta W^0(\infty, 0, 1) - b\Bigg). \qquad (38)\end{aligned}$$

Observe also that if Player-type i^0 bids b_{ss}, she gets $\bar{U}^0\left(b_{ss}\right)$. Let $\underline{\underline{b}}^0 \in \arg\max_b \underline{\underline{U}}^0(b)$. Let $\bar{b}^0 \in \arg\max_b \bar{U}^0(b)$. A semi-separating equilibrium in this case only exists if $\underline{\underline{U}}^0(\underline{\underline{b}}^0) \leq \bar{U}^0\left(b_{ss}\right)$ as otherwise a type-zero player can profitably deviate by bidding $\underline{\underline{b}}^0$. Suppose that $b_{ss} \leq \bar{b}^0$. (Actually, it can be shown that any candidate semi-separating equilibrium in which $\bar{b}^0 < b_{ss}$ rests on unintuitive out-of-equilibrium beliefs.) Let $b^{sup} \equiv \max\{b : \bar{U}^0(b) = \bar{U}^0(b_{ss})\}$. I thus consider the situation which is depicted in Figure 2. (In Figure 2 it is implicitly assumed that $b^{sup} < \frac{1}{2}(\xi^1 - c)$. This, however, is not crucial.) Figure 2 is identical to my previous one. It is assumed that $\underline{U}^1$ reaches its maximum in Point e and that $\bar{U}^1$ goes through Point d. For the sake of simplicity, I did not include both functions in the figure. Nor did I include $\underline{U}^0(b)$. Actually, from above we know that $\underline{U}^0(b)$ always lies between $\underline{\underline{U}}^0(b)$ and $\bar{U}^0(b)$ and reaches its maximum for a value of $b \in [\underline{\underline{b}}^0, \bar{b}^0]$. As d and e possess the same ordinate, type-one players are indifferent between bidding b_{ss} and bidding $\frac{1}{2}(\xi^1 - c)$. Let $x \equiv \Pr(b_i = b_{ss}|s_i = 1)$. Let

$$\begin{aligned}\Omega^3 \equiv &\Bigg\{(\nu, p, c, \delta, \rho^0, \rho^1, x) : \text{ Assumptions 1 to 3 hold, } \mu^0(\infty) < c < \mu^1(0), \\ &(1 - \nu)(1 - \rho^0) = \nu(1 - \rho^1), l_i\left(b_{ss}\right) < \tilde{l}, \bar{U}^1(b_{ss}) = \underline{U}^1\left(\frac{1}{2}\left(\xi^1 - c\right)\right), \\ &\underline{\underline{U}}^0(\underline{\underline{b}}^0) \leq \bar{U}^0\left(b_{ss}\right), b_{ss} \leq \bar{b}^0, x \in (0, 1)\Bigg\}.\end{aligned}$$

Ω^3 is non-empty: The vector $\left(\frac{1}{2}, 0.53, \frac{1}{2}, 0.8, 0.6, 0.6, 0.1900156763\right)$, for example, is an element of Ω^3.

I now analyze which out-of-equilibrium beliefs support such a semi-separating equilibrium. First, observe that $\forall b < b_{ss}$, $T^0(b) = T^1(b) = \{\emptyset\}$. Next, suppose that Player i submits a bid $b \in (b_{ss}, b^{sup}] \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$. On the basis of (37) and (38), and using Lemma 7, it is easy to check that

$$\left.\frac{\partial \bar{U}^0}{\partial b}\right|_{b \geq 0 \text{ and } b \notin \left\{b_{ss}, \frac{1}{2}(\xi^1 - c)\right\}} < \left.\frac{\partial \bar{U}^1}{\partial b}\right|_{b \geq 0 \text{ and } b \notin \left\{b_{ss}, \frac{1}{2}(\xi^1 - c)\right\}}.$$

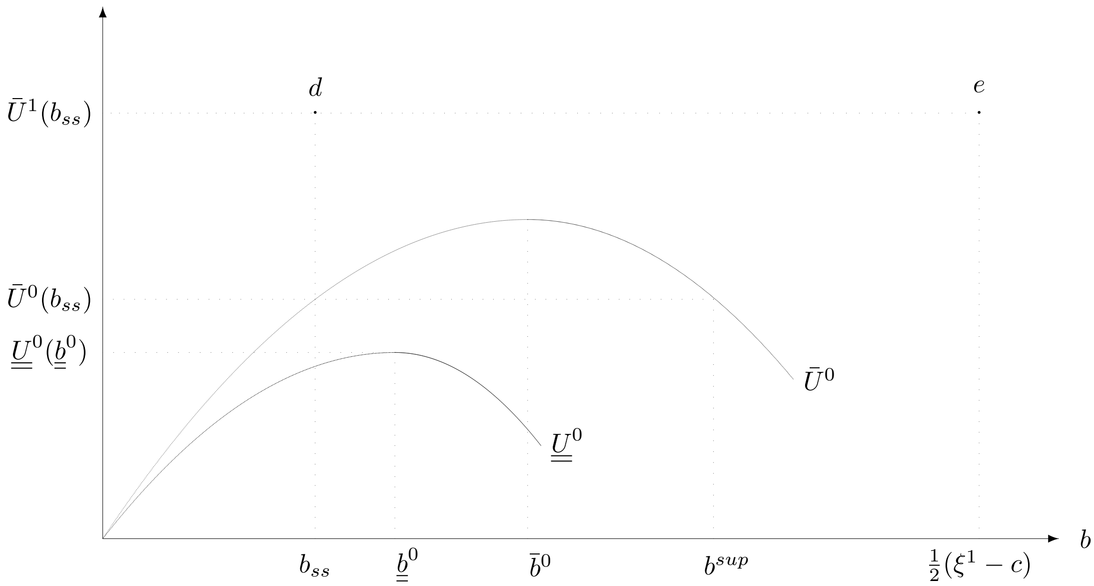

Figure 2: A semi-separating equilibrium.

These inequalities imply that $\forall b \in (b_{ss}, b^{sup}] \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$, $T^1(b) = \left[0, \tilde{l}\right)$. As $\left[0, \tilde{l}\right)$ is also $\in T^0(b)$ $\forall b \in (b_{ss}, b^{sup}] \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$, the out-of-equilibrium belief

$$\Pr\left(s_i = 1 \middle| s_{-i}, b_i \in (b_{ss}, b^{sup}] \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}\right) = \Pr(s_i = 1|s_{-i})\ \forall s_{-i}$$

is divine. Observe, however, that if Player $-i$ updates his beliefs using $l_i = 1$, both Player-types lose from submitting $b \in (b_{ss}, b^{sup}] \setminus \left\{\frac{1}{2}(\xi^1 - c)\right\}$ as Player-type i^1 gets $\underline{U}^1(b) < \underline{U}^1\left(\frac{1}{2}(\xi^1 - c)\right) = \bar{U}^1(b_{ss})$ and as Player-type i^0 gets $\underline{\underline{U}}^0(b) < \bar{U}^0(b_{ss})$. Suppose now that Player i deviates and submits a bid $b > b^{sup}$. As $T^0(b) = \{\emptyset\}$ $\forall b > b^{sup}$, the out-of-equilibrium belief $\Pr(s_i = 1|s_{-i}, b_i > b^{sup}) = 1\ \forall s_{-i}$ is divine. As above, this out-of-equilibrium belief ensures that both player-types cannot gain from this deviation. Hence, if $(\nu, p, c, \delta, \rho^0, \rho^1, x) \in \Omega^3$, there exists a semi-separating equilibrium which is supported by divine out-of-equilibrium beliefs.

I now argue that there does not exist a non-coordinating equilibrium in cases (*iv*) and (*v*). In both cases $B_e^1 \setminus B_e^0 = \{\emptyset\}$. Recall from Lemma 9 that such equilibria only exist if $\frac{1}{2}(\xi^1 - c) \in B_e^0 \cap B_e^1$ and if $l_i\left(\frac{1}{2}(\xi^1 - c)\right) > \tilde{l}$. Hence, in both candidate equilibria type-zero players get $\underline{\underline{U}}^0\left(\frac{1}{2}(\xi^1 - c)\right)$ when they bid $\frac{1}{2}(\xi^1 - c)$. Recall also from Equation 35 that Player-type i^0's interim payoff when $b_i = \frac{1}{2}(\xi^1 - c)$ is equal to

$$\underline{\underline{u}}^0 \equiv \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) \middle| s_i = 0\right) \delta W^0\left(l_{-i}, 0, \lambda_{-i}^1\right),$$

where λ_{-i}^1 is computed such that $\Delta^1\left(l_{-i}, 0, \lambda_{-i}^1\right) = 0$. Let $\underline{u}^1$ denote the interim payoff of a type-one player if she bids $b = \frac{1}{2}(\xi^1 - c)$. Observe that

$$\underline{u}^1 > \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) \middle| s_i = 1\right) \delta W^1\left(l_{-i}, 0, 1\right),$$

as type-one players also earn a positive payoff in case Player $-i$ does not win his tract. Observe that

$$\Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) \middle| s_i = 0\right) < \Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c) \middle| s_i = 1\right).$$

Recall from Lemma 7 that $W^0\left(l_{-i}, 0, 1\right) \leq W^1\left(l_{-i}, 0, 1\right)$. Hence, $\underline{\underline{u}}^0 < \underline{u}^1$. As $\frac{1}{2}\underline{\underline{u}}^0 \in \arg\max_b \underline{\underline{U}}^0(b)$, as $\frac{1}{2}\underline{u}^1 = \frac{1}{2}(\xi^1 - c) \in \arg\max_b \underline{U}^1(b)$, and as $\underline{\underline{U}}^0$ is unimodal, I conclude that

$$\left.\frac{\partial \underline{\underline{U}}^0}{\partial b}\right|_{b = \frac{1}{2}(\xi^1 - c)} < \left.\frac{\partial \underline{U}^1}{\partial b}\right|_{b = \frac{1}{2}(\xi^1 - c)} = 0.$$

Recall from above that $\forall b \geq 0$, $\underline{\underline{U}}^0(b) \leq \underline{U}^0(b) \leq \bar{U}^0(b)$. These insights, however, imply that Player-type i^0 has a profitable deviation: She can bid slightly less than $\frac{1}{2}(\xi^1 - c)$ and—independent of the specified out-of-equilibrium beliefs—achieve a higher payoff.

There exist thus, at most, two different equilibria when $\mu^0(\infty) < c < \mu^1(0)$: a separating and a semi-separating one. Let $e \in \{sep, ss\}$ refer either to the separating or to the semi-separating equilibrium. Suppose δ is close to one. From above, we know that a separating equilibrium then exists. Suppose the semi-separating equilibrium also exists. I now argue that the difference between both equilibria's bidding strategies is then close to zero.

Recall that $x \equiv \Pr(b_i = b_{ss} | s_i = 1)$. Let $\bar{U}^1_{ss}(b)$ denote the value of $\bar{U}^1(b)$ in the semi-separating equilibrium, i.e. when $\bar{U}^1(b)$ is given by (37). Recall that $b_{ss} \equiv \min\left\{b : \bar{U}^1_{ss}(b) = \underline{U}\left(\frac{1}{2}\left(\xi^1 - c\right)\right)\right\}$. Let $\bar{U}^1_{sep}(b)$ denote the value of $\bar{U}^1(b)$ in the separating equilibrium, i.e. when $\bar{U}^1(b)$ is given by (36). Recall that $b^0 \equiv \min\left\{b : \bar{U}^1_{sep}(b) = \underline{U}\left(\frac{1}{2}\left(\xi^1 - c\right)\right)\right\}$. Observe that $\bar{U}^1_{ss}(b)$ is decreasing in x, that $\lim_{x\to 0} \bar{U}^1_{ss}(b) = \bar{U}^1_{sep}(b)$, and that $\lim_{x \to 1} \bar{U}^1_{ss}(b) = \underline{U}^1(b)$. Those insights imply that b_{ss} is increasing in x and that $\lim_{x\to 0} b_{ss} = b^0$. Let $\left(\bar{b}^0\right)_{sep}$ refer to the value of $\bar{b}^0$ in the separating equilibrium. Recall from the body of the text that

$$\left(\bar{b}^0\right)_{sep} \in \arg\max_b b\left(\Pr\left(b_{-i} = b^1 > r \,\middle|\, s_i = 0, r < b\right)\delta W^0(\infty, 0, 1) - b\right).$$

Observe that $\left(\bar{b}^0\right)_{sep} = \min\left\{b^1, \frac{1}{2}\Pr(s_{-i} = 1 | s_i = 0)\delta W^0(\infty, 0, 1)\right\}$. Call $\left(\bar{b}^0\right)_{ss}$ the value of $\bar{b}^0$ in the semi-separating equilibrium. Recall from above that $\left(\bar{b}^0\right)_{ss} \in \arg\max_b \bar{U}^0(b)$ where $\bar{U}^0(b)$ is given by 38. Observe that $\left(\bar{b}^0\right)_{ss} \leq \left(\bar{b}^0\right)_{sep}$. Recall also that $B^0_{sep} = \left\{\min\left\{b^0, \left(\bar{b}^0\right)_{sep}\right\}\right\}$ and that $B^0_{ss} = \{b_{ss}\}$. Recall that any candidate semi-separating equilibrium in which $b_{ss} > \left(\bar{b}^0\right)_{ss}$ rests on unintuitive out-of-equilibrium beliefs. As

$$b^0 = b_{ss}|_{x=0} < b_{ss}|_{x>0} \leq \left(\bar{b}^0\right)_{ss} \leq \left(\bar{b}^0\right)_{sep},$$

both equilibria only co-exist if $B^0_{sep} = \left\{b^0\right\}$. Recall that the semi-separating equilibrium only exists if $l_i\left(b_{ss}\right) < \tilde{l}$. Recall also that $\tilde{l}$ is implicitly defined such that $\mu^1\left(\tilde{l}\right) - c = \delta W^1\left(\tilde{l}, 0, 1\right)$. Observe that $\lim_{\delta\to 1} \tilde{l} = 0$. Hence, in any candidate semi-separating equilibrium, $\lim_{\delta \to 1} x = 0$. The last claim of Proposition 3 then follows from the fact that both $\tilde{l}$ and b_{ss} are continuous in x and from my earlier finding that $\lim_{x\to 0} b_{ss} = b^0$.

In the last subsection of this Appendix, I will prove that if signals are sufficiently precise, there exists a unique equilibrium outcome in which no types distort their bids. The explanations provided in this subsection, along with the ones provided in the body of the text, prove Proposition 3.

Equilibrium outcomes when $\xi^0 < c < \mu^0(\infty)$

Observe that, in this parameter range, Player-type i^1 always faces a positive payoff from drilling at time one, i.e $c < \mu^1(l_{-i}) \forall l_{-i}$. This is equivalent to

$$0 < \Delta^1(l_{-i}, 0, 0) \ \forall l_{-i}. \tag{39}$$

Let $\hat{l}$ be implicitly defined such that $\mu^0\left(\hat{l}\right) = c$. As $\mu^0(1) = \xi^0 < c$, as $c < \mu^0(\infty)$ and as μ^0 is increasing in l_i, $\hat{l} \in (1, \infty)$. As $\tilde{l} < 1$, $\tilde{l} < \hat{l}$.

Suppose Player-type i^1 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b) < \tilde{l}$. The following inequalities then hold:

$$0 < \Delta^1(l_i, 0, 1) \text{ and } \Delta^0(l_i, 0, 0) < 0. \tag{40}$$

Suppose also that $b_{-i} \neq \frac{1}{2}(\xi^1 - c)$. The following inequalities then hold:

$$0 < \Delta^1(l_{-i}, 0, 1) \text{ and } \Delta^0(l_{-i}, 0, 0) < 0. \tag{41}$$

Inequalities 39 and 41, combined with Lemma 8, allow me to conclude that her continuation payoff is then equal to $\mu^1(l_{-i}) - c$. Suppose that $b_{-i} = \frac{1}{2}(\xi^1 - c)$. Recall from Lemma 9 that l_{-i} is then no less than $\tilde{l}$. Suppose that

$l_{-i} \in \left(\tilde{l}, \hat{l}\right)$. The following inequalities then hold:

$$\Delta^1(l_{-i}, 0, 1) < 0 \text{ and } \Delta^0(l_{-i}, 0, 0) < 0. \tag{42}$$

Inequalities 39, 40, and 42, combined with Lemma 8, allow me to conclude that she then gets a continuation payoff equal to $\delta W^1(l_{-i}, 0, 1)$. Hence, if Player-type i^1 bids b, if Player $-i$ updates his beliefs using a likelihood $l_i(b) < \tilde{l}$ and if $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \in \left(\tilde{l}, \hat{l}\right)$, her unconditional and net expected payoff is:

$$\begin{aligned} \bar{U}^1(b) \quad &\equiv \quad b\left(\int_{(B_e^0 \cup B_e^1)\setminus\{\frac{1}{2}(\xi^1 - c)\}} \left(\mu^1(l_{-i}) - c\right) dH^1(b_{-i})\right. \\ &+ \quad \Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right) > r \middle| s_i = 1, r < b\right) \delta W^1(l_{-i}, 0, 1) \\ &+ \quad \left.\Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right) < r \middle| s_i = 1, r < b\right) \left(\mu^1(l_{-i}) - c\right) - b\right). \end{aligned} \tag{43}$$

Suppose now that $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \geq \hat{l}$. In this case $0 \leq \Delta^0(l_{-i}, 0, 0)$. This inequality, combined with (39) and Lemma 8, allows me to conclude that her interim payoff is then equal to $\xi^1 - c$. Recall from Lemma 9 that her interim payoff is also equal to $\xi^1 - c$ when $l_i \geq \tilde{l}$. Hence, if $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \geq \hat{l}$ or if Player $-i$ updates his beliefs using a $l_i \geq \tilde{l}$, her unconditional and net expected payoff is:

$$\underline{U}^1(b) \equiv b\left(\xi^1 - c - b\right). \tag{44}$$

Suppose Player-type i^0 bids b, that Player $-i$ updates his beliefs using a likelihood $l_i(b) < \tilde{l}$ and that $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \in \left(\tilde{l}, \hat{l}\right)$. Suppose also that $b_{-i} \neq \frac{1}{2}(\xi^1 - c)$. Inequalities 39, 40, and 41 then hold. It then follows from Lemma 8 that her continuation payoff is equal to $\delta W^0(l_{-i}, 0, 1)$. Suppose now that $b_{-i} = \frac{1}{2}(\xi^1 - c)$. Inequalities 39, 40, and 42 then hold. It then follows from Lemma 8 that her continuation payoff is also equal to $\delta W^0(l_{-i}, 0, 1)$. Hence, in this case her unconditional and net expected payoff is equal to

$$\bar{\bar{U}}^0(b) \equiv b\left(\int \Pr(r < b_{-i} | r < b) \delta W^0(l_{-i}, 0, 1)\ dH^0(b_{-i}) - b\right). \tag{45}$$

Suppose Player-type i^0 bids b, that Player $-i$ updates his beliefs using a likelihood $l_i(b) < \tilde{l}$ and that $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \geq \hat{l}$. If $b_{-i} \neq \frac{1}{2}(\xi^1 - c)$, as in my previous paragraph, she gets a continuation payoff equal to $\delta W^0(l_{-i}, 0, 1)$. If $b_{-i} = \frac{1}{2}(\xi^1 - c)$, $\Delta^0(l_{-i}, 0, 0) \geq 0$. Observe that Inequalities 39, 40, and 42 then also hold. On the basis of those inequalities, and of Lemma 8, I conclude that her continuation payoff is equal to $\mu^0(l_{-i}) - c$. Hence, in this case her unconditional and net expected payoff is equal to

$$\begin{aligned} \bar{U}^0(b) \quad &\equiv \quad b\left(\int_{(B_e^0 \cup B_e^1)\setminus\{\frac{1}{2}(\xi^1 - c)\}} \Pr(r < b_{-i} | r < b) \delta W^0(l_{-i}, 0, 1)\ dH^0(b_{-i})\right. \\ &+ \quad \left.\Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right) \middle| s_i = 0\right) \left(\mu^0(l_{-i}) - c\right) - b\right). \end{aligned} \tag{46}$$

Suppose Player-type i^0 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b) = \tilde{l}$. The following equality and inequality then hold:

$$\Delta^1(l_i, 0, 1) = 0 \text{ and } \Delta^0(l_i, 0, 0) < 0. \tag{47}$$

If $b_{-i} \neq \frac{1}{2}(\xi^1 - c)$, Inequality 41 also holds and, using Lemma 8, her continuation payoff is then equal to $\delta W^0(l_{-i}, 0, a(b_{-i}))$ where $a(b_{-i}) \in [0, 1]$. Suppose that $b_{-i} = \frac{1}{2}(\xi^1 - c)$. The first inequality presented in (42) then holds. This inequality, combined with (39) and (47) and with Lemma 8, allows me to conclude that her continuation payoff is then equal to $\delta W^0\left(l_{-i}, 0, \lambda^1_{-i}\right)$ where λ^1_{-i} is computed such that $\Delta^1\left(l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right), 0, \lambda^1_{-i}\right) = 0$.

Hence, her unconditional and net expected payoff is equal to

$$\begin{aligned}\tilde{U}^0(b) &\equiv& b\Bigg(\int_{(B_e^0\cup B_e^1)\setminus\{\frac{1}{2}(\xi^1-c)\}} \Pr(r<b_{-i}|r<b)\delta W^0(l_{-i},0,a(b_{-i}))\ dH^0(b_{-i}) \\ &+& \Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)>r\Big|\, s_i=0,r<b\right)\delta W^0\left(l_{-i},0,\lambda_{-i}^1\right) \\ &+& \Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)<r\Big|\, s_i=0,r<b\right)\max\left\{0,\mu^0(l_{-i})-c\right\}-b\Bigg).\end{aligned} \tag{48}$$

Suppose Player-type i^0 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b)\in\left(\tilde{l},\hat{l}\right)$. The following inequalities then hold:

$$\Delta^1(l_i,0,1)<0 \text{ and } \Delta^0(l_i,0,0)<0. \tag{49}$$

If $b_{-i}\neq\frac{1}{2}(\xi^1-c)$, Inequalities 39 and 41 also hold and, using Lemma 8, her continuation payoff is then equal to zero. If $b_{-i}=\frac{1}{2}(\xi^1-c)$, $\Delta^1\left(l_{-i},0,1\right)<0<\Delta^1(l_{-i},0,0)$. These inequalities, combined with (39), (49) and with Lemma 8, allow me to conclude that her continuation payoff is then equal to $\delta W^0\left(l_{-i},0,\lambda_{-i}^1\right)$. Hence, her unconditional and net expected payoff is equal to

$$\begin{aligned}\underline{\underline{U}}^0(b) &\equiv& b\Bigg(\Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)>r\Big|\, s_i=0,r<b\right)\delta W^0\left(l_{-i},0,\lambda_{-i}^1\right) \\ &+& \Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)<r\Big|\, s_i=0,r<b\right)\max\left\{0,\mu^0(l_{-i})-c\right\}-b\Bigg).\end{aligned} \tag{50}$$

Suppose Player-type i^0 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b)\geq\hat{l}$. The following inequalities then hold:

$$\Delta^1(l_i,0,1)<0 \text{ and } \Delta^0(l_i,0,0)\geq 0. \tag{51}$$

If $b_{-i}\neq\frac{1}{2}(\xi^1-c)$, Inequalities 39 and 41 also hold and, using Lemma 8, her continuation payoff is then equal to $\delta W^0(l_{-i},\lambda_{-i}^0,1)$ where λ_{-i}^0 is determined such that $\Delta^1\left(l_{-i},\lambda_{-i}^0,1\right)=0$. If $b_{-i}=\frac{1}{2}(\xi^1-c)$, $\Delta^1\left(l_{-i},0,1\right)<0<\Delta^1(l_{-i},0,0)$. These inequalities, combined with (39), (51) and with Lemma 8, allow me to conclude that her continuation payoff is then equal to $\delta W^0\left(l_{-i},0,\lambda_{-i}^1\right)$. Hence, her unconditional and net expected payoff is equal to

$$\begin{aligned}\underline{U}^0(b) &\equiv& b\Bigg(\int_{(B_e^0\cup B_e^1)\setminus\{\frac{1}{2}(\xi^1-c)\}} \Pr(r<b_{-i}|r<b)\delta W^0(l_{-i},\lambda_{-i}^0,1)\ dH^0(b_{-i}) \\ &+& \Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)>r\Big|\, s_i=0,r<b\right)\delta W^0\left(l_{-i},0,\lambda_{-i}^1\right) \\ &+& \Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)<r\Big|\, s_i=0,r<b\right)\max\left\{0,\mu^0(l_{-i})-c\right\}-b\Bigg).\end{aligned} \tag{52}$$

We now know enough to characterize the set of equilibrium outcomes. We need to consider five candidate non-coordinating equilibria in which:

$$\begin{aligned}&(i) && B_e^0\setminus B_e^1\neq\{\emptyset\}, && B_e^0\cap B_e^1\neq\{\emptyset\}, && B_e^1\setminus B_e^0\neq\{\emptyset\},\\ &(ii) && B_e^0\setminus B_e^1\neq\{\emptyset\}, && B_e^0\cap B_e^1=\{\emptyset\}, && B_e^1\setminus B_e^0\neq\{\emptyset\},\\ &(iii) && B_e^0\setminus B_e^1=\{\emptyset\}, && B_e^0\cap B_e^1\neq\{\emptyset\}, && B_e^1\setminus B_e^0\neq\{\emptyset\},\\ &(iv) && B_e^0\setminus B_e^1\neq\{\emptyset\}, && B_e^0\cap B_e^1\neq\{\emptyset\}, && B_e^1\setminus B_e^0=\{\emptyset\},\\ &(v) && B_e^0\setminus B_e^1=\{\emptyset\}, && B_e^0\cap B_e^1\neq\{\emptyset\}, && B_e^1\setminus B_e^0=\{\emptyset\}.\end{aligned}$$

Consider a candidate non-coordinating equilibrium in which $B_e^0\cap B_e^1\neq\{\emptyset\}$ and in which $B_e^1\setminus B_e^0\neq\{\emptyset\}$. It follows from Lemma 9 that such a candidate equilibrium only exists if $\frac{1}{2}(\xi^1-c)\in B_e^1\setminus B_e^0$. Furthermore, in such a candidate equilibrium $l_{-i}\left(\frac{1}{2}(\xi^1-c)\right)=\infty>\hat{l}$. It then follows from above that $\forall l_i$ Player-type i^1's interim payoff is equal to ξ^1-c. It is thus a best response for her to bid $\frac{1}{2}\left(\xi^1-c\right)$ with probability one. Hence, a non-coordinating equilibrium in cases (i) and (iii) does not exist.

I now consider a candidate non-coordinating equilibrium in which (ii) holds. As $B_e^0\cap B_e^1=\{\emptyset\}$, this case corresponds to a separating equilibrium. In such an equilibrium $l_{-i}\left(\frac{1}{2}(\xi^1-c)\right)=\infty>\hat{l}$. Thus, such an equilibrium only exists if $B_e^1\setminus B_e^0=\left\{\frac{1}{2}(\xi^1-c)\right\}$. Furthermore, $T^1(b)=\{\emptyset\}\ \forall b\neq\frac{1}{2}(\xi^1-c)$.

Suppose Player-type i^0 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b)<\tilde{l}$. Observe that in such an equilibrium $\left(B_e^0\cup B_e^1\right)\setminus\left\{\frac{1}{2}(\xi^1-c)\right\}=B_e^0$. Observe also that $\forall b\in B_e^0$, $l_{-i}(b)=0$ and that $\delta W^0(0,0,1)=0$. It then follows from (46) that her unconditional and net expected payoff is

$$\bar{U}^0=b\left(\Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)\middle|\,s_i=0\right)\left(\mu^0(\infty)-c\right)-b\right).$$

Let $b^0\equiv\frac{1}{2}\Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)\middle|\,s_i=0\right)\left(\mu^0(\infty)-c\right)$.

Suppose that Player-type i^0 submits bid b and that Player $-i$ updates his beliefs using a likelihood $l_i(b)\in\left[\tilde{l},\hat{l}\right)$. Following an identical logic as the one I explained in my previous paragraph, Equations 48 and 50 boil down to

$$\begin{aligned}\tilde{U}^0(b)\quad&\equiv\quad b\left(\Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)>r\middle|\,s_i=0,r<b\right)\delta W^0\left(\infty,0,\lambda_{-i}^1\right)\right.\\&+\quad\left.\Pr\left(b_{-i}=\frac{1}{2}\left(\xi^1-c\right)<r\middle|\,s_i=0,r<b\right)\left(\mu^0(\infty)-c\right)-b\right).\end{aligned}$$

It follows from Lemma 6 that $\delta W^0\left(\infty,0,\lambda_{-i}^1\right)>\mu^0(\infty)-c$. Hence, $\tilde{U}^0(b)>\bar{U}^0(b)\ \forall b>0$.

Recall from (52) that Player-type i^0's unconditional and net expected payoff if Player $-i$ updates his beliefs using a $l_i\geq\hat{l}$ is equal to $\underline{U}^0(b)$. Observe that $\forall b>0$, $\underline{U}^0(b)>\tilde{U}^0(b)$. Let $b^{sup}\equiv\max\left\{b:\underline{U}^0(b)=\bar{U}^0\left(b^0\right)\right\}$. Suppose that $b^{sup}\leq\frac{1}{2}(\xi^1-c)$. For a graphical representation of $\bar{U}^0$, $\tilde{U}^0$, and $\underline{U}^0$, see Figure 3. Observe that a separating equilibrium does not exist when $\frac{1}{2}(\xi^1-c)<b^{sup}$, as type-zero players can then profitably deviate by bidding $\frac{1}{2}(\xi^1-c)$. For the sake of simplicity, I did not include U^1 in the figure. Recall, however, that U^1 goes through the origin and that it reaches its maximum in Point e. Observe that a separating equilibrium only exists if $B_e^0=\left\{b^0\right\}$ as otherwise a type-zero player can profitably deviate by bidding b^0. Let

$$\begin{aligned}\Omega^4\quad\equiv\Big\{\left(\nu,p,c,\delta,\rho^0,\rho^1\right):\quad&\text{Assumptions 1 to 3 hold, }\xi^0<c<\mu^0(\infty),\\&(1-\nu)(1-\rho^0)=\nu(1-\rho^1),b^{sup}\leq\frac{1}{2}(\xi^1-c)\Big\}.\end{aligned}$$

I now argue that if $\left(\nu,p,c,\delta,\rho^0,\rho^1\right)\in\Omega^4$, there exists a separating equilibrium. Beliefs in this equilibrium are updated as follows: $l_i=0$ when $b\leq b^{sup}$ and $l_i=\infty$ when $b>b^{sup}$. Note that those beliefs imply divine out-of-equilibrium beliefs as $T^1(b)=\{\emptyset\}\ \forall b\neq\frac{1}{2}(\xi^1-c)$ and as $T^0(b)=\{\emptyset\}\ \forall b>b^{sup}$. Player-type i^0 cannot gain from deviating as $\forall b\leq b^{sup}$, she gets $\bar{U}^0(b)<\bar{U}^0\left(b^0\right)$. I am left to show that $b^{sup}\leq\frac{1}{2}(\xi^1-c)$ when the discount factor δ is sufficiently high. To see this, observe that $\lim_{\delta\to1}\lambda_{-i}^0=0$ and that $\lim_{\delta\to1}\lambda_{-i}^1=0$. This implies that $\lim_{\delta\to1}\delta W^0(\infty,0,\lambda_{-i}^1)=\mu^0(\infty)-c$ and that $\lim_{\delta\to1}\delta W^0(0,\lambda_{-i}^0,1)=0$. Hence, $\lim_{\delta\to1}\underline{U}^0=\lim_{\delta\to1}\tilde{U}^0=\lim_{\delta\to1}\bar{U}^0$. Therefore, $\lim_{\delta\to1}b^{sup}<\frac{1}{2}(\xi^1-c)$.

I now tackle the existence of a non-coordinating equilibrium in which (iv) holds. Recall that in this case $B_e^0\setminus B_e^1\neq\{\emptyset\}$, $B_e^0\cap B_e^1\neq\{\emptyset\}$, and $B_e^1\setminus B_e^0=\{\emptyset\}$. From Lemma 9 we know that $\frac{1}{2}\left(\xi^1-c\right)\in B_e^0\cap B_e^1$.

Suppose there exists a $b'\in\left\{B_e^0\cap B_e^1\right\}\setminus\left\{\frac{1}{2}\left(\xi^1-c\right)\right\}$. As type-one players must be indifferent between submitting $\frac{1}{2}\left(\xi^1-c\right)$ and b', such an equilibrium only exists if b' gives rise to an interim payoff different from ξ^1-c. On the basis of (43) and (44), we know that this only happens if $l_{-i}\left(\frac{1}{2}\left(\xi^1-c\right)\right)\in\left(\tilde{l},\hat{l}\right)$. It then follows

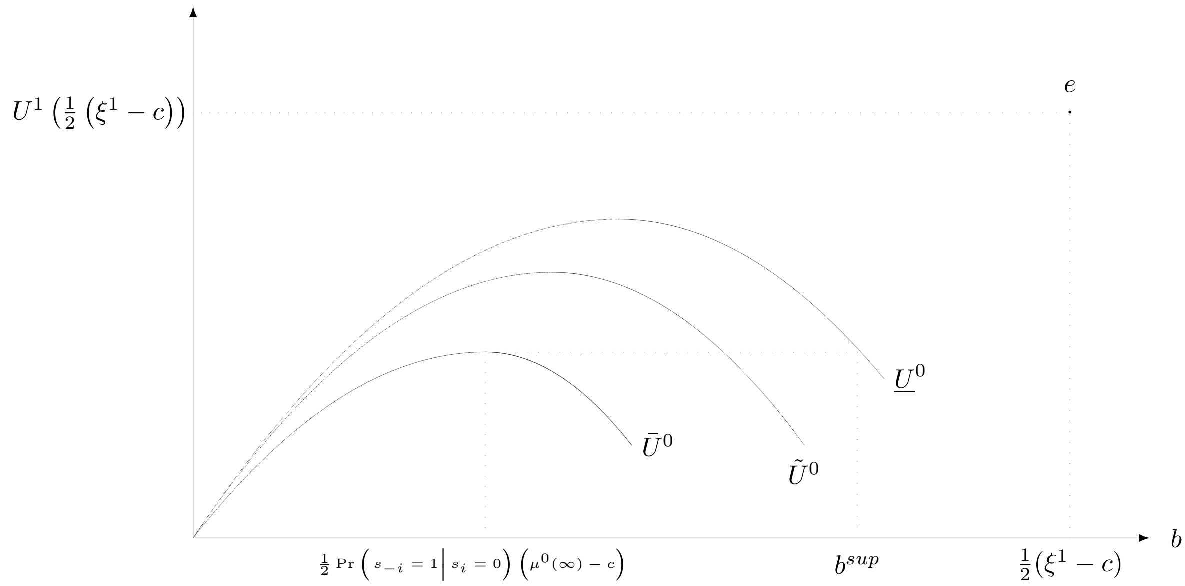

Figure 3: A separating equilibrium when $c < \mu^0(\infty)$.

from (50) that if Player-type i^0 bids $\frac{1}{2}\left(\xi^1 - c\right)$, she gets

$$\underline{\underline{U}}^0\left(\frac{1}{2}\left(\xi^1-c\right)\right) = b\left(\Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right) > r \middle| s_i = 0, r < b\right)\delta W^0\left(l_{-i}, 0, \lambda^1_{-i}\right) - b\right).$$

If Player $-i$ updates his beliefs using a likelihood $l_i < \tilde{l}$, or a $l_i = \tilde{l}$, or a $l_i \geq \hat{l}$, she respectively gets $\bar{\bar{U}}^0$, $\tilde{U}^0$, and $\underline{U}^0$. Observe that $\forall b > 0$, $\bar{\bar{U}}^0(b)$, $\tilde{U}^0(b)$, and $\underline{U}^0(b)$ are no less than $\underline{\underline{U}}^0(b)$. Using an identical reasoning as the one I explained in cases (iv) and (v) of my previous subsection,

$$\left.\frac{\partial \underline{\underline{U}}^0}{\partial b}\right|_{b=\frac{1}{2}\left(\xi^1-c\right)} < \left.\frac{\partial \underline{U}^1}{\partial b}\right|_{b=\frac{1}{2}\left(\xi^1-c\right)} = 0.$$

Player-type i^0 can thus profitably deviate: She can bid slightly less than $\frac{1}{2}\left(\xi^1 - c\right)$ and—independent of the specified out-of-equilibrium beliefs—achieve a higher payoff. Hence, such an equilibrium only exists if $B^0_e \cap B^1_e = \left\{\frac{1}{2}\left(\xi^1 - c\right)\right\}$.

Consider such a semi-separating equilibrium and suppose that $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \geq \hat{l}$. (Using an identical reasoning as the one I explained in my previous paragraph, such an equilibrium cannot exist if $l_{-i}\left(\frac{1}{2}(\xi^1 - c)\right) \in \left(\tilde{l}, \hat{l}\right)$.) Recall from above that $T^1(b) = \{\emptyset\}\ \forall b \neq \frac{1}{2}\left(\xi^1 - c\right)$. Suppose that Player-type i^0 bids b and that Player $-i$ updates his beliefs using a $l_i(b) < \tilde{l}$. Observe that in this candidate equilibrium $\left(B^0_e \cup B^1_e\right)\setminus\left\{\frac{1}{2}(\xi^1 - c)\right\} = B^0_e$, that $l_{-i}(b) = 0\ \forall b \in B^0_e$, and that $\delta W^0(0, 0, 1) = 0$. It then follows from 46 that she gets

$$\bar{U}^0(b) = b\left(\Pr\left(b_{-i} = \frac{1}{2}(\xi^1 - c)\middle| s_i = 0\right)\left(\mu^0(l_{-i}) - c\right) - b\right).$$

Suppose that Player-type i^0 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b) \in \left[\tilde{l}, \hat{l}\right)$. It then follows from (48) and (50) that her unconditional and net expected payoff is equal to

$$\begin{aligned}\underline{\underline{U}}^0(b) \quad = \quad & b\left(\Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right) > r\middle| s_i = 0, r < b\right)\delta W^0\left(l_{-i}, 0, \lambda^1_{-i}\right)\right. \\ + \quad & \left.\Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right) < r\middle| s_i = 0, r < b\right)\left(\mu^0(l_{-i}) - c\right) - b\right).\end{aligned}$$

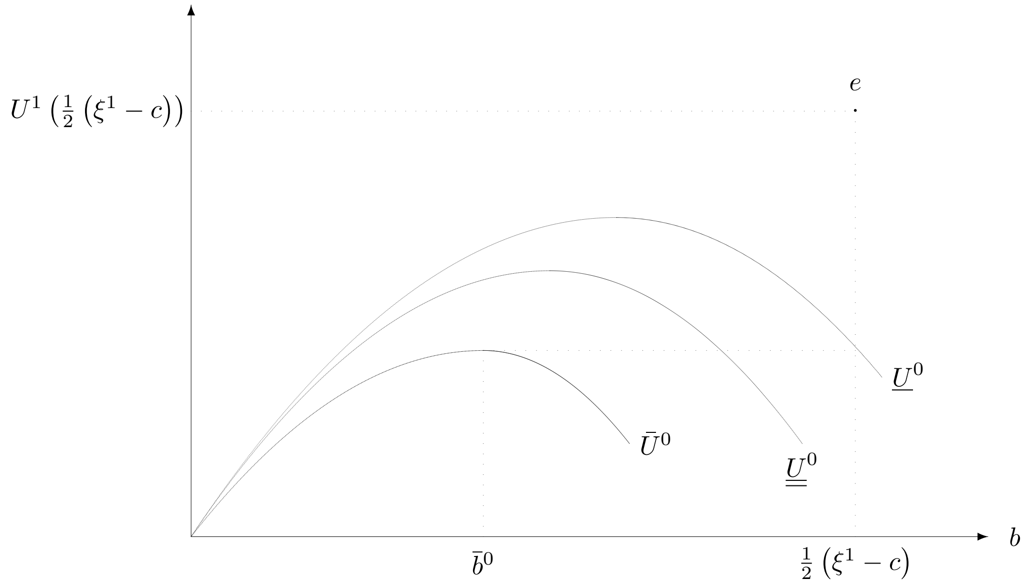

Figure 4: A semi-separating equilibrium when $c < \mu^0(\infty)$.

From Lemma 6 we know that $\delta W^0\left(l_{-i}, 0, \lambda^1_{-i}\right) > \mu^0(l_{-i}) - c$ and thus $\forall b > 0$, $\underline{\underline{U}}^0(b) > \bar{U}^0(b)$. Suppose that Player-type i^0 bids b and that Player $-i$ updates his beliefs using a likelihood $l_i(b) \geq \hat{l}$. She then gets $\underline{U}^0(b)$. It is immediate from (52) that $\forall b > 0$, $\underline{U}^0(b) > \underline{\underline{U}}^0(b)$. Observe that in this candidate equilibrium if Player-type i^0 bids $b \in B^0_e \setminus B^1_e$, she gets $\bar{U}^0(b)$. Let $\bar{b}^0 \in \arg\max_b \bar{U}^0(b)$. Observe that $\bar{b}^0 < \frac{1}{2}(\xi^1 - c)$. Observe also that Player-type i^0 can always bid $\bar{b}^0$ and—depending on $l_i(\bar{b}^0)$—achieve a payoff no less than $\bar{U}^0(\bar{b}^0)$. Hence, such a candidate equilibrium only exists if $B^0_e \backslash B^1_e = \left\{\bar{b}^0\right\}$. If she submits a bid $b = \frac{1}{2}\left(\xi^1 - c\right)$, she gets $\underline{U}^0\left(\frac{1}{2}\left(\xi^1 - c\right)\right)$. Therefore, such an equilibrium only exists if $\bar{U}^0(\bar{b}^0) = \underline{U}^0\left(\frac{1}{2}\left(\xi^1 - c\right)\right)$. A graphical representation of this equilibrium is provided in Figure 4. As above, U^1 does not appear in that figure. Recall, however, that U^1 goes through the origin and that it reaches its maximum in Point e. Let $x \equiv \Pr\left(b_i = \frac{1}{2}\left(\xi^1 - c\right) | s_i = 0\right)$. Recall that I consider a semi-separating equilibrium in which $l_{-i}\left(\frac{1}{2}\left(\xi^1 - c\right)\right) \geq \hat{l}$. Observe that this inequality can be rewritten as $\mu^0\left(\frac{1}{x}\right) \geq c$. Let

$$\begin{aligned}\Omega^5 \equiv \Bigg\{(\nu, p, c, \delta, \rho^0, \rho^1, x) : \quad & \text{Assumptions 1 to 3 hold, } \xi^0 < c < \mu^0(\infty), \mu^0\left(\frac{1}{x}\right) \geq c \\ & (1-\nu)(1-\rho^0) = \nu(1-\rho^1),\ \bar{U}^0\left(\bar{b}^0; x\right) = \underline{U}^0\left(\frac{1}{2}\left(\xi^1 - c\right); x\right)\Bigg\}.\end{aligned}$$

Ω^5 is non-empty. Consider the vector $(\nu, p, c, \delta, \rho^0, \rho^1) = (0.5, 0.55, 0.49, 0.58, 1, 1)$. This vector satisfies Assumptions 1 to 3. It also satisfies the inequalities $\xi^0 < c < \mu^0(\infty)$ and the equality $(1-\nu)(1-\rho^0) = \nu(1-\rho^1)$. Furthermore, it can be shown that there exists then a $x \in (0, 0.11)$ such that $\mu^0\left(\frac{1}{x}\right) \geq c$ and such that $\bar{U}^0\left(\bar{b}^0; x\right) = \underline{U}^0\left(\frac{1}{2}\left(\xi^1 - c\right); x\right)$. I now argue that if $(\nu, p, c, \delta, \rho^0, \rho^1, x) \in \Omega^5$, there exists a semi-separating equilibrium in which type-zero players randomize between $\bar{b}^0$ and $\frac{1}{2}\left(\xi^1 - c\right)$ and in which type-one players bid $\frac{1}{2}\left(\xi^1 - c\right)$ with probability one. As x is chosen such that $\bar{U}^0(\bar{b}^0; x) = \underline{U}^0\left(\frac{1}{2}\left(\xi^1 - c\right); x\right)$, I am left to show that there exist divine out-of-equilibrium beliefs which support those bidding strategies. Suppose beliefs are updated as follows: $l_i = 0$ when $b_i < \frac{1}{2}(\xi^1 - c)$ and $l_i = \frac{1}{x}$ when $b_i \geq \frac{1}{2}\left(\xi^1 - c\right)$. Note that those beliefs imply divine out-of-equilibrium beliefs as $T^1(b) = \{\emptyset\}\ \forall b \neq \frac{1}{2}\left(\xi^1 - c\right)$ and as $T^0(b) = \{\emptyset\}\ \forall b > \frac{1}{2}\left(\xi^1 - c\right)$. Furthermore, those beliefs imply that Player-type i^0 cannot gain from bidding $b < \frac{1}{2}\left(\xi^1 - c\right)$ and $b \neq \bar{b}^0$ as she then gets $\bar{U}^0(b) < \bar{U}^0(\bar{b}^0)$. Recall from case (ii) that $\lim_{\delta \to 1} \bar{U}^0 = \lim_{\delta \to 1} \underline{U}^0$. Therefore, there exists a $\tilde{\delta}(\nu, p, c, \rho^0, \rho^1) < 1$

such that for all $x \in [0,1]$ and for all $\delta > \tilde{\delta}(\nu, p, c, \rho^0, \rho^1)$, $\bar{U}^0(\bar{b}^0) > \underline{U}^0\left(\frac{1}{2}\left(\xi^1 - c\right)\right)$. Hence, if the discount factor δ is sufficiently high, this semi-separating equilibrium does not exist.

Recall that $b^0 \equiv \frac{1}{2}\Pr(s_{-i} = 1|s_i = 0)\left(\mu^0(\infty) - c\right)$. Hence, $\bar{b}^0\Big|_{x=0} = b^0$. Observe that

$$\bar{b}^0 = \frac{1}{2}\left\{\xi^0\left(p + (1-p)x\right)(1-c) - \left(1-\xi^0\right)\left((1-p) + px\right)c\right\},$$

and that $\frac{\partial \bar{b}^0}{\partial x} = \frac{1}{2}\left\{\xi^0(1-p)(1-c) - (1-\xi^0)pc\right\}$. As $c > \xi^0$,

$$\frac{\partial \bar{b}^0}{\partial x} < \frac{1}{2}\xi^0\left(1-\xi^0\right)\left((1-p) - p\right) < 0.$$

Hence, $\bar{b}^0 < b^0$. Let $LHS \equiv (1-x)\bar{b}^0 + x\frac{1}{2}\left(\xi^1 - c\right)$. Observe that $LHS = b^0$ when $x = 0$. It follows from above that $\frac{\partial LHS}{\partial x} > 0$. Type-zero players on average thus overbid, i.e. $(1-x)\bar{b}^0 + x\frac{1}{2}\left(\xi^1 - c\right) > b^0$.

I now prove that there does not exist a non-coordinating equilibrium in which (v) holds. Recall that in this case $B^0_e \setminus B^1_e = \{\emptyset\}$, $B^0_e \cap B^1_e \neq \{\emptyset\}$, and $B^1_e \setminus B^0_e = \{\emptyset\}$. Using an identical reasoning as the one I explained in case (iv), such an equilibrium only exists if $B^0_e \cap B^1_e = \left\{\frac{1}{2}\left(\xi^1 - c\right)\right\}$. Hence, in this candidate equilibrium, $l_{-i}\left(\frac{1}{2}\left(\xi^1 - c\right)\right) = 1$. As $1 < \hat{l}$, it also follows from my analysis in case (iv) that Player-type i^0 can profitably deviate by bidding slightly less than $\frac{1}{2}\left(\xi^1 - c\right)$.

Recall that $b^0 \equiv \frac{1}{2}\Pr(s_{-i} = 1|s_i = 0)\left(\mu^0(\infty) - c\right)$ and that

$$\bar{b}^0 \equiv \frac{1}{2}\Pr\left(b_{-i} = \frac{1}{2}\left(\xi^1 - c\right)\Big| s_i = 0\right)\left(\mu^0\left(l_{-i}\left(\frac{1}{2}\left(\xi^1 - c\right)\right)\right) - c\right).$$

The main insights of this subsection are summarized in the proposition below.

Proposition 7 *Suppose that $\xi^0 < c < \mu^0(\infty)$. Then, either:*

1. *there exists a separating equilibrium with no bid distortion, i.e. type-one players bid $\frac{1}{2}\left(\xi^1 - c\right)$ while type-zero players bid b^0, or*
2. *there exists a semi-separating equilibrium in which only type-zero players distort their bids, i.e. with probability $1-x$ they bid $\bar{b}^0 < b^0$ and with probability x they bid $\frac{1}{2}\left(\xi^1 - c\right) > b^0$, or*
3. *there does not exist a non-coordinating equilibrium.*

In the semi-separating equilibrium type-zero players on average overbid, i.e. $(1-x)\bar{b}^0 + x\frac{1}{2}\left(\xi^1 - c\right) > b^0$. Finally, if the discount factor δ is high enough, there exists a unique equilibrium outcome in which no types distort their bids.

The sufficiently-precise-signals case

I am left to argue that there exists a unique equilibrium outcome when signals are sufficiently precise. Observe that u^1 is bounded below by $\xi^1 - c$. Hence, type-one players, by bidding $\frac{1}{2}(\xi^1 - c)$, guarantee that their unconditional and net expected payoff U^1 is bounded below by $\frac{1}{4}(\xi^1 - c)^2$. Observe that her interim payoff is bounded above by $\delta\Pr(V_{-i} = 1|s_i = 1)(\Pr(V_i = 1|s_i = 1, V_{-i} = 1) - c)$. Let $\underline{b}$ be the lowest bid such that

$$\frac{1}{4}\left(\xi^1 - c\right)^2 = \underline{b}\left(\delta\Pr(V_{-i} = 1|s_i = 1)(\Pr(V_i = 1|s_i = 1, V_{-i} = 1) - c) - \underline{b}\right).$$

Observe that $\underline{b} > 0$ and that, in any undominated strategy, type-one players submit a bid no less than $\underline{b}$.

Observe also that, in any undominated strategy, type-zero players bid below

$$\Pr(s_{-i} = 1|s_i = 0)\delta W^0(\infty, 1, 1) + \Pr(s_{-i} = 0|s_i = 0)\delta W^0(0, 1, 1) \equiv \bar{u}^0.$$

It follows from Lemma 2 that $\lim_{p\to 1}\bar{u}^0 = 0 < \lim_{p\to 1}\underline{b}$. By continuity, there exists a $\tilde{p} < 1$ such that $\forall p \in [\tilde{p}, 1)$, $\bar{u}^0 < \underline{b}$. Hence, if $\tilde{p} < p$, in any undominated strategy, type-zero players bid strictly less than type-one players.

Hence, in any equilibrium players bid as if signals instead of bids were disclosed. This separating equilibrium is supported, for example, by the following beliefs: $l_i = 0$ if $b_i \leq \bar{u}^0$, $l_i = \infty$ if $b_i > \bar{u}^0$. Those beliefs imply divine out-of-equilibrium beliefs as $T^1(b) = \{\emptyset\}$ $\forall b < \underline{b}$, and as $T^0(b) = \{\emptyset\}$ $\forall b > \bar{u}^0$.■