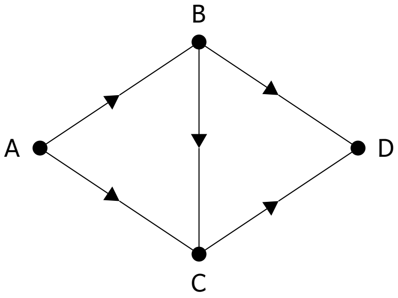
The Drivers' Problem



- $N \geq 2$ drivers want to go from A to D in the least amount of time.
- There are 3 alternative routes: ABD, ACD, and ABCD.
- The amount of time each driver spends on a given segment (AB, BD, AC, CD, BC) depends on the number of drivers using it.

Allocations

- An allocation is a triple of nonnegative integers that add up to N.
- Denote it with $(N_{\text{ABD}}, N_{\text{ACD}}, N_{\text{ABCD}})$.
- An allocation is socially optimal if it minimizes the aggregate time spent by drivers on the road.
- An allocation is a Nash equilibrium if no driver can gain by unilaterally changing its proposed route.
- For this problem, finding optimal allocations and Nash equilibria can be easily automated with, eg, a spreadsheet.

Time Costs

Segment	Time cost
AB	$2\,N_{\mathrm{AB}}$
BD	$7 + N_{\mathrm{BD}}$
AC	$7 + N_{\mathrm{AC}}$
CD	$2\,N_{\mathrm{CD}}$
BC	N_{BC}

- The time costs of the different segments, as function of the drivers taking them, are given in the above table.
- With these costs, the time per driver for each route is:

$$T_{\mathrm{ABD}} = 7 + 3\,N_{\mathrm{ABD}} + 2\,N_{\mathrm{ABCD}}$$
$$T_{\mathrm{ACD}} = 7 + 3\,N_{\mathrm{ACD}} + 2\,N_{\mathrm{ABCD}}$$
$$T_{\mathrm{ABCD}} = 2\,N_{\mathrm{ABD}} + 2\,N_{\mathrm{ACD}} + 5\,N_{\mathrm{ABCD}}$$

Example with $N = 4$

$(N_{ABD}, N_{ACD}, N_{ABCD})$	$(T_{ABD}, T_{ACD}, T_{ABCD})$	Agg time
$(4,0,0)$	$(19,-,-)$	76
$(3,1,0)$	$(16,10,-)$	58
$(3,0,1)$	$(18,-,11)$	65
$(2,2,0)$	$(13,13,-)$	52 Opt
$(2,1,1)$	$(15,12,11)$	53
$(2,0,2)$	$(17,-,14)$	62
$(1,3,0)$	$(10,16,-)$	58
$(1,2,1)$	$(12,15,11)$	53
$(1,1,2)$	$(14,14,14)$	56 Nash
$(1,0,3)$	$(16,-,17)$	67
$(0,4,0)$	$(-,19,-)$	76
$(0,3,1)$	$(-,18,11)$	65
$(0,2,2)$	$(-,17,14)$	62
$(0,1,3)$	$(-,16,17)$	67
$(0,0,4)$	$(-,-,20)$	80