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A discrete-bid second-price sealed-bid auction with complete information

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Two bidders, 1 and 2, compete in order to get an object using the following auction mechanism. Each bidder will confidentially write her bid in a piece of paper and give this to the auctioneer. The auctioneer will then award the object to the higher bidder, who will have to pay the lower bid (the "second price") for it. In case of a tie, the auctioneer flips a fair coin to decide who gets the object.

Let v_i denote the maximum amount that bidder i, $i \in \{1,2\}$, is willing to pay in order to get the object (the bidder's value for the object). If the respective bids are (b_1, b_2), then the utility of bidder i, taking $j \neq i$, is:

$$u_i(b_1, b_2) = \begin{cases} v_i - b_j, & \text{if } b_i > b_j; \\ \frac{1}{2}(v_i - b_j), & \text{if } b_i = b_j; \\ 0, & \text{if } b_i < b_j. \end{cases}$$

We will assume that both values and bids must be integer numbers between 0 and 100:

$$v_1, v_2, b_1, b_2 \in \{0, 1, 2, \dots, 99, 100\}.$$

The values are assumed to be common knowledge. In order to find all Nash equilibria, we will first characterize the *best response* functions of the players. Taking again $i \neq j$, we have that:

$$\mathrm{BR}_i(b_j) = \begin{cases} \{b_i : b_i \leq b_j - 1\}, & \text{if } b_j \geq v_i + 1; \\ \{b_i : 0 \leq b_i \leq 100\}, & \text{if } b_j = v_i; \\ \{b_i : b_i \geq b_j + 1\}, & \text{if } b_j \leq v_i - 1. \end{cases} = \begin{cases} \{0, 1, \dots, b_j - 1\}, & \text{if } b_j \geq v_i + 1; \\ \{0, 1, \dots, 100\}, & \text{if } b_j = v_i; \\ \{b_j + 1, b_j + 2, \dots, 100\}, & \text{if } b_j \leq v_i - 1. \end{cases}$$

From here on, we will take $v_1 = 60$ and $v_2 = 50$. Contrasting with first-price auctions, in second-price auctions (with complete information) it is possible for the player with the lower value to win the object in equilibrium.

We can distinguish two types of Nash equilibria:

- *Equilibria in which bidder 1 gets the object*: In this case, we must have $b_1 \geq b_2 + 1$. In order for player 1 to be willing to win the object, it must be the case that $b_2 \leq 60$. In order for 2's strategy to be a best response, it must be the case that $b_1 \geq 50$. Any pair (b_1, b_2) satisfying those three inequalities will be an equilibrium. For example, $(b_1, b_2) = (65, 60)$ or $(b_1, b_2) = (50, 40)$.
- *Equilibria in which bidder 2 gets the object*: In this case, we must have $b_1 + 1 \leq b_2$. In order for player 2 to be willing to win the object, it must be the case that $b_1 \leq 50$. In order for 1's strategy to be a best response, it must be the case that $b_2 \geq 60$. Any pair (b_1, b_2) satisfying those three inequalities will be an equilibrium. For example, $(b_1, b_2) = (50, 60)$ or $(b_1, b_2) = (0, 100)$.

Exercise. Show that there are no Nash equilibria in which $b_1 = b_2$. Can you find values v_1 and v_2 between, say, 50 and 100, such that this is no longer true?