Instituto Tecnológico Autónomo de México
Maestría en Economía
Microeconomía Aplicada II (Eco-31112), 2015
Lista de ejercicios 4

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1. Let $\Omega = (0,1]$ with uniform distribution. Let X and Y be simple random variables defined by: $X(\omega) = 10\, I_{(0,1/6]}(\omega) + 50\, I_{(1/6,2/3]}(\omega) + 90\, I_{(2/3,1]}$; $Y(\omega) = 20\, I_{(0,1/2]} + 80\, I_{(1/2,1]}$. Given $x \in \mathbb{R}$, let D_x be the constant random variable given by $D_x(\omega) = x$ for all ω.

(i) Find the distributions and expectations of the random variables $X + D_{50}$, $Y + D_{50}$, and $S = X + Y$.

(ii) Find the induced (marginal) distributions p_X and p_Y on $\mathbb{R}$.

(iii) Compute the expectations of X and Y from their probability distributions on $(0,1]$ and also from their induced distributions p_X and p_Y on the real numbers, and check that the results are the same in both cases.

(iv) Suppose that the wealth of an individual consists of two financial assets that are represented by the random variables X and Y. Suppose that the individual's Bernoulli utility over wealth is $u(z) = \sqrt{(}z)$. Compute the expected utility of the total wealth of the individual.

2. Let $\Omega = (0,1]$ with uniform distribution.

(i) Consider the simple distribution p equal to 10 with probability $\frac{1}{4}$, and to 20 with probability $\frac{3}{4}$. Give three different random variables that have this distribution.

(ii) Let $X(\omega) = 20\, I_{(0,1/2]}(\omega) + 40\, I_{(1/2,1]}(\omega)$, and $Y = 10\, I_{(0,1/3]}(\omega) + 30\, I_{(1/3,2/3]}(\omega) + 50\, I_{(2/3,1]}(\omega)$. Find the joint distribution of (X,Y), and show that they are not independent random variables. Let $\mu_X = \mathbb{E}(X)$ and $\mu_Y = \mathbb{E}(Y)$. The variance of X is defined as $\operatorname{Var}(X) := \mathbb{E}\{(X-\mu_X)^2\}$, the covariance of X and Y as $\operatorname{Cov}(X,Y) := \mathbb{E}\{(X-\mu_X)\,(Y-\mu_Y)\}$, and the variance of the sum of X and Y as $\operatorname{Var}(X+Y) = \mathbb{E}\{[(X-\mu_X)+(Y-\mu_Y)]^2\}$. Find μ_X and μ_Y. Show that, in general (no matter what the joint distribution is), $\mathbb{E}\{X+Y\} = \mu_X + \mu_Y$, and $\operatorname{Var}(X+Y) = \operatorname{Var}(X) + \operatorname{Var}(Y) + 2\operatorname{Cov}(X,Y)$ (thus, the magnitude of the variance of the sum compared to the sum of variances depends on the sign of the covariance), and verify those results for the present case.

(iii) Let $X(\omega) = 20\, I_{(0,1/2]}(\omega) + 40\, I_{(1/2,1]}(\omega)$, and $Y = 10\, I_{(0,1/6]}(\omega) + 30\, I_{(1/6,1/3]}(\omega) + 50\, I_{(1/3,2/3]}(\omega) + 30\, I_{(2/3,5/6]}(\omega) + 10\, I_{(5/6,1]}(\omega)$. Find the joint distribution of (X,Y), and show that they are independent random variables. Find μ_X, μ_Y, $\operatorname{Var}(X)$, $\operatorname{Var}(Y)$, $\operatorname{Cov}(X,Y)$ and $\operatorname{Var}(X+Y)$.

3. Let $\Omega = (0,1]$ with uniform distribution, and let $X = 5\, I_{(0,1/3]} + 10\, I_{(1/3,2/3]} + 20\, I_{(2/3,1]}$, and $Y = 20\, I_{(0,1/2]} + 5\, I_{(1/2,1]}$.

(i) Construct a random variable R, taking on the values 0 and 1 with respective probabilities 1/4 and 3/4, that is independent of X.

(ii) Construct a random variable S, taking on the values 0 and 1 with respective probabilities 1/4 and 3/4, that is independent of both X and Y.