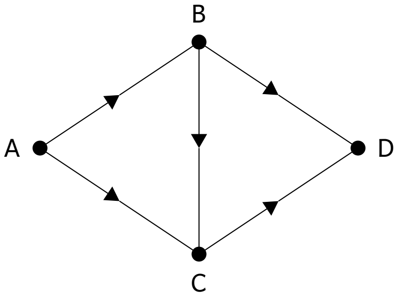
The Drivers' Problem



- $N \geq 2$ drivers want to go from A to D in the least amount of time.
- There are 3 alternative routes: ABD, ACD, and ABCD.
- The amount of time each driver spends on a given segment (AB, BD, AC, CD, BC) depends on the number of drivers using it.

Allocations

- An allocation is a triple of nonnegative integers that add up to N.
- Denote it with $(N_{\mathrm{ABD}}, N_{\mathrm{ACD}}, N_{\mathrm{ABCD}})$.
- An allocation is socially optimal if it minimizes the aggregate time spent by drivers on the road.
- An allocation is a Nash equilibrium if no driver can gain by unilaterally changing its proposed route.
- For this problem, finding optimal allocations and Nash equilibria can be easily automated with, eg, a spreadsheet.

Time Costs

Segment	Time cost
AB	$2\,N_{\text{AB}}$
BD	$7+N_{\text{BD}}$
AC	$7+N_{\text{AC}}$
CD	$2\,N_{\text{CD}}$
BC	N_{BC}

- The time costs of the different segments, as function of the drivers taking them, are given in the above table.
- With these costs, the time per driver for each route is:

$$T_{\text{ABD}} = 7 + 3\,N_{\text{ABD}} + 2\,N_{\text{ABCD}}$$

$$T_{\text{ACD}} = 7 + 3\,N_{\text{ACD}} + 2\,N_{\text{ABCD}}$$

$$T_{\text{ABCD}} = 2\,N_{\text{ABD}} + 2\,N_{\text{ACD}} + 5\,N_{\text{ABCD}}$$

Example with $N = 4$

$(N_{ABD}, N_{ACD}, N_{ABCD})$	$(T_{ABD}, T_{ACD}, T_{ABCD})$	Agg time
$(4, 0, 0)$	$(19, -, -)$	76
$(3, 1, 0)$	$(16, 10, -)$	58
$(3, 0, 1)$	$(18, -, 11)$	65
$(2, 2, 0)$	$(13, 13, -)$	52 Opt
$(2, 1, 1)$	$(15, 12, 11)$	53
$(2, 0, 2)$	$(17, -, 14)$	62
$(1, 3, 0)$	$(10, 16, -)$	58
$(1, 2, 1)$	$(12, 15, 11)$	53
$(1, 1, 2)$	$(14, 14, 14)$	56 Nash
$(1, 0, 3)$	$(16, -, 17)$	67
$(0, 4, 0)$	$(-, 19, -)$	76
$(0, 3, 1)$	$(-, 18, 11)$	65
$(0, 2, 2)$	$(-, 17, 14)$	62
$(0, 1, 3)$	$(-, 16, 17)$	67
$(0, 0, 4)$	$(-, -, 20)$	80