Gains from Trade

Rahul Giri*
An obvious question that we should ask ourselves is: Is trade beneficial; are there gains from trade? In order to better understand this question, and ultimately answer it, we will need some theoretical structure.

1 Closed Economy OR Autarky

Consider an economy, which can produce two goods - good X and good Y. The economy is in autarky, i.e. it does not trade (exchange goods X and Y) with any country. Then the equilibrium would be characterized by the following conditions:

1. Consumers maximize utility: \( \frac{p_x}{p_y} = MRS = -\frac{\Delta Y}{\Delta X} \). Here is an example:

\[
\max_{\{X_c, Y_c\}} U(X_c, Y_c) = X_c^\delta Y_c^{1-\delta},
\]

s.t. \( p_x X_c + p_y Y_c \leq I \).

2. Producers maximize profits: \( \frac{p_x}{p_y} = MRT = -\frac{\Delta Y}{\Delta X} \). Here is an example:

\[
\max_{\{L_x, K_x\}} \pi_x = p_x X_p - wL_x - rK_x,
\]

s.t. \( X_p = K_x^\alpha L_x^{1-\alpha} \),

and

\[
\max_{\{L_y, K_y\}} \pi_y = p_y Y_p - wL_y - rK_y,
\]

s.t. \( Y_p = K_y^\beta L_y^{1-\beta} \).

3. Markets for the two goods clear, i.e. the demand for each good equals its supply or consumption of each good equals its production: \( X_c = X_p \) and \( Y_c = Y_p \). Also, markets for the factor inputs clear, i.e. all inputs are fully employed. In our example this would mean that \( L_x + L_y = L \) and \( K_x + K_y = K \), where \( L \) and \( K \) are the total amount of labor and capital, respectively, available.

Figure 1 depicts this autarky equilibrium. The country’s production possibilities are given by the production possibility frontier \( YAX \). Producers maximize profit at point \( A \), i.e. the slope of the tangent to the production possibility frontier at point \( A \) (given by MRT) is equal to the price ratio, \( p_a \) (which is \( \frac{p_x}{p_y} \)). Also, the consumers maximize their utility at point \( A \), i.e. the slope of the tangent to the indifference curve \( U_a \) at point \( A \) is equal to the price ratio \( p_a \). Lastly, at point \( A \), the consumption of each good equals its production.
2 Open Economy OR Trading Economy

Now, let us allow this economy to trade at a fixed world price ratio, which is denoted by $p^* = p_x^*/p_y^*$. So, the producers and consumers choose amounts of good $X$ and $Y$ to be produced and consumed optimally given the world price ratio. Thus, the equilibrium condition is:

1. Consumers maximize utility: $\frac{p_x^*}{p_y^*} = MRS = -\frac{\Delta Y}{\Delta X}$
2. Producers maximize profits: $\frac{p_x^*}{p_y^*} = MRT = -\frac{\Delta Y}{\Delta X}$

The more important difference between autarky and trade is in terms of market clearing condition. In the presence of trade the economy is no longer constrained to consume what it produces. It can consume more than what it produces of a certain good by importing it; and similarly, it can consume less than what it produces of a good by exporting it. It is this change which results in gains from trade. So, instead of the market clearing condition we have the balanced trade condition. The balanced trade condition states that the value of sales of a country on the world market must equal its purchases. Note that the market clearing for factor inputs remains unchanged because we assume that factor inputs cannot move across countries.

$$p_x^* (X_c - X_p) + p_y^* (Y_c - Y_p) = 0$$

The term $(X_c - X_p)$ denotes the excess demand for good $X$. If this is negative, it implies that the country consumes less of good $X$ than it produces, i.e. it exports good $X$. Likewise, if this is positive it implies that the country imports good $X$. Thus, another way to state the balanced
trade condition is that the value of exports equals the value of imports. Yet another way to express the balanced trade condition is: the value of production must be equal to value of consumption.

\[ p^*_x X_p + p^*_y Y_p = p^*_x X_c + p^*_y Y_c \]

Figure 2 depicts the equilibrium with trade. Given the world price ratio, the producers maximize profits at point \( B \), whereas the consumers maximize utility at point \( C \). Therefore, the economy imports good \( X \) and exports good \( Y \).

![Figure 2: Trade Equilibrium](image)

Note that if the world prices happen to be equal to the country’s autarky prices, then the trading equilibrium would be identical to the autarky equilibrium.

### 2.1 Determination of World Prices

As always, prices are determined by the forces of demand and supply. In the case of a trading world, world prices are determined by the ‘world demand’ for and ‘world supply’ of each good. When the autarky price ratio, \( p_a \), is greater than the world price ratio, \( p^* \), it implies that the relative price of good \( X \) (relative price of good \( Y \)) in the country is higher (lower) than that in the world economy. This would mean the country would import good \( X \) and export good \( Y \). In other word the excess demand for good \( X \) is positive while the excess demand for good \( Y \) is negative. Analogously, if \( p_a < p^* \), then good \( X \) is exported (excess demand is negative) while good \( Y \) is imported (excess demand is positive). This analysis is depicted in Figure 3.

Note that if the world price ratio equals the autarky (or domestic) price ratio then the excess demand is zero. The shape of the excess demand curve is determined by the response of producers.
and consumers when the world price ratio deviates from the autarky price ratio. When $p^* < p_a$, producers will move resources away from the production of good X and towards the production of Y, thereby increasing the excess demand for X. Consumers, on the other hand, would demand more of good X as a result of the substitution effect and income effect - X is relatively cheaper at the world price ratio and the real income increases due the lower relative price of X. The two imply that the excess demand curve is negatively sloped when $p^* < p_a$. Furthermore, the concavity of the production possibility frontier implies that more and more units of X would have to be given up in order to produce one unit of Y. Similarly, the convexity of the indifference curve would imply a larger and larger increase in consumption of X for each unit decline in consumption of Y. This explains the convex shape of the excess demand curve when $p^* < p_a$.

However, when $p^* > p_a$, the analysis is not so simple. The reason is that now the income effect increases the demand for both X and Y, whereas the substitution effect reduces the consumption of X.

3 General Equilibrium: Introducing A Second Country

Now, let us introduce a second country, which we are going to call ‘foreign country’, F. We call the original (existing) country ‘home’, H. In Figure 4 the excess demand curve of country F is denoted by $E_X^F$ and that of country H is denoted by $E_X^H$. International general equilibrium is established at that world price ratio where the excess demand for good X in country H is exactly offset by the excess demand for good X in country F. This happens at $p^*$. Notice that equilibrium world price ratio lies between the autarky price ratios of the two countries. This result holds when
markets are perfectly competitive. Country H, which has $p_a < p^*$, will be able to find more buyers in country F and hence will export good X. Country F, which has $p_a^* > p^*$, will face less scarcity of good X as it can import it from country H.

Figure 4: International General Equilibrium

Notice that in order to determine international general equilibrium we do not need to worry about market clearing for good Y. Since in equilibrium it must be that each country satisfies its trade balance condition, which implies that

$$p_x^* E_x^H + p_y^* E_y^H = 0 \Rightarrow p_x^* E_x^H = -p_y^* E_y^H,$$

$$p_x^* E_x^F + p_y^* E_y^F = 0 \Rightarrow p_x^* E_x^F = -p_y^* E_y^F,$$

and since

$$p_x^* E_x^H = -p_x^* E_x^F \quad OR \quad E_x^H = -E_x^F \Rightarrow p_y^* E_y^H = -p_y^* E_y^F \Rightarrow E_y^H = -E_y^F,$$

we have that market for good Y also clears. This is nothing but the ‘Walras’ Law’.

4 Gains from Trade

Having developed the theoretical structure, we are now in a position to answer the question we had posed in the beginning: Is trade beneficial; are there gains from trade? And the answer to the question is YES.

To see this, let us look at Figure 5, which shows the autarky and trade equilibria for one country. Point A is the autarky equilibrium; $p_a$ is the autarky price ratio; and $U_a$ is the autarky
utility level. Now consider trade equilibria for two different world price ratios - $p^*_1$ and $p^*_2$. For both price ratios the level of utility achieved is $U_t$. Though the diagram has been drawn so that the same free trade utility level is achieved for both price ratios, you can see it for yourself that any price ratio other than the autarky price ratio would result in a higher level of utility. Therefore, trade makes the country better off. If the world price ratio equals the autarky price ratio then the country is no worse under trade off than in autarky.

In case of a two country world, the two countries experience mutual gains from trade. As shown earlier in Figure 4, in a two country world the equilibrium world price ratio lies between the two countries’ autarky price ratios. And since we just showed that a country gains by trading at any price ratio other than the autarky price ratio, it implies that the two countries would experience mutual gains from trade. Note that the gains from trade are independent of the direction of trade, i.e. whether a country exports good $X$ and imports good $Y$ or vice-versa does not have any effect on the gains from trade.

![Figure 5: International General Equilibrium](image)

The formal proof uses the equilibrium conditions, i.e. firms maximize profits and consumers maximize utility. So, given the world price ratio $p^*$, if firms in a country choose to produce $X^*_p$ and $Y^*_p$, then profit maximization ensures that for any other feasible production bundle $(X^f_p, Y^f_p)$, which also includes the autarky bundle $(X^a_p, Y^a_p)$, the value of the equilibrium production bundle must exceed the value of any other feasible production bundle, including that of the autarky bundle.

$$p^*_x X^*_p + p^*_y Y^*_p \geq p^*_x X^f_p + p^*_y Y^f_p$$
The balanced trade condition implies that value of production equals the value of consumption. Imposing this on both sides of the above equation implies that at the world price ratio the value of consumption under trade must be greater than or equal to the value of consumption of any other feasible consumption bundle.

\[ p_x^* x^t + p_y^* y^t \geq p_x^* x^f + p_y^* y^f \]

As a result the free-trade consumption bundle \((X^t_c, Y^t_c)\) is preferred to the autarky consumption bundle \((X^a_c, Y^a_c)\). This is also called the **Gains-from-Trade Theorem**.