

SOLUTION TO FINAL - FALL, 2014

ECO-13101 Economia Internacional I (International Trade Theory)

December 12, 2014

Question 1 (50 points)

1. (10 points) The representative consumer in country j solves the following utility maximization exercise:

$$\begin{aligned} \max_{\{C_{ij}\}} U_j &= \sum_{i=1}^N n_i (C_{ij})^{(\sigma-1)/\sigma} , \\ \text{s.t.} \quad \sum_{i=1}^N n_i p_{ij} C_{ij} &= w_j \bar{L}_j . \end{aligned}$$

The first-order condition with-respect-to C_{ij} is

$$n_i \frac{(\sigma-1)}{\sigma} (C_{ij})^{-1/\sigma} - \lambda_j n_i p_{ij} = 0 ,$$

where λ_j is the lagrange multiplier for the consumer in country j . The first-order condition with-respect-to the lagrange multiplier just gives the budget constraint of the consumer in country j . The first-order condition above implies that

$$C_{ij} = \left[\frac{\sigma}{(\sigma-1)} \lambda_j p_{ij} \right]^{-\sigma} .$$

Substituting this for C_{ij} in the budget constraint gives

$$\begin{aligned} \left[\frac{\sigma}{(\sigma-1)} \lambda_j \right]^{-\sigma} \sum_{i=1}^N n_i (p_{ij})^{1-\sigma} &= w_j \bar{L}_j , \\ \Rightarrow \left[\frac{\sigma}{(\sigma-1)} \lambda_j \right]^{-\sigma} &= \frac{w_j \bar{L}_j}{\sum_{i=1}^N n_i (p_{ij})^{1-\sigma}} . \end{aligned}$$

Substituting for $\left[\frac{\sigma}{(\sigma-1)} \lambda_j \right]^{-\sigma}$ in the expression for C_{ij} gives

$$C_{ij} = \frac{w_j \bar{L}_j}{\sum_{i=1}^N n_i (p_{ij})^{1-\sigma}} (p_{ij})^{-\sigma} .$$

Since $P_j = \left(\sum_i^N n_i (p_{ij})^{1-\sigma} \right)^{1/(1-\sigma)}$, it implies that $P_j^{1-\sigma} = \sum_i^N n_i (p_{ij})^{1-\sigma}$. Using this gives the required demand function:

$$C_{ij} = \frac{w_j \bar{L}_j}{P_j} \left(\frac{p_{ij}}{P_j} \right)^{-\sigma} .$$

2. (15 points) The profit function of a product produced in country i is

$$\pi_i = p_i X_i - (\alpha + \beta_i X_i) w_i .$$

Given that every product produced by country i is unique and is sold to every country in the world (including itself), the output of every product must be equal to the world demand for it.

$$X_i = \sum_{j=1}^N C_{ij} ,$$

which, given the expression for C_{ij} (derived above in (1)), implies that

$$X_i = \sum_{j=1}^N \frac{w_j \bar{L}_j}{P_j} \left(\frac{p_{ij}}{P_j} \right)^{-\sigma} .$$

Since every product is sold for the same price $p_{ij} = p_i$

$$X_i = p_i^{-\sigma} \sum_{j=1}^N \frac{w_j \bar{L}_j}{P_j^{1-\sigma}} .$$

Substitute this in the profit function.

$$\pi_i = p_i^{1-\sigma} \sum_{j=1}^N \frac{w_j \bar{L}_j}{P_j^{1-\sigma}} - \left(\alpha + \beta_i p_i^{-\sigma} \sum_{j=1}^N \frac{w_j \bar{L}_j}{P_j^{1-\sigma}} \right) w_i .$$

Maximising this with respect to p_i gives the first order condition

$$(1 - \sigma) p_i^{-\sigma} \sum_{j=1}^N \frac{w_j \bar{L}_j}{P_j^{1-\sigma}} + \sigma \beta_i w_i p_i^{-\sigma-1} \sum_{j=1}^N \frac{w_j \bar{L}_j}{P_j^{1-\sigma}} = 0 ,$$

which can be re-arranged to

$$(1 - \sigma) + \sigma \frac{\beta_i w_i}{p_i} = 0 .$$

Thus, the price charged for every product produced in country i

$$p_i = \frac{\sigma}{\sigma - 1} \beta_i w_i .$$

The total cost of production is $w_i L_i = w_i \alpha + \beta_i X_i w_i$. Differentiating this with respect to X_i gives the marginal cost to be $\beta_i w_i$. Since $\sigma > 1$ it implies that $\sigma/(\sigma - 1) > 1$, which means that price is greater than the marginal cost, i.e. $p_i > \beta_i w_i$. The mark-up, therefore, is $\sigma/(\sigma - 1)$.

3. (10 points) Due to free entry of firms, the profit of each firm is driven to zero, i.e. $p_i = AC_i$, where AC_i is the average cost of producing a product in country i .

$$AC_i = \frac{w_i \bar{L}_i}{X_i} = w_i \left(\frac{\alpha}{X_i} + \beta_i \right) .$$

For zero profits

$$p_i = AC_i ,$$

$$\frac{\sigma}{(\sigma - 1)} \beta_i w_i = w_i \left(\frac{\alpha}{X_i} + \beta_i \right) ,$$

Solving this for X_i give

$$\Rightarrow X_i = \frac{\alpha(\sigma - 1)}{\beta_i} .$$

Thus,

$$L_i = \alpha + \beta_i \frac{\alpha(\sigma - 1)}{\beta_i} = \alpha\sigma .$$

Since every product produced in country i uses this amount of labor the total amount of labor used is $n_i L_i$ and it has to be equal to the endowment of labor in country i (\bar{L}_i). Thus,

$$n_i L_i = \bar{L}_i \Rightarrow n_i \alpha\sigma = \bar{L}_i ,$$

$$n_i = \frac{\bar{L}_i}{\sigma\alpha} .$$

4. Now we incorporate trade costs and derive the gravity equation.

- (a) (10 points) With trade costs, the demand, in country j , for products imported from country i is given by:

$$C_{ij} = \frac{w_j \bar{L}_j}{P_j} \left(\frac{\tau_{ij} p_i}{P_j} \right)^{-\sigma} ,$$

Multiplying the quantity demanded by the price paid by country j gives the expenditure on a product imported from country i .

$$\tau_{ij} p_i C_{ij} = t_{ij} = w_j \bar{L}_j \left(\frac{\tau_{ij} p_i}{P_j} \right)^{1-\sigma} . \quad \mathcal{B}$$

Since all products imported from country i have the same expenditure, the total expenditure by country j on products of country i is $T_{ij} = n_i t_{ij}$.

$$T_{ij} = n_i Y_j \left(\frac{\tau_{ij} p_i}{P_i} \right)^{1-\sigma} . \quad \mathcal{C}$$

$$Y_j = w_j \bar{L}_j .$$

(b) (15 points) Substituting for n_i in the expression for T_{ij} gives

$$T_{ij} = \frac{\bar{L}_i}{\sigma \alpha} Y_j \left(\frac{\tau_{ij} p_i}{P_i} \right)^{1-\sigma},$$

$$\Rightarrow T_{ij} = \frac{w_i \bar{L}_i}{\sigma \alpha w_i} Y_j \left(\frac{\tau_{ij} p_i}{P_i} \right)^{1-\sigma},$$

$$\Rightarrow T_{ij} = \frac{Y_i Y_j}{\sigma \alpha w_i} \left(\frac{\tau_{ij} p_i}{P_i} \right)^{1-\sigma}.$$

*-8 not given
-4-5 major mistakes*

Since $p_i = \frac{\sigma}{\sigma-1} \beta_i w_i \Rightarrow w_i = \frac{\sigma-1}{\sigma} \frac{p_i}{\beta_i}$. Thus,

$$T_{ij} = \frac{Y_i Y_j}{\sigma \alpha} \frac{\sigma}{(\sigma-1)} \frac{\beta_i}{p_i} \left(\frac{\tau_{ij} p_i}{P_i} \right)^{1-\sigma},$$

$$\Rightarrow T_{ij} = \frac{\beta_i}{\alpha(\sigma-1)} \frac{Y_i Y_j}{p_i^\sigma} \left(\frac{\tau_{ij}}{P_i} \right)^{1-\sigma}.$$

An increase in τ_{ij} causes trade costs to increase. Thus, it makes importing products from country i more expensive. Since $\sigma > 1$, an increase in τ_{ij} reduces the demand for imported products, and also reduces the expenditure on imported products (T_{ij}). A higher σ will result in a greater decline in T_{ij} for a given change in τ_{ij} . This is because a higher σ implies a greater elasticity of substitution between products. Thus, when the products from country i become more expensive, due to an increase in τ_{ij} , country j consumer shifts to cheaper products from other countries (including products from itself) by a greater extent when σ is larger.

Question 2 (40 points)

1. (20 points) Cutoff Productivity for Exporting: All firms with a φ s.t. $\Pi_x(\varphi) \geq 0$ will export.

$$\therefore \Pi_x(\varphi_x) = 0$$

$$\Rightarrow \left(\frac{(\sigma-1) \varphi_x P}{\sigma} \right)^{\sigma-1} \cdot \frac{R}{\sigma} - f_x = 0$$

Remember for $\varphi = \underline{\varphi}$

$$\Pi_d(\varphi) = 0$$

$$\Rightarrow \left(\frac{(\sigma-1) \varphi P}{\sigma} \right)^{\sigma-1} \cdot \frac{R}{\sigma} - f = 0$$

$$\Rightarrow \left(\frac{(\sigma-1) P}{\sigma} \right)^{\sigma-1} \cdot \frac{R}{\sigma} = \frac{f}{(\varphi)^{\sigma-1}}.$$

$$\Rightarrow \left(\frac{\varphi_x}{\varphi} \right)^{\sigma-1} \cdot f = f_x$$

$$\Rightarrow \varphi_x = \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \cdot \varphi$$

We will assume that $\tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} > 1$ so that

$$\underline{\varphi}_x > \underline{\varphi}$$

Thus we get the following partitions:

- (i) $\varphi < \underline{\varphi}$ → do not enter the domestic market
- (ii) $\underline{\varphi} \leq \varphi < \underline{\varphi}_x$ → enter the domestic market but do not export
- (iii) $\varphi \geq \underline{\varphi}_x$ → enter both domestic and foreign market

Probability of Exporting: Conditional on survival, the probability of exporting is simply

$$prob_x = \frac{1 - G(\underline{\varphi}_x)}{1 - G(\underline{\varphi})}$$

Average profit of a firm: Conditional on survival a firm's average profit is

$$\Pi(\tilde{\varphi}) = \Pi_d(\tilde{\varphi}) + \Pi_x(\tilde{\varphi}_x) \cdot prob_x$$

We know that

$$\Pi_d(\varphi) = 0 \Rightarrow r(\varphi) = \sigma f$$

And since $\frac{r_d(\tilde{\varphi})}{r(\varphi)} = \left(\frac{\tilde{\varphi}(\varphi)}{\varphi} \right)^{\sigma-1}$

$$\begin{aligned} \Rightarrow r_d(\tilde{\varphi}) &= \left(\frac{\tilde{\varphi}(\varphi)}{\varphi} \right)^{\sigma-1} \cdot \sigma f \\ \therefore \Pi_d(\tilde{\varphi}) &= \left[\left(\frac{\tilde{\varphi}(\varphi)}{\varphi} \right)^{\sigma-1} - 1 \right] f = \kappa(\varphi) f \end{aligned}$$

Similarly, among the firms that export

$$\Pi_x(\tilde{\varphi}_x) = \left[\left(\frac{\tilde{\varphi}_x(\varphi_x)}{\varphi_x} \right)^{\sigma-1} - 1 \right] f_x = \kappa(\varphi_x) f_x$$

Thus,

(ZCP)

$$\Pi(\tilde{\varphi}) = \left[\left(\frac{\tilde{\varphi}(\varphi)}{\varphi} \right)^{\sigma-1} - 1 \right] f + prob_x \cdot \left[\left(\frac{\tilde{\varphi}_x(\varphi_x)}{\varphi_x} \right)^{\sigma-1} - 1 \right] f_x = \kappa(\varphi) f + prob_x \kappa(\varphi_x) f_x ,$$

where

$$(a) \quad prob_x = \frac{1 - G(\varphi_x)}{1 - G(\varphi)}$$

And since

$$(b) \quad \frac{r_x(\varphi_x)}{r_d(\varphi)} = \tau^{1-\sigma} \left(\frac{\varphi_x}{\varphi} \right)^{\sigma-1} = \frac{f_x}{f}$$

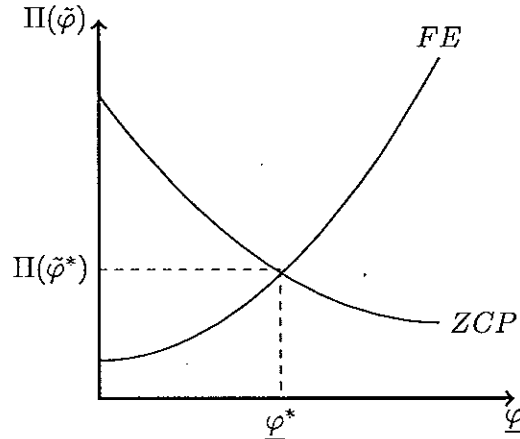
$$\Rightarrow \quad \varphi_x = \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \cdot \varphi$$

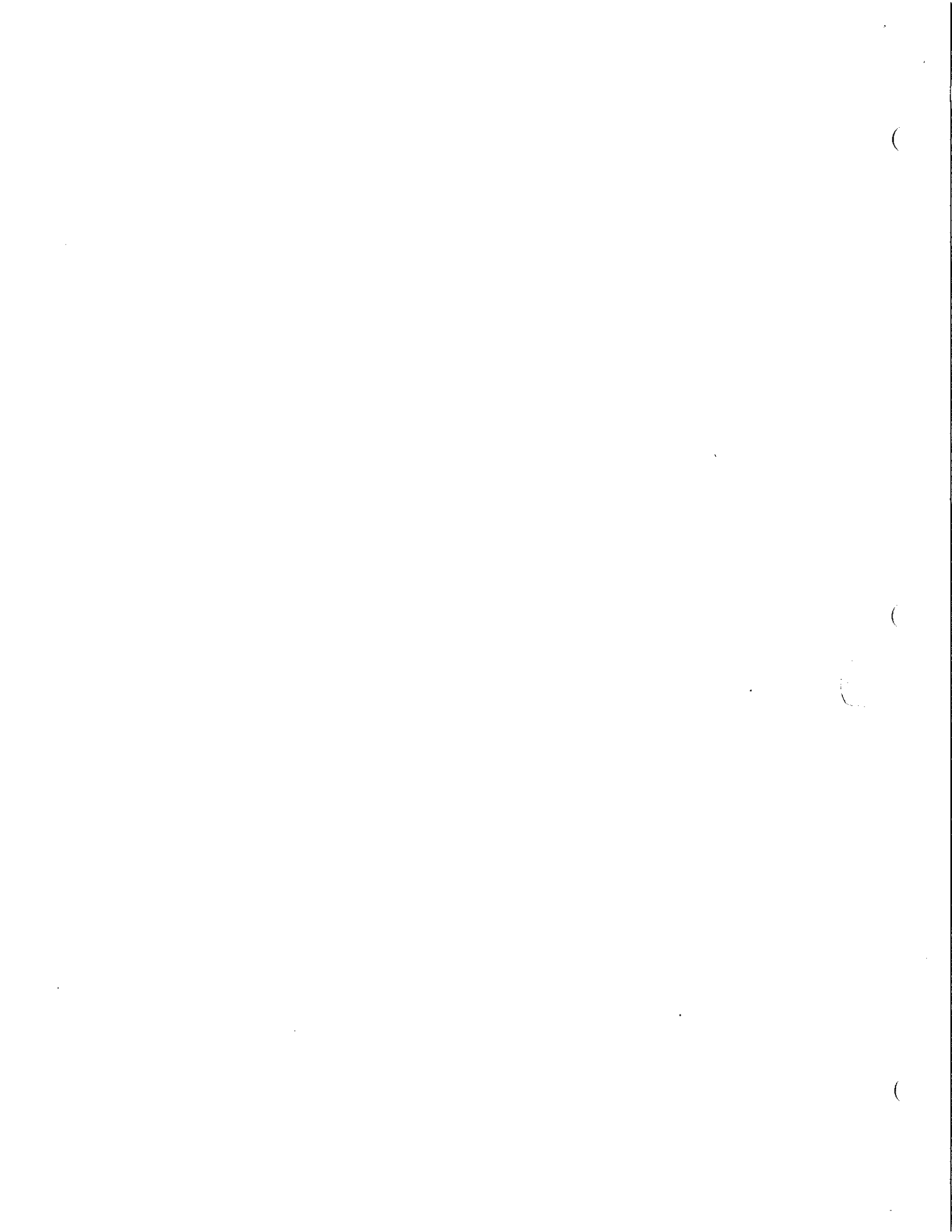
Free entry: due to free entry, the value from entry is driven to zero in equilibrium

$$V^e = \underbrace{(1 - G(\varphi))}_{\text{Probability of entry}} \cdot \underbrace{\Pi(\varphi)}_{\text{Ex-ante profits conditional on which include expected profits from exporting}} - \underbrace{f_e}_{\text{sunk cost of entry}}$$

$$(FE) \quad \therefore V^e = 0 \Rightarrow \Pi(\tilde{\varphi}) = \frac{f_e}{1 - G(\varphi)}$$

ZCP and FE provide a system of two equations in two unknowns. Equilibrium cut-off productivity - φ^* - and the equilibrium average profits - $\Pi(\tilde{\varphi}^*)$ - solve the ZCP and FE. Equilibrium cut-off productivity - φ^* - also implies the equilibrium cut-off productivity for exporting - φ_x^* .





(2)

$$\Rightarrow \frac{\partial j(\psi^*)}{\partial \tau} f = - \frac{\partial j(\psi_n^*)}{\partial \tau} f_x$$

$$\frac{\partial j(\psi^*)}{\partial \tau} = \frac{\partial j(\psi^*)}{\partial \psi^*} \frac{\partial \psi^*}{\partial \tau}$$

Similarly $\frac{\partial j(\psi_n^*)}{\partial \tau} = \frac{\partial j(\psi_n^*)}{\partial \psi_n^*} \frac{\partial \psi_n^*}{\partial \tau}$

Since $\psi_n^* = \psi^* \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}}$

$$\Rightarrow \frac{\partial \psi_n^*}{\partial \tau} = \frac{\partial \psi^*}{\partial \tau} \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} + \psi^* \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma-1}}$$

$$\Rightarrow \frac{\partial \psi_n^*}{\partial \tau} = \frac{\partial \psi^*}{\partial \tau} \left(\frac{\psi_n^*}{\psi^*} \right) + \frac{\psi_n^*}{\tau} \quad \text{--- (i)}$$

$$\left[\frac{\partial \psi^*}{\partial \tau} \cdot \frac{\partial j(\psi^*)}{\partial \psi^*} \cdot f = - \frac{\partial j(\psi_n^*)}{\partial \psi_n^*} \cdot \frac{\partial \psi_n^*}{\partial \tau} \cdot f_x \right] \quad \text{--- (A)}$$

$$\frac{\partial \psi^*}{\partial \tau} \cdot \frac{\partial j(\psi^*)}{\partial \psi^*} \cdot f = - \frac{\partial j(\psi_n^*)}{\partial \psi_n^*} \cdot f_x \left[\frac{\partial \psi^*}{\partial \tau} \left(\frac{\psi_n^*}{\psi^*} \right) + \frac{\psi_n^*}{\tau} \right]$$

$$\Rightarrow \frac{\partial \psi^*}{\partial \tau} \left[\frac{\partial j(\psi^*)}{\partial \psi^*} f + \frac{\partial j(\psi_n^*)}{\partial \psi_n^*} f_x \cdot \frac{\psi_n^*}{\psi^*} \right] = - \frac{\partial j(\psi_n^*)}{\partial \psi_n^*} f_x \cdot \frac{\psi_n^*}{\tau}$$

$$\Rightarrow \frac{\partial \psi^*}{\partial \tau} = - \frac{\psi_n^*}{\tau} \cdot \frac{\frac{\partial j(\psi_n^*)}{\partial \psi_n^*} f_x \cdot \psi_n^*}{f \cdot \psi^* \cdot \frac{\partial j(\psi^*)}{\partial \psi^*} + f_x \cdot \psi_n^* \cdot \frac{\partial j(\psi_n^*)}{\partial \psi_n^*}} \quad \text{--- (ii)}$$

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$$\text{Now } j(\underline{y}^*) = k(\underline{y}^*) (1 - G(\underline{y}^*)) \quad (3)$$

$$\Rightarrow \frac{\partial j(\underline{y}^*)}{\partial \underline{y}^*} = \frac{\partial k(\underline{y}^*)}{\partial \underline{y}^*} (1 - G(\underline{y}^*)) - k(\underline{y}^*) \frac{\partial G(\underline{y}^*)}{\partial \underline{y}^*}$$

It is given to us that

$$\frac{\partial k(\underline{y}^*)}{\partial \underline{y}^*} = \frac{k(\underline{y}^*) g(\underline{y}^*)}{1 - G(\underline{y}^*)} - \frac{(\sigma - 1) [k(\underline{y}^*) + 1]}{\underline{y}^*}$$

$$\Rightarrow \frac{\partial j(\underline{y}^*)}{\partial \underline{y}^*} = \left[\frac{k(\underline{y}^*) g(\underline{y}^*)}{1 - G(\underline{y}^*)} - \frac{(\sigma - 1) [k(\underline{y}^*) + 1]}{\underline{y}^*} \right] (1 - G(\underline{y}^*)) - k(\underline{y}^*) g(\underline{y}^*)$$

$$= \cancel{k(\underline{y}^*) g(\underline{y}^*)} - \frac{(\sigma - 1) (1 - G(\underline{y}^*)) [k(\underline{y}^*) + 1]}{\underline{y}^*} - \cancel{k(\underline{y}^*) g(\underline{y}^*)}$$

$$\Rightarrow \frac{\partial j(\underline{y}^*)}{\partial \underline{y}^*} = - \frac{(\sigma - 1) (1 - G(\underline{y}^*)) [k(\underline{y}^*) + 1]}{\underline{y}^*} < 0 \quad \text{--- (iii)}$$

Analogously $\frac{\partial j(\underline{y}_x^*)}{\partial \underline{y}_x^*} < 0 \quad \rightarrow \text{(iv)}$

$$\Rightarrow \frac{\partial j(\underline{y}^*)}{\partial \underline{z}} < 0 \quad \left(\text{Using (ii) \& (iv)} \right) \quad \text{--- (v)}$$

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Substituting (ii) in (i)

(4)

~~$$\frac{\partial \underline{y}_x^*}{\partial \tau} = \left(\frac{\underline{y}_x^*}{\underline{y}^*} \right) \left(\frac{-\underline{y}^*}{\tau} \right) \cdot \frac{f_x \underline{y}_x^* \cdot \frac{\partial j(\underline{y}_x^*)}{\partial \underline{y}_x^*}}{f \cdot \underline{y}^* \frac{\partial j(\underline{y}^*)}{\partial \underline{y}^*} + f_x \underline{y}_x^* \frac{\partial j(\underline{y}_x^*)}{\partial \underline{y}_x^*}}$$~~

From (A) we know that

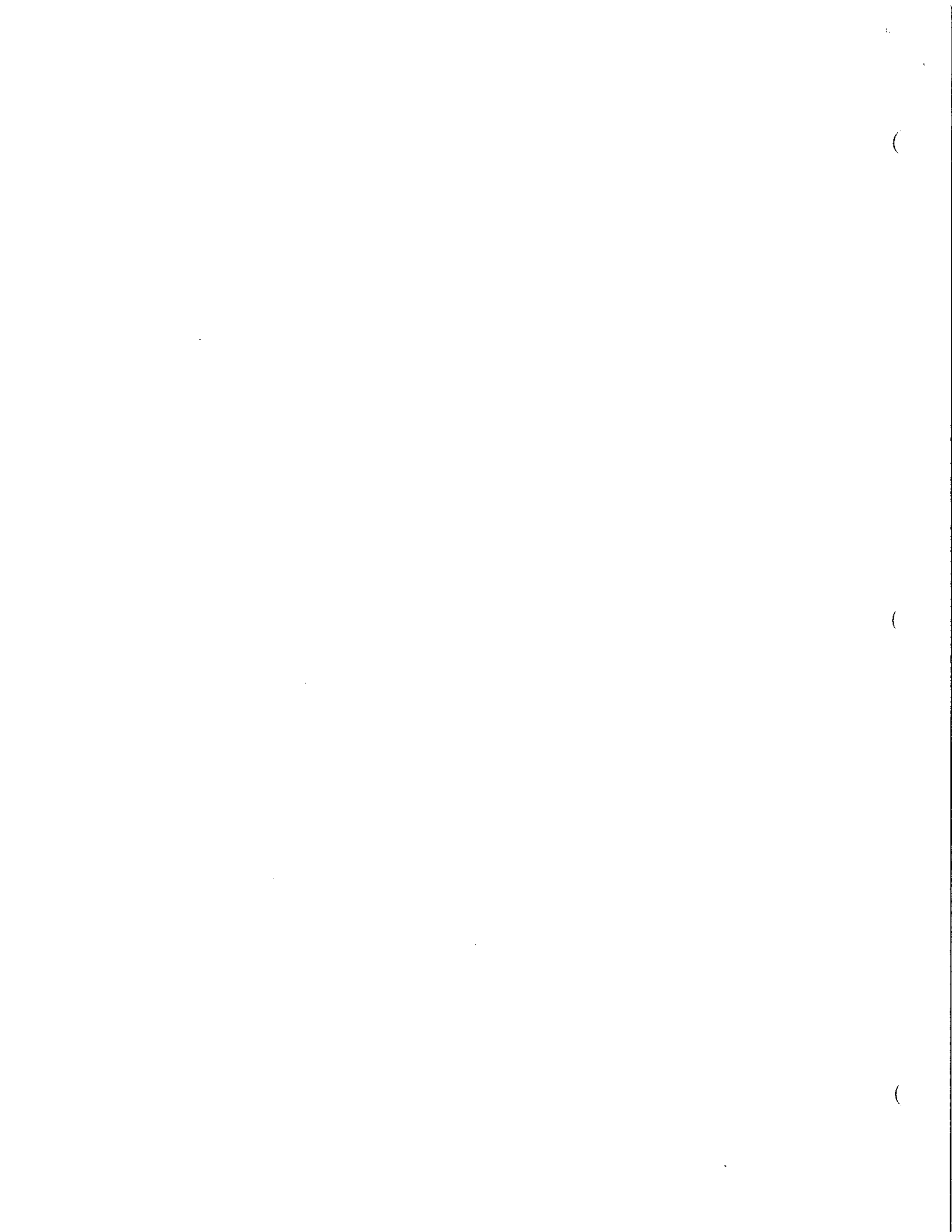
$$\frac{\partial \underline{y}^*}{\partial \tau} \cdot \frac{\partial j(\underline{y}^*)}{\partial \underline{y}^*} \cdot f = - \frac{\partial j(\underline{y}_x^*)}{\partial \underline{y}_x^*} \cdot \frac{\partial \underline{y}_x^*}{\partial \tau} \cdot f_x$$

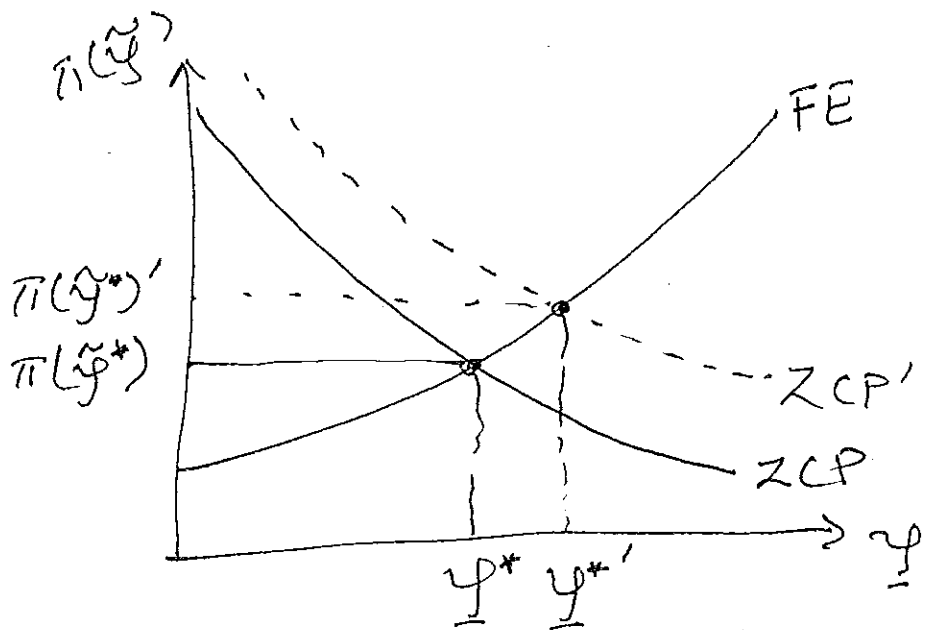
$$\Rightarrow \frac{\partial \underline{y}_x^*}{\partial \tau} = - \frac{f}{f_x} \cdot \frac{\frac{\partial j(\underline{y}^*)}{\partial \underline{y}^*}}{\frac{\partial j(\underline{y}_x^*)}{\partial \underline{y}_x^*}} \cdot \frac{\partial \underline{y}^*}{\partial \tau}$$

Using (iii), (iv) & (v) we get that

$$\frac{\partial \underline{y}_x^*}{\partial \tau} > 0 \quad \text{--- (vi)}$$

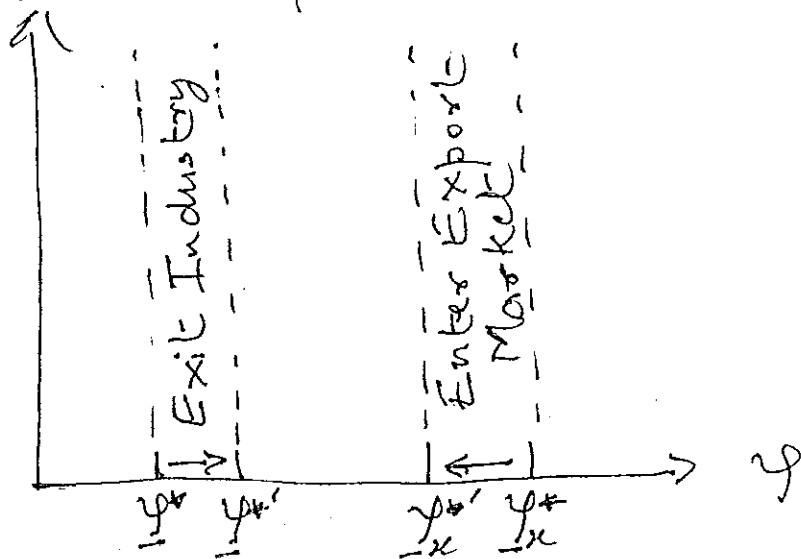
Intuition: A decrease in τ will shift the ZCP curve up & hence result in an increase in the cut-off productivity \underline{y}^* . The ZCP curve shifts up because a lower trade cost will increase average profits because of increase in profits from exporting.





However, the new exporting cut-off productivity will be lower. This is because the marginal cost of exporting is lower due to the lower τ & this allows lower productivity firms to be able to export profitably, ~~new~~.

Overall, a reduction in trade cost forces the least productive firms to exit but allows new lower (relative to the initial open economy equilibrium) productivity firms to enter the export market.



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