Investing in Human Capital: 
The Efficiency of Covenants Not to Compete \(^1\)

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Abstract

Covenants not to compete (CNCs) are used in employment contracts to prevent employees from working for other employers. The legal enforcement of CNCs varies across jurisdictions in the U.S.: some states ban them (notably, California) while a majority of other states enforce CNCs when they reasonably protect a legitimate interest of the employer. The discrepancy in the legal policy regarding CNCs is reflected in an academic debate over the economic efficiency of these covenants. One side argues that CNCs are bad because they restrict labor mobility; the other side argues that the restriction on the movement of workers is good because it prevents workers from appropriating their employers’ human capital investments (and CNCs thereby encourage such investment). This paper addresses together the two objectives of ex post (labor mobility) and ex ante (human capital investment) efficiency. It compares CNCs with the alternative contract breach remedies of specific performance and liquidated damages. A given CNC may be analyzed as a hybrid that adopts specific performance with respect to attempted movements to employers within its scope and liquidated damages equal to zero with respect to movements outside its scope. Among the results of the paper is the finding that, where a CNC can be renegotiated, first-best performance and first-best investment can be induced. The appropriate choice of the CNC scope can balance perfectly the overinvestment tendency of specific performance against the underinvestment effect caused by zero liquidated damages. Contracting parties, however, have the incentive to agree to excessively broad CNCs that enable them to extract rents from prospective new employers within the CNC scope. The law should be wary of this incentive in policing CNCs.
1 Introduction

The economic objectives of labor mobility and human capital investment are in tension with each other. When labor is mobile, human capital moves to its highest valued use, but employers are discouraged from investing in training their workers because the investment payoffs are captured by future employers.\(^1\) In the language of incomplete contracts economics, this is the tension between *ex post* and *ex ante* efficiency. Attaining efficiency in both respects faces added obstacles in the case of employment contracts because of the nature of human capital investment by an employer (that is, human capital is portable in the hands of the worker and valuable in other uses) and because workers can invoke various legal defenses to avoid contract liability (notably, the discharge of debts in bankruptcy).

Covenants not to compete (CNCs) are common in employment contracts.\(^2\) A CNC forbids the worker to compete against the employer or to work for a competitor, either during or after the term of employment. It is frequently enforced by injunction: a court order of compliance that is backed by jail sanction instead of money damages. The purpose of a CNC is usually to protect the employer’s investment in human capital, as well as in other intangibles in which legal property rights protection is weak. The same purpose motivates CNCs in many joint ventures, partnerships and franchise agreements.

The legal enforcement of CNCs varies widely across jurisdictions in the U.S.: some states ban them (notably, California) while other states enforce CNCs when they reasonably protect a legitimate interest of the employer (for example, Massachusetts). Although a majority of states take the latter approach, there is a significant variation in the judicial interpretation of what constitutes a sufficient employer interest and, in light of this interest, the reasonable breadth of a CNC.

The discrepancy in the legal treatment of CNCs is reflected in an academic debate over the economic efficiency of these covenants. One side argues that CNCs are bad because they restrict labor mobility (e.g. Hyde (1998), Gilson (1999)); the other side argues that they are good because they protect human capital investment by restricting the movement of workers (e.g. Rubin and Shedd (1981), Trebilcock (1986) and Lester (2001)). For example, Rubin and Shedd (1981) note that various legal obstacles prevent workers from financing their own general training, including prohibitions against indentured servitude and assignment of wages, and the discharge of liabilities under bankruptcy law.\(^3\) An employer might finance the general training

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\(^1\)Similar arguments have been raised by, for example, Rajan and Zingales (1998), Coy (2002), and Kesner (2002).
\(^2\)See Whitmore (1990), Schwab and Thomas (2000), and Kaplan and Stromberg (2001).
\(^3\)The distinction between specific and general human capital comes from Becker (1993), who
and then recoup its investment by subsequently paying a wage less than the value of the worker’s contribution. However, this recovery is unavailable if the worker is free to move and take his new skills to a competitor (or threaten to do so). The worker cannot credibly commit \textit{ex ante} to refrain from moving because courts do not specifically enforce a worker’s employment promise and because the worker can avoid damages liability by, for instance, filing for bankruptcy. Therefore, Rubin and Shedd (1981) argue that the CNC is an effective alternative remedy that protects the employer’s general investment by restricting the worker’s outside opportunities. However, they do not address circumstances in which it may be \textit{ex post} efficient for the worker to leave the firm in favor of another employer.

In this paper, we address together the two objectives of \textit{ex post} (labor mobility) efficiency and \textit{ex ante} human capital investment efficiency. Section 2 presents a simple example of an incomplete employment contract that anticipates the subsequent entry of two alternative employers. We provide a more general model in Section 3. In each section, we solve for the first-best \textit{ex post} performance result (for whom should the worker optimally work) and the first-best \textit{ex ante} investment solution (how much should the employer invest in increasing the value of human capital). Human capital investment differs from the specific investment addressed in incomplete contracts models: although it is made by the buyer (the employer), the value may be taken by the seller (the worker) and used in other relationships.\footnote{We show that the CNC addresses the ex ante efficiency problem with respect to this type of investment.} Consistent with convention in the incomplete contracts literature, we examine separately the cases of no renegotiation and costless renegotiation. We compare in each case the performance of the CNC to the alternative breach remedies of specific performance and liquidated damages. Our model analyzes implicitly the scenario in which the worker is free to move to another employer, by setting liquidated damages to zero. The defining features of the CNC is that it is typically enforced by injunction and is contingent on the worker’s choice among alternative employment.

In the absence of renegotiation, each remedy is expected either to induce excessive switching to other employers (liquidated damages) or to deter efficient breach (specific performance). The CNC is a hybrid remedy that effectively combines specific performance when the worker seeks to move within the CNC scope, with liquidated damages equal to zero when the worker moves outside the covenant. The argued that workers should finance their own general human capital.

\footnote{This feature of human capital investment in employment relationships precludes the solutions to \textit{ex post} and \textit{ex ante} efficiency proposed by, among others, Che and Hausch (1999), Edlin and Reichelstein (1996), Hermalin and Katz (1993), and Noldeke and Schmidt (1995).}
CNC both prevents efficient movement to employers within its scope and permits inefficient movement to firms outside the covenant. Nevertheless, we show in Section 2 that a CNC may often be superior to the alternative contract breach remedies, including even a contract which raises no obstacles to worker mobility (i.e. liquidated damages equal to zero). The contracting parties have the appropriate incentives to agree to the CNC clause that maximizes ex post efficiency, unlike the incentives to provide for excessively high liquidated damages that extract rents from the entrants (demonstrated in Aghion and Bolton (1987) and Chung (1992)).

We then present a costless renegotiation model in which ex post efficiency is thereby assured. We show that optimal investment is possible if the scope of the CNC balances offsetting over- and under-investment effects. The overinvestment tendency is caused by the prospect of bargaining to permit the worker to move within the CNC. If it is efficient for the worker to shift to a firm covered by the CNC, the restriction enables the employer to extract a payment that reflects its expected private return from investment, rather than simply the return that the investment will in fact yield in the service of the new employer. Overinvestment results similarly from the renegotiation of specific performance. A CNC is different from those remedies, however, because it does not impede the worker from moving to a firm outside the CNC. The prospect that the worker may threaten to do so in order to hold-up the initial employer, together with the inability of the employer to recover its investment when the worker in fact moves, leads to underinvestment. By appropriately selecting the scope of the CNC, the parties can balance the over- and underinvestment distortions to yield the first-best investment solution.

Unfortunately, however, the parties cannot be relied on to reach this efficient result because part of the efficiency loss from overinvestment is externalized to future employers who lie within the CNC. Therefore, the initial employer and worker will tend to agree to inefficiently broad covenants. This result is analogous to the finding that contract parties may agree to supercompensatory liquidated damages in order to extract from entrants a larger portion of their surplus (see Spier and Whinston, 1995).

Our model applies to CNC restrictions either during or after the employment term.\textsuperscript{5} As a matter of form, a post-employment CNC is not a remedy for breach because the worker has no outstanding obligation to work for the employer at a contract wage. However, in our analysis, the post-employment CNC has the same effect as the CNC breach remedy. The worker is free to move to an employer outside the CNC, but she must negotiate a release from the initial employer before working.

\textsuperscript{5}The courts require that the CNC be ancillary to a legitimate contract or relationship; otherwise, a naked CNC is per se unenforceable (Farnsworth (1999), page 332).
for an employer within the CNC. If renegotiation is costless, the worker will move to her highest valued use. If that is with the initial employer, they will reach an agreement to extend the original employment term. As in the case when CNCs bind the worker during the employment term, the scope of the post-employment covenant can be set to yield the optimal \textit{ex ante} investment incentives, but the parties will be tempted to draft overly broad restrictions.

In Section 4, we discuss the legal implications of our results. Even in those jurisdictions that currently enforce CNCs, the courts are inclined to limit CNCs to circumstances in which the employer has legitimate interests in trade secrets, customer lists or relations and other confidential information. Our analysis suggests that the courts should expand the recognized interests to include both specific and general training. Moreover, the normative economic basis for judicial restraint in enforcing the terms of a CNC is not the often cited concern with restricting labor mobility, but rather the incentive for parties to employment agreements to contract for inefficiently broad CNCs that externalize the costs of investment to future employers. The courts should be watchful of evidence of such overreaching.

We conclude in Section 5 and present several avenues for extending our model.

2 A Three Firm Model

We first construct a model with four parties: a worker (W) contracts initially with a firm (Firm 0), but may later decide to seek employment with one of two other firms (Firm 1 or Firm 2). Of these two potential new employers, Firm 1 shares the most commonality with Firm 0, perhaps because it operates in the same industry and/or geographical region.

2.1 Model Assumptions

At $t = 0$, W promises to work for Firm 0 at time $T$. In turn, Firm 0 agrees to pay W a non-negative wage, $P$, at $T$ if W performs, and may also pay W a (non-negative) signing bonus $B$ at $t = 0$. We assume that Firm 0 will not breach this promise.\(^6\) Once the contract is agreed upon, Firm 0 invests $I$ to increase the value of the worker’s output. This investment may involve training in technical or managerial expertise and sharing of knowledge accumulated by the firm.

The value of the worker’s performance to Firm 0 at $T$ is given by $V_0(I, \theta) = k_0 I^\alpha + \beta_0 (\theta - .5)$, where $\theta$ is the value of a random state variable at $T$, which is

\(^6\)Alternatively, we could assume that Firm 0 is subject to expectations damages if it breaches, and that it is always able to make the resulting damages payment.
uniformly distributed between 0 and 1; $0 < \alpha < 1$ is a decreasing-returns-to-scale parameter for investment; $k_0$ is a positive sensitivity parameter with respect to investment; and $\beta_0$ is the sensitivity of the output to $\theta$.

At $T$, $W$ may decide to seek employment at $Firm$ 1 or $Firm$ 2 rather than perform for $Firm$ 0.\footnote{These firms may have also been around at $t = 0$. Rather than needlessly complicate the analysis by modelling the competition for $W$’s services at the initial time of contracting, we implicitly assume that $Firm$ 0 was successful at contracting with $W$ under the terms given above.} The contract between $W$ and $Firm$ 0 must specify a breach remedy to address this possibility. Under specific performance, the court would enforce the contract by an injunction compelling $W$ to work for $Firm$ 0. The injunction is backed by a sanction of jail sentence which we assume to be of infinite cost to $W$.ootnote{As a matter of current law, specific performance is not enforceable against an employee and liquidated damages are not enforced to the extent that they exceed the expected loss caused by the breach to the nonbreaching party. We noted earlier that the enforcement of CNCs is also limited. To present a normative comparison of the effects of the three remedies, however, we assume that the court will enforce without intervention the remedy chosen by the parties.} Under liquidated damages, the court would order $W$ to pay $Firm$ 0 a non-negative amount $D$ should she leave to work for one of the other firms.\footnote{While liquidated damages represent explicit payments to be made to a firm upon breach, the loss of unvested stock or stock option grants represents an implicit cost to $W$ of leaving her original employer that may be a component of $D$.} If $D = 0$, there would be no breach penalty and $W$ would be free to leave to work for another firm.

A CNC stipulates remedies that are contingent on the firm which $W$ seeks to join. A CNC is a negative covenant - a promise to refrain from working for a defined set of employers. Like specific performance, it is enforced by an injunction. In the three firm model in this section, we assume that $W$ is restricted from working for $Firm$ 1, the firm that is the more similar of the two new employers to $Firm$ 0, while $W$ is free to work for $Firm$ 2.

Should $W$ work for $Firm$ 1 or $Firm$ 2 at $T$, the value of the output associated with $W$ has a functional form analogous to that for $Firm$ 0: $V_i(I, \theta) = k_i I^\alpha + \beta_i(\theta - .5), i = 1, 2$. The sensitivity parameter, $k_i$, reflects the impact of investment on the value of $W$’s output should she work for $Firm$ $i$, and thus measures the specificity (and generality) of $Firm$ 0’s investment with respect to any other given employer. The investment in training will be tailored to $Firm$ 0 and thus will have its strongest impact on this firm, but the worker will be able to apply some of her acquired skills and knowledge to another employer, presumably affecting the output of companies in the same industry and/or geographical area in which it competes more significantly than the output of other companies. Thus, the sensitivities can be ranked as follows: $k_0 > k_1 > k_2$. We believe that this flexible specification appropriately reflects the fact that investments in human capital are unlikely to be
either purely general or purely specific (see Ehrenberg and Smith (2000)).

In order to ensure that there are $\theta$ values for which it would be optimal for $W$ to work for either Firm 1 or Firm 2 (which makes the model more interesting and realistic), we specify that $\beta_1 > \beta_0 > \beta_2$, i.e. Firm 1 has a comparative advantage for high $\theta$ values, while Firm 2 benefits in a relative sense from low $\theta$ values.

The following assumption specifies what is observable by the contracting parties, and what is verifiable by the courts.

**Assumption 1** The values $V_i(I, \theta)$, $i = 0, 1, 2$ are observable to $W$ and to all firms. However, these values, as well as $I$ and $\theta$, are not verifiable by the courts. Courts can only verify the following: the wage $P$ (and whether it has been paid); the liquidated damages amount $D$; which firm $W$ ultimately works for at $T$; and, where a CNC is specified, which firms are within the scope of the CNC (i.e., which firms $W$ is restricted from working for).

There are practical considerations regarding capital constraints of the employee, and opportunities for her to judgment proof her assets, i.e., to shield her assets from creditors. This leads us to make the following assumption.

**Assumption 2** $W$ is capital constrained so that she cannot finance her own training as it is received or make an up-front payment to Firm 0 (in exchange for a higher wage $P$ at $T$). $W$ may also be judgment proof, and thus cannot credibly bond her performance or be relied on to make large damages payments. $D$ and $P$ are jointly bounded above by the restriction that $P + D \leq \max_i V_i(I, \theta) \forall \theta$.

Assumption 2 precludes a liquidated damages remedy that specifies overly large damages in some states. Thus, while it may appear that a CNC could be replicated by a contingent damages contract that specifies a high level of damages for certain firms and zero damages for all other firms, such a contingent damages contract would require damages that are sometimes too high to be collectable. In addition, damages that are contingent on the outside option chosen by $W$ might appear as penalties rather than compensatory damages, and thus may not be enforced by the court.

Finally, we assume that all agents are risk-neutral and that the discount rate used to bring back the value at the date of performance to the date of contracting is equal to zero. These assumptions merely simplify the model exposition, without altering the nature of the results.
2.2 First-best Solution

The first-best solution represents the socially efficient investment by Firm 0 at \( t = 0 \) as well as the efficient worker performance at \( T \) that a central planner would choose.\(^{10}\) This will serve as the benchmark for our subsequent analysis. At \( T \), \( W \) should work for the firm that has the highest value associated with her output. This will depend on the realization of the uncertainty \( \theta \) at \( T \), the initial investment \( I \), and the values of \( k \) and \( \beta \) for the three firms.

Figure 1 illustrates the \( \text{ex post} \) performance decision for \( W \) given the following parameter values: \( \alpha = 0.5, k_0 = 16, k_1 = 4, k_2 = 2, \beta_0 = 50, \beta_1 = 290, \beta_2 = -200, \) and \( I = 23.3. \)\(^{11}\) The range of \( \theta \) values can be separated into three distinct regions: \((0, \theta_2)\), where it would be optimal for \( W \) to work for Firm 2, \((\theta_2, \theta_1)\), where \( W \) should work for Firm 0, and \((\theta_1, 1)\), where \( W \)'s output is maximized by working for Firm 1.\(^{12}\) One could loosely think of this as corresponding to a situation where under “normal” economic conditions (mid-range \( \theta \) values), \( W \) should work for a large stable firm that prospers under such conditions, while under a higher growth scenario (high \( \theta \)), \( W \) should work for a smaller firm that could better leverage \( W \)'s talents, and under a low growth scenario (low \( \theta \)), \( W \) should work for a company in a counter-cyclical or more stable industry. For instance, a software engineer working for a large technology company in the early 1990s might have been better employed by a start-up during the late 1990s, and in subsequent years by a company in the defense industry.

Let \( V^{FB}(I) \) denote the present value at \( t = 0 \) of the expected value derived from \( W \)'s efficient performance at \( T \), net of the investment in human capital, \( I \), and the wage, \( P \), that the central planner would pay \( W \). (Recall that \( \theta \) is uniformly distributed between 0 and 1, and that the discount rate is equal to zero).

\[
V^{FB}(I) = \int_0^{\theta_2} V_2(I, \theta) d\theta + \int_{\theta_2}^{\theta_1} V_0(I, \theta) d\theta + \int_{\theta_1}^1 V_1(I, \theta) d\theta - I - P
\]  

(1)

The first-best investment at \( t = 0 \), \( I^{FB} \), can be derived by determining the investment level at which the marginal benefit of an additional unit of investment is

\(^{10}\)This solution would also reflect the perspective of an investor holding a well-diversified portfolio of stocks, as long as the portfolio includes all three companies. For instance, this investor would want a successful manager to work for the firm in this investor’s portfolio that can best capitalize on the manager’s expertise.

\(^{11}\)While these parameter values have been chosen to clearly illustrate the ability of a CNC to lead to first-best investment, it will become clear in Section 3 that the general nature of our results are not parameter dependent.

\(^{12}\)Given the parameter values assumed, \( \theta_2 = .23 \) and \( \theta_1 = .74. \)
Figure 1: Values associated with W’s output at T at the three firms. The bold line represents first-best performance for the different θ realizations. The parameters used in this example are: α = .5, k₀ = 16, k₁ = 4, k₂ = 2, β₀ = 50, β₁ = 290, β₂ = −200, and I = 23.3.

equal to one, or equivalently, by setting the derivative of V^{FB} in equation (1) equal to zero. Assuming that α = .5, the following expression for I^{FB} can be readily obtained:

\[
I^{FB} = \left( \frac{.25(k_1 + k_2)}{1 - .5 \left( \frac{(k_0 - k_1)^2}{(β_0 - β_1)} - \frac{(k_0 - k_2)^2}{(β_0 - β_2)} \right)} \right)^2
\]  

Given the parameter values shown earlier, I^{FB} = 23.3.

2.3 Performance and Investment Assuming No Renegotiation

We now examine the investment and performance behavior of Firm 0 and W, respectively, once they have negotiated their initial contract at t = 0. We first look at the case where the contract cannot be renegotiated at T after θ is revealed. While it is unlikely that renegotiation would be completely infeasible, this can be viewed as an analytically tractable limiting case of costly renegotiation. Without renegotiation, performance at T will be inefficient in some states of the world, i.e. W will work for some firm other than the one given by the first-best solution for some values of θ. This may be the result of inefficient breach (leaving to work for Firm 1 or Firm 2 when it is efficient to continue to work for Firm 0), or inefficient lack of breach (working for Firm 0 when W should be working for one of the two other employers). The incidence of either of these two inefficiencies will depend on
the breach remedy specified in the contract.

Given Assumption 1, courts cannot calculate expectation or reliance damages, and cannot enforce a perfect state contingent contract. Thus, we focus on the following three remedial provisions in our analysis: specific performance of W’s promise to work for Firm 0; liquidated damages (and, if $D = 0$, the special case of no sanction for breach); and injunctive relief under a CNC.

Under specific performance, W will be bound to work for Firm 0 regardless of the realization of $\theta$. Contract performance will thus be first-best only when $\theta \in (\theta_2, \theta_1)$. For all other $\theta$ values, there will be inefficient lack-of-breach, and such inefficiency will be significant if the other two firms are much better able to capitalize on W’s expertise under particular $\theta$ realizations than is Firm 0. Since Firm 0’s output is more sensitive to the investment, $I$, than the output of the other two firms, Firm 0 has an incentive to choose an optimal investment level, $I^{SP}$, which will be higher than the first-best level $I^{FB}$. While intuitive for the three-firm model here, this result will be proved more formally in Section 3. We define $V^{SP}(I)$ as the value of the contract to Firm 0 at $t = 0$ assuming specific performance, i.e., the present value of the expected profit from W’s output, net of $P$, $I$, and the signing bonus $B^{SP}$ that Firm 0 would pay W at $t = 0$ in exchange for agreeing to the limitations imposed by the specific performance remedy:

$$V^{SP}(I) = \int_0^1 V_0(I, \theta) d\theta - P - B^{SP} - I$$

Firm 0 will maximize its value from the contract with W by selecting $I^{SP}$ such that $\frac{dV^{SP}(I^{SP})}{dI} = 0$. Assuming that $\alpha = .5$, it is straightforward to show that $\frac{dV^{SP}(I)}{dI} = .5k_0I^{-5} - 1$, and thus $I^{SP} = (.5k_0)^2$. Given $k_0 = 16$, $I^{SP} = 64.0$, which is indeed greater than $I^{FB} = 23.3$. As has been documented in prior studies on contract remedies in the absence of renegotiation, specific performance leads to both inefficient lack of breach and overinvestment relative to the first-best solution.\textsuperscript{13}

Under a liquidated damages rule, W can breach her contract with Firm 0 to go work for Firm $i$ ($i = 1, 2$), but must pay damages of $D$ if she leaves. For the purposes of this subsection, where we assume no renegotiation, it is sufficient to assume that as long as $V_i(I, \theta) > P + D$, Firm $i$ would offer W at least $P + D$ to entice her to leave Firm 0. As illustrated in Figure 2a (where we use the values $P = 40$ and $D = 10$), there are two $\theta$ regions where W would leave: for $\theta < \theta_2^D$, W would leave to work for Firm 2, and for $\theta > \theta_1^D$, W would leave to work for Firm 1. While breach will be efficient when $0 \leq \theta \leq \theta_2$ and $\theta_1 \leq \theta \leq 1$, inefficient breach

\textsuperscript{13}Of course, given the ex post inefficient lack of breach, $I^{SP}$ is the efficient investment level from Firm 0’s perspective, though it differs from $I^{FB}$. 

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Figure 2: Optimal performance of $W$ assuming either: (a) a liquidated damages remedy, or (b) a CNC. Optimal performance is indicated by the bold lines. The lightly shaded regions indicate inefficient breach; the darkly shaded region indicates inefficient lack of breach. In addition to the parameters shown in the caption to Figure 1, we assume that $P = 40$ and $D = 10$.

will occur in the regions $(\theta_2, \theta_2^D)$ and $(\theta_1^D, \theta_1)$ (shown as lightly shaded regions in Figure 2a).

We define $V^{LD}(I)$ as the value of the contract to Firm 0 at $t = 0$ assuming liquidated damages, i.e., the present value of the profits when $W$ works for Firm 0 and of the damages $D$ when $W$ works for another firm, net of $I$ and the signing bonus $B^{LD}$:

$$V^{LD}(I) = \int_0^{\theta_2^D} Dd\theta + \int_{\theta_2^D}^{\theta_1^D} (V_0(I, \theta) - P)d\theta + \int_{\theta_1^D}^1 Dd\theta - B^{LD} - I$$ (4)

Note that Firm 0 will directly profit from its investment only if $\theta_2^D < \theta < \theta_1^D$, i.e. if $W$ does not breach her contract, but will also receive $D$ should $W$ choose to leave. This leads Firm 0 to underinvest relative to the first-best solution, since some of the return from the investment is not internalized by Firm 0. To demonstrate this formally, we examine the relative benefit of an additional unit of investment in the first-best case versus in the case where a contract specifies a liquidated damages remedy.\footnote{Unlike in the first-best and specific performance cases, a closed-form expression for the optimal investment level, $I^{LD}$, is not easily attainable here. Therefore, we show here indirectly that $I^{LD} < I^{FB}$.} Using the shorthand notation $V'_i$ to denote the derivative of $V_i(I, \theta)$ with respect to $I$, the difference between the derivatives of $V^{LD}(I)$ and $V^{FB}(I)$ (shown in (1) and (4)) with respect to $I$ can be derived using Leibniz’s Theorem for differentiation of an integral:
\[
\frac{d(V^{LD}(I) - V^{FB}(I))}{dI} = \int_{\theta_2}^{\theta_1} -V'_2 d\theta + \int_{\theta_2}^{\theta_1^D} -V'_0 d\theta + \int_{\theta_1}^{1} -V'_1 d\theta + (P + D - V_0(I, \theta_2^D)) \frac{d\theta_2^D}{dI} - (P + D - V_0(I, \theta_1^D)) \frac{d\theta_1^D}{dI}
\]

(5)

\(V'_i\) is positive for all \(i\) (and all \(I\) and \(\theta\) values), and thus the first four terms in (5) are negative. It is relatively straightforward to show that \(\frac{d\theta_2^D}{dI} > 0\) and \(\frac{d\theta_1^D}{dI} < 0\), and since \(V_0(I, \theta_i^D) > P + D\) for \(i = 1, 2\), the last two terms are also negative. Therefore, the benefit of an additional unit of investment under liquidated damages is lower than under the first-best solution, and Firm 0 will underinvest. Given the parameters shown earlier, and using a simple numerical technique to find the investment level, \(I^{LD}\), that maximizes \(V^{LD}(I)\), the optimal investment level is found to be \(I^{LD} = 9.0\), which is indeed significantly less than \(I^{FB} = 23.3\).

A CNC combines features of both the specific performance and liquidated damages remedies just presented, since it restricts \(W\) from working for some firms (Firm 1 in our three-firm model), while allowing her to work for other firms (Firm 2, with \(D = 0\)). With respect to performance of \(W\) at \(T\), there will be inefficient breach if \(\theta_2 < \theta < \theta_0^2\) (lightly shaded area in Figure 2b), and inefficient lack of breach if \(\theta_1 < \theta < 1\) (darkly shaded area in Figure 2b). Note that \(\theta_0^2\) is simply \(\theta_2^D\) for \(D = 0\), and that it is higher than \(\theta_2^D\) values for \(D > 0\).

We define \(V^{CNC}(I)\) as the value of the contract with a CNC to Firm 0 at \(t = 0\) (i.e., the present value of profits from \(W\)’s employment at Firm 0, net of \(I\) and the signing bonus \(B^{CNC}\) that Firm 0 pays \(W\) to get her to agree to include the CNC provision).

\[
V^{CNC}(I) = \int_{\theta_2^D}^{1} (V_0(I, \theta) - P) d\theta - B^{CNC} - I
\]

(6)

While performance is inefficient as compared to the first-best solution, the hybrid nature of the CNC is desirable in terms of improving the efficiency of Firm 0’s investment decision at \(t = 0\). Since specific performance leads to overinvestment, while liquidated damages lead to underinvestment, there will be offsetting inefficient investment tendencies as a result of the hybrid nature of the CNC. We numerically solve for the investment level, \(I^{CNC}\) that maximizes \(V^{CNC}(I)\), given that an explicit closed-form solution for \(I^{CNC}\) is unattainable, and the sign of the difference between the derivatives of \(V^{CNC}(I)\) and \(V^{FB}(I)\) with respect to \(I\) depends on the specific parameter values chosen (unlike in the specific performance or liquidated damages
cases). Given the parameters specified earlier for our example, $I^{CNC} = 24.9$, which is reasonably close to the first-best solution (23.3), and certainly much closer to first-best than the investment level that would be selected under either specific performance or liquidated damages remedies.

Another important feature of the CNC is that, since there are no damages to be paid when $W$ goes to work for Firm 2, a CNC is not undermined by the risk that $W$ is judgement proof. Furthermore, in the absence of renegotiation, a CNC does not face the social welfare problem that authors such as Aghion and Bolton (1987) and Chung (1992) have pinned on liquidated damages, namely that $W$ and Firm 0 have the incentive to choose high liquidated damages in order to extract value from future potential employers who will attempt to bid for $W$’s services. Thus, when renegotiation is impossible, courts should be less hostile to CNCs than to liquidated damages provisions.

### 2.4 Investment Assuming Renegotiation

We continue to analyze performance and investment under different breach remedies, but we now assume that $W$ can renegotiate her contract with Firm 0 once $\theta$ is observed, but before performing at $T$. She will choose to do so if another firm offers her compensation that exceeds her contract wage, $P$, plus the damages payment $D$ that she would have to pay to leave Firm 0. Either Firm 0 will respond with a competitive offer, or $W$ will leave to join the other firm. While the breach remedy will affect the circumstances under which renegotiation will take place, the firm that $W$ will end up working for ultimately matches the first-best solution regardless of the remedy, given that renegotiation is assumed to be costless. However, the investment incentives will differ across the three remedy cases we investigate. We show below that only a CNC can lead to first-best investment under conditions where $W$ may be judgement proof and capital constrained. However, we also argue that Firm 0 and $W$ have an incentive when contracting at $t = 0$ to widen the CNC, i.e., to increase restrictions on $W$’s mobility beyond the socially optimal scope. Thus, courts may be justified in carefully enforcing CNCs when renegotiation is possible.

When specific performance is the breach remedy, there is no incentive for Firm 0 to renegotiate the terms of $W$’s contract when $\theta_2 < \theta < \theta_1$, i.e. where $V_0(I, \theta) > V_1(I, \theta)$ and $V_0(I, \theta) > V_2(I, \theta)$. However, if either $V_2(I, \theta) > V_0(I, \theta)$ or $V_1(I, \theta) > V_0(I, \theta)$, then there are gains attainable through renegotiation. We denote the renegotiation surplus as $S = \max_i (V_i(I, \theta) - V_0(I, \theta))$. There are three parties effectively involved in the negotiation at $T$: $W$, Firm 0, and Firm $j$, where $j = \arg\max_i V_i(I, \theta)$. While $W$ may negotiate with each of the two firms separately,
all that is necessary for our analysis is that there is some three-way surplus sharing $(\pi_0^{SP}, \pi_j^{SP}, \pi_W^{SP})$ such that each $\pi$ is non-negative and $\pi_0^{SP} + \pi_j^{SP} + \pi_W^{SP} = 1$. At $t = 0$, Firm 0 must select the investment level $I^{SP}$ that maximizes:

$$V^{SP}(I) = \int_0^{\theta_2} (V_0(I, \theta) + \pi_0^{SP}(V_2(I, \theta) - V_0(I, \theta))d\theta + \int_{\theta_2}^{\theta_1} V_0(I, \theta)d\theta$$

$$+ \int_{\theta_1}^{1} (V_0(I, \theta) + \pi_0^{SP}(V_1(I, \theta) - V_0(I, \theta))d\theta - P - B^{SP} - I$$

In order to determine whether the optimal investment level under specific performance will be above the first-best level, as expected, we must show that the incremental benefit of an additional unit of investment to Firm 0 under specific performance exceeds the incremental benefit under the first-best case. Once again denoting the derivative of $V_i(I, \theta)$ with respect to $I$ as $V_i'$ for simplicity, we have:

$$\frac{d(V^{SP}(I) - V^{FB}(I))}{dI} = \int_0^{\theta_2} (\pi_0^{SP} - 1)(V_2' - V_0')d\theta + \int_{\theta_1}^{1} (\pi_0^{SP} - 1)(V_1' - V_0')d\theta$$

This derivative is non-negative since $V_0' > V_1'$ and $V_0' > V_2'$ for all $I$ and $\theta$. Intuitively, changes in $I$ have a larger impact on $V_0$ than on $V_1$ or $V_2$, and therefore the surplus decreases in the regions $(0, \theta_2)$ and $(\theta_1, 1)$ as $I$ increases. Firm 0 only internalizes a portion $\pi_0^{SP}$ of the lost surplus, and therefore will overinvest.

Thus, while renegotiation leads to the first-best performance solution in terms of $W$ always working for the employer who values her output the most, the investment level selected by Firm 0 will be higher than the first-best solution. Using the parameter values specified earlier, together with $\pi_0^{SP} = .4$, the optimal investment under specific performance in our example can be numerically determined to be $I^{SP} = 55.0$, which is indeed greater than the first-best solution. Note, however, that while Firm 0 does commit more investment at $t = 0$ than prescribed by the first-best solution, the degree of overinvestment is not as severe as it is in the case where there is no renegotiation (where $I^{SP} = 64.0$). Since Firm 0 receives some of the surplus, and since the marginal increase in the surplus with respect to $I$, $V_i' - V_0'$, is negative, this will mitigate the incentive to overinvest, particularly if Firm 0 has

\[\text{For instance, } W \text{ may first solicit an offer from Firm } j \text{ of } V_j - \pi_j^{SP}S, \text{ and then will agree to pay Firm } 0 \text{ } V_0 - P + \pi_0^{SP}S \text{ in exchange for releasing her to work for Firm } j. \text{ Net of her payment to Firm } 0, W \text{ will earn } P + \pi_W^{SP}S, \text{ while Firm } j \text{ will earn a profit of } \pi_j^{SP}S. \text{ We assume for sake of simplicity that } \pi_1^{SP} = \pi_2^{SP}.\]

\[\text{Note that } V_0(I, \theta_j) = V_j(I, \theta_j) \text{ for } j = 1, 2, \text{ and thus the terms in (8) that would involve } \frac{d\theta_j}{dt} (\text{from Leibniz’s Theorem for differentiation of an integral)} \text{ disappear.}\]
significant power in the negotiation (i.e., \((\pi_0^{SP} - 1)\) in (8) is close to zero).

As in the case of specific performance, first-best performance is also obtained when renegotiation is superimposed on the liquidated damages remedy. With liquidated damages, renegotiation will occur if \(V_i(I, \theta) > P + D\) and yet \(V_0(I, \theta) > V_i(I, \theta)\) \(i.e.\), in the regions \((\theta_2, \theta_2^D)\) and \((\theta_1^D, \theta_1)\). \(W\) will threaten to quit to go work for another firm and \(Firm 0\) will then agree to pay \(W\) a higher wage in order to prevent her from quitting, thus losing a portion of its investment. This is the classic hold-up problem of Hart and Moore (1988) and Williamson (1975).

The surplus that \(W\) and \(Firm 0\) attempt to capture through their negotiation is \(S_i = V_0(I, \theta) - V_i(I, \theta), i = 1, 2\). We assume that this is split between them according to the sharing rule \(\pi_{LD}^{W}\) and \(\pi_{LD}^0\) (both non-negative and \(\pi_{LD}^{W} + \pi_{LD}^0 = 1\)). Since \(Firm i\) will be willing to offer \(W\) a wage as high as \(V_i(I, \theta)\), \(Firm 0\) must pay \(W\) an amount equal to \(V_i(I, \theta) - D\) plus its share of the surplus \((\pi_{LD}^{W}(V_0(I, \theta) - V_i(I, \theta)))\). This leaves \(Firm 0\) with \(\pi_{LD}^0(V_0(I, \theta) - V_i(I, \theta)) + D\). Under the liquidated damages rule, \(Firm 0\) must thus select \(I\) so as to maximize the following objective function:

\[
V_{LD}(I) = \int_0^{\theta_2} Dd\theta + \int_{\theta_2}^{\theta_2^D} (\pi_{LD}^{W}(V_0(I, \theta) - V_2(I, \theta)) + D)d\theta \\
+ \int_{\theta_2}^{\theta_1^D} (V_0(I, \theta) - P)d\theta + \int_{\theta_1^D}^{\theta_1} (\pi_{LD}^{W}(V_0(I, \theta) - V_1(I, \theta)) + D)d\theta \\
+ \int_{\theta_1}^{1} Dd\theta - B_{LD} - I \tag{9}
\]

As in the case of no renegotiation, the direction of inefficient investment can be determined by looking at the difference between the derivatives of \(V_{LD}\) and \(V_{FB}\) with respect to \(I\).

\[
\frac{d(V_{LD}(I) - V_{FB}(I))}{dI} = \int_0^{\theta_2} -V_2'(d\theta + \int_{\theta_2}^{\theta_2^D} (\pi_{LD}^{W}(V_0'(I, \theta) - V_2'(I, \theta)) - V_0'(d\theta \\
+ \int_{\theta_2}^{\theta_1^D} (\pi_{LD}^{W}(V_0'(I, \theta) - V_1'(I, \theta)) - V_0'(d\theta \\
+ \int_{\theta_1^D}^{\theta_1} -V_1'(d\theta \\
+ (\pi_{LD}^{W}(V_0(I, \theta_2^D) - V_2(I, \theta_2^D)) - V_0(I, \theta_2^D) + P + D)\frac{d\theta_2^D}{dI} \\
- (\pi_{LD}^{W}(V_0(I, \theta_1^D) - V_1(I, \theta_1^D)) - V_0(I, \theta_1^D) + P + D)\frac{d\theta_1^D}{dI} \\
+ D\left(\frac{d\theta_2}{dI} - \frac{d\theta_1}{dI}\right) \tag{10}
\]
Since $V'_i > 0$ for all $i$, the first and fourth terms are clearly negative, and, since $\pi_0^{LD}(V'_0 - V'_i) - V'_0 = -(\pi_0^{LD}V'_i + (1 - \pi_0^{LD})V'_0)$, the second and third terms are negative as well. Since $\pi_0^{LD}(V_0(I, \theta) - V_i(I, \theta)) - V_0(I, \theta) + P + D = -(\pi_0^{LD}V_i(I, \theta) + (1 - \pi_0^{LD})V_0(I, \theta)) + P + D \leq -V_i(I, \theta) + P + D = 0$ at $\theta_i^{D}$, and, as noted earlier, $\frac{d\theta_i^{D}}{dI} > 0$ and $\frac{d\theta_i^{D}}{dI} < 0$, the fifth and sixth terms are also negative (as long as $\pi_0^{LD} < 1$, otherwise they are equal to zero). Finally, it is easy to show that $\frac{d\theta_i^{D}}{dI} < 0$ and $\frac{d\theta_i^{D}}{dI} > 0$, and thus the last term is negative. Thus, $\frac{d(V^{LD} - V^{FB})}{dI} < 0$, indicating that Firm 0 will underinvest, as expected.

Given the parameters specified earlier, together with $\pi_0^{LD} = 0.4$, one can compute that the optimal investment level $I^{LD} = 9.4$. While this is indeed less than $I^{FB} = 23.3$, it is marginally higher than in the case where renegotiation was not allowed, since Firm 0 gets a portion of the surplus in the regions $(\theta_2, \theta_2^{D})$ and $(\theta_1^{D}, \theta_1)$ and thus has some additional incentive to invest.

We turn now to the CNC that restricts $W$ from working for Firm 1 but allows her to leave without penalty to work for Firm 2. Recall that the CNC acts as a hybrid remedy combining features present under each of specific performance and liquidated damages, and as result the CNC may balance the offsetting tendencies to over- and under-invest. Indeed, we now show that when renegotiation is possible, efficient ex ante investment and ex post performance can both be obtained.\textsuperscript{17}

Renegotiation will occur under the following two circumstances: in the region $(\theta_2, \theta_2^{D})$, Firm 0 will renegotiate with $W$ so that she continues to work for Firm 0, and the two can share the surplus $V_0(I, \theta) - V_2(I, \theta)$ (with Firm 0’s share of the surplus denoted by $\pi_0^{LD}$); in the region $(\theta_1, 1)$, Firm 0 will release $W$ from her CNC obligation, and $W$, Firm 0, and Firm 1 will split the surplus $V_1(I, \theta) - V_0(I, \theta)$ (with Firm 0’s share of the surplus denoted by $\pi_0^{SP}$ in this three-party negotiation). Given these renegotiations, Firm 0’s selection of an optimal investment level will maximize the following objective function:

$$V^{CNC}(I) = \int_{\theta_2}^{\theta_2^{D}} \pi_0^{LD}(V_0(I, \theta) - V_2(I, \theta))d\theta + \int_{\theta_2^{D}}^{\theta_1} (V_0(I, \theta) - P)d\theta + \int_{\theta_1}^{1} (V_0(I, \theta) - P + \pi_0^{SP}(V_1(I, \theta) - V_0(I, \theta)))d\theta - B^{CNC} - I \quad (11)$$

\textsuperscript{17}We could also model post-employment CNC restrictions in this model by letting $P = D = 0$ and setting aside the remedy of specific performance: that is, the employer would have no obligation to pay a wage $P$, and the worker would have no obligation to perform. This covenant only makes sense in this model if the parties can renegotiate, thereby permitting the employer to recover a portion of the value of her investment.
Trying to sign the derivative of $V^{CNC}(I) - V^{FB}(I)$ with respect to $I$, as we did for the other breach remedies, leads to an ambiguous result in general. This should be expected as the CNC combines the overinvestment tendency associated with specific performance and the underinvestment incentive associated with liquidated damages. However, the following explicit expression for $I^{CNC}$ can be found by setting the derivative of equation (11) with respect to $I$ equal to zero:

$$I^{CNC} = \left( \frac{P^{(1-(\pi_0^{LD})/(\beta_2^2)) - k_0 - k_2) + .5(k_0 + \pi_0^{SP}(k_1 - k_0))}{2 - \frac{\pi_0^{LD}(k_0-k_2)^2}{(\beta_0-\beta_2)} - \frac{\pi_0^{FD}(k_0-k_1)^2}{(\beta_1-\beta_0)} - \frac{k_2}{\beta_2^2}(\pi_0^{LD}k_2 + (1 - \pi_0^{LD})(2k_0 - \frac{3k_0k_2}{\beta_2}))} \right)^2$$

(12)

Substituting the parameter values assumed earlier into equation (12), we find $I^{CNC} = 23.3 = I^{FB}$. Thus, by restricting $W$ from working for only one of the two other firms, the first-best investment solution can be attained.\(^{18}\)

While this leads to efficient investment, Firm 0 and $W$ will not necessarily set the scope of the CNC in this manner. They will tend to agree to an inefficiently broad CNC. In this example, it is easy to demonstrate that Firm 0 and $W$ have an incentive to include both firms in the CNC restriction.\(^{19}\)

If $W$ is restricted from working for Firm 2 (as well as Firm 1), there will be a different distribution of profits at $T$ than under the single-firm CNC restriction if $\theta$ is in either $(0, \theta_2)$ or $(\theta_2, \theta_0^2)$. In the latter region, rather than $W$ having an outside option to work for Firm 2, which she can use to obtain a portion of $V_0(I, \theta) - V_2(I, \theta)$, there would be no negotiation since $W$ would be restricted from leaving. In exchange for $W$’s agreement to include Firm 2 in the scope of the CNC, Firm 0 will pay $W$ a larger up-front bonus, $B^{CNC}$, to compensate $W$ for the loss of her hold-up opportunity. The total value of the contract would rise because underinvestment would be avoided in this region.

However, in the region $(0, \theta_2)$, if $W$ and Firm 0 agree to include Firm 2 in the scope of the CNC, there would be a three-way negotiation that would include Firm 0, rather than just a two-way negotiation between $W$ and Firm 2. There are

\(^{18}\)The parameter values were chosen here so as to carefully demonstrate that a CNC can induce the efficient level of investment by balancing the hold-up problem due to the potential departure of $W$ to Firm 2, and the overinvestment problem associated with the restriction preventing $W$ from working for Firm 1. While the contrasting incentives may not perfectly balance each other out for other parameter values, there will nonetheless be a tendency for the CNC to lead to more efficient incentives than under the alternative remedies. Furthermore, in the next section we will show that an optimal scope for the CNC can be obtained under general conditions, and thus our results are not parameter dependent.

\(^{19}\)This turns the CNC into specific performance in this example, because there are only two other employers.
two consequences of this altered negotiation game, one distributional and the other related to investment efficiency. First, it is quite possible that $W$ and Firm 0 can extract more of the surplus ($V_2(I, \theta) - V_0(I, \theta)$) than $W$ could on her own. Thus, including Firm 2 in the CNC's scope might shift some of the surplus towards the original contracting parties, but nevertheless yields \textit{ex-post} efficiency. Second, $W$ and Firm 0 could appropriate some of the surplus they would otherwise share with Firm 2 by inefficiently increasing the level of investment (since $V'_0 > V'_2$ and thus the surplus $V_2(I, \theta) - V_0(I, \theta)$ is negatively related to $I$).\footnote{If Firm 1 and Firm 2 could invest in anticipation of the possible hiring of $W$ at $T$, then an overly broad CNC would lead them to invest less than the socially desirable amount. However, this type of investment is outside the scope of our model.}

Our analysis thus indicates that while courts should recognize CNCs as having the potential to improve investment incentives, they should also scrutinize the CNCs to ensure that they are not set with too broad a scope based on the contracting parties' incentive to expropriate wealth from third parties who offer the possibility of \textit{ex post} optimal performance.

\section{The General Model}

We now generalize the model from the previous section, relaxing three restrictive features of that model. First, we allow for an arbitrarily large number, $N$, rather than just two, potential new employers at $T$. Second, we require that the value functions $V(I, \theta)$ satisfy only some reasonably general conditions (provided below), rather than specifying a particular functional form. Third, we incorporate a general distribution $f(\theta)$ (with the cumulative distribution denoted as $F(\theta)$), rather than restricting $\theta$ to be uniformly distributed.\footnote{While we could also readily allow $\theta$ to represent a vector of uncertainties, rather than just a single uncertainty, we limit its dimensionality to one to simplify the exposition.}

In the three-firm example, the parameters of the problem were carefully selected to ensure that limiting the scope of the CNC to include only one firm yielded the exact balancing of offsetting investment incentives required to obtain the first-best solution. We show now that with an arbitrarily large $N$, one can select a scope $n$ for the CNC (i.e., restricting $W$ from working for $n$ firms) such that the first-best solution can be attained regardless of parameter values, functional form of the output values, or type of uncertainty distribution. We demonstrate this result only for the case where renegotiation is allowed, and thus both investment and performance will be efficient.\footnote{Analogous results for the case of no renegotiation can be obtained by substituting $\pi_0^{SP} = \pi_0^{LD} = 0$ into the expressions shown in this section, i.e. assuming that Firm 0 has no negotiating power.}
The following assumption about the value functions \( V_i(I, \theta) \) ensures that investment in human capital will be positive, but finite:

**Assumption 3** For all \( i = 0, \ldots, N \), \( V_i(I, \theta) \) is bounded, increasing, twice differentiable, and strictly concave in \( I \).

We continue to assume that the impact of the investment by Firm 0 on \( W \)'s output will be largest when \( W \) works for Firm 0, i.e., \( V_0'(I, \theta) > V_i'(I, \theta) \) \( \forall \theta, i = 1, \ldots, N \). In the limiting case where the training received by \( W \) is purely specific, all \( V_i'(I, \theta) = 0 \) for \( i > 0 \), and the condition clearly holds. If there is a general component to investment, this assumption remains reasonable as long as there is a measurable firm-specific component of investment which increases the marginal value, \( V_0'(I, \theta) \), relative to \( V_i'(I, \theta) \). For exposition purposes, it is also useful to order the firms consecutively in decreasing order of \( V_i'(I, \theta) \). This will likely reflect decreasing commonality with Firm 0: Firm 1 will be most like Firm 0 in terms of its operations and composition of assets; Firm 2 is next most similar; and so on, with Firm \( N \) sharing the least commonality with Firm 0. The cutoff point, \( n \), that determines the scope of the CNC may thus be based on factors such as industry or geographic location that determine the commonality between firms (or, in the language of the doctrine, Firm 0’s closest competitors).

We define \( \Theta \) as the set of all \( \theta \) values, and \( \Phi(I) \) as the decomposition of \( \Theta \) into \( N + 1 \) subsets \( (\Theta_0, \Theta_1, \ldots, \Theta_N) \), where \( \Theta_i \) is the (possibly null) subset of \( \theta \) values for which \( W \) optimally works for Firm \( i \), i.e., where \( i = \text{argmax}_j V_j(I, \theta) \). \( \Phi \) thus represents the efficient “performance” policy at \( T \), which will be independent of the type of breach remedy specified in the contract, given that we are assuming costless renegotiation. \( \Phi \) is, though, a function of \( I \), since the investment level will affect the relative values of \( W \)'s output at each of the \( N + 1 \) firms.

Given both first-best performance, \( \Phi \), and investment, \( I^{FB} \), we can express \( V^{FB}(I^{FB}) \) as follows:

\[
V^{FB}(I^{FB}) = \sum_{i=0}^{N} \int_{\theta \in \Theta_i} V_i(I^{FB}, \theta) dF(\theta) - P - I
\]  

(13)

First-best investment, \( I^{FB} \), is determined such that at the margin the benefit and cost of an additional unit of investment are equal, and thus \( \frac{dV^{FB}(I^{FB})}{dI} = 0 \).

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23It is actually sufficient for our analysis that \( \int_{\theta_0}^{\theta_i} (V_0' - V_i') dF(\theta) > 0 \), where \((\theta_0, \theta_i)\) represents the region where \( i = \text{argmax}_j V_j(I, \theta) \), i.e., where Firm \( i \) dominates all other firms in terms of the value it would receive from \( W \)'s output. This is a sufficient, but not necessary, condition for a CNC to lead to an efficient investment solution. In the unlikely case where this inequality were to be violated over many regions of \( \theta \) values, then all remedies that we explore will lead to underinvestment, including specific performance.
We now more formally revisit the three remedial provisions analyzed in the previous section. We show that only a CNC can lead to first-best investment, and that the CNC must have a carefully selected scope \((n^{FB})\) in order to achieve the efficient investment level. However, we will also show, this time in the setting of the more general model, that the initial contracting parties have an incentive to select a broader scope than the socially efficient optimum (i.e. \(n > n^{FB}\)), and thus courts may be justified in carefully enforcing CNCs when renegotiation is possible.

We consider first the remedy of specific performance. When \(\theta \in \Theta_i, i = 1, \ldots, N\) (i.e. \(i = \arg\max_j V_j(I, \theta)\), and \(i \neq 0\), Firm 0 should negotiate a severance agreement with \(W\), who will in turn negotiate a new employment contract with Firm \(i\). The three parties will split the surplus \(V_i(I, \theta) - V_0(I, \theta)\) according to the sharing rule \((\pi^{SP}_W, \pi^{SP}_0, \pi^{SP}_i)\), where each \(\pi\) is bounded below by 0 and above by 1. This general specification of the surplus sharing rule encompasses a range of possible labor market competitiveness for \(W\)’s services (e.g. if the market is highly competitive, then \(\pi^{SP}_i\) will be close to zero).

The value of the contract to Firm 0 at \(t = 0\), \(V^{SP}(I)\), is as follows:

\[
V^{SP}(I) = \sum_{i=0}^{N} \int_{\theta \in \Theta_i} \left( V_0(I, \theta) + \pi^{SP}_0(V_i(I, \theta) - V_0(I, \theta)) \right) dF(\theta) - P - B^{SP} - I \tag{14}
\]

**Proposition 1** When the terms of the contract specify specific performance by \(W\), the investment level, \(I^{SP}\), chosen by Firm 0 to maximize \(V^{SP}(I)\) will be strictly larger than the socially efficient level \(I^{FB}\), as long as Firm 0 does not have all the negotiation power (i.e., \(\pi^{SP}_0 < 1\)).

The proof of this proposition is relegated to the Appendix, along with the proofs of all subsequent propositions. The intuition behind Firm 0’s overinvestment is the same as that presented in Section 2. When \(W\) moves to an entrant firm, some of the social value of the investment is lost. Yet, Firm 0 does not internalize this loss because it can recover the full value of its investment from the entrant. This leads Firm 0 to overinvest.

Turning now to the case of liquidated damages, recall from the three-firm example in Section 2 that there were three distinct scenarios to consider. When \(\theta \in \Theta_i, i \neq 0\), it will be optimal for \(W\) to leave Firm 0 to go to work for Firm \(i\), and Firm 0 will simply receive \(D\) in these regions. Now, consider the following two distinct subsets of \(\Theta_0\). For \(\theta\) values where \(V_0 > \max_{j \neq 0} V_j(I, \theta) > P + D\), \(W\) will

\[\text{In contrast, if Firm 0 captured all the surplus because of its bargaining power (\(\pi^{SP}_0 = 1\)), it would bear the efficiency loss, and therefore would not overinvest.}\]
threaten to leave to join Firm $i$ ($i = \arg\max_j V_j(I, \theta)$), and Firm 0 will be forced to renegotiate with her, leaving Firm 0 with a payoff of $\pi_{0D}(V_0(I, \theta) - V_i(I, \theta)) + D$ in these regions, which we denote $\Theta_{0i}$ ($i = 1, \ldots, N$). For those $\theta$ values in $\Theta_0$ where $V_0 > P + D > V_j(I, \theta) \ \forall \ j \neq 0$, there will be no renegotiation of the original terms of the contract; this region will be denoted as $\Theta_{00}$. The value to Firm 0 of its contract with $W$ can thus be expressed as:

$$V^{LD}(I) = \sum_{i=1}^{N} \int_{\theta \in \Theta_{i}} DdF(\theta) + \int_{\theta \in \Theta_{00}} (V_0(I, \theta) - P) dF(\theta) + \sum_{i=1}^{N} \int_{\theta \in \Theta_{0i}} (\pi_{0D}(V_0(I, \theta) - V_i(I, \theta)) + D) dF(\theta) - B^{LD} - I$$

(15)

Firm 0 is subject to a positive externality problem when $\theta \in \Theta_{i}$, and a hold-up problem when $\theta \in \Theta_{0i}$, for all $i \neq 0$.25

**Proposition 2** When the breach remedy is liquidated damages, the optimal investment level, $I^{LD}$, chosen by Firm 0 will be strictly lower than the socially efficient level $I^{FB}$.

Turning now to the CNC, we define the set of firms which $W$ is restricted from working for as $\Omega = 1, \ldots, n$, and the set of firms which $W$ is free to work for as $\Delta = n + 1, \ldots, N$. As pointed out earlier, a CNC can be viewed as a hybrid of specific performance for the subset of firms $i \in \Omega$, and liquidated damages (where $D = 0$) for the complement subset of firms $i \in \Delta$. The hybrid nature of the CNC results in six distinct categories of subsets of $\Theta$ to consider (we denote $V_i(I, \theta)$ here simply as $V_i$):

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25 Recall that Assumption 2 imposes an upper bound on $P + D$. 


Case I: \[ \max_{i \in \Delta} V_i > \max_{j=0,i \in \Omega} V_j \quad (\theta \in \Theta_{\delta}) \]

Case II: \[ V_0 = \max_{i=0,...,N} V_i \quad \text{and} \quad \max_{i \in \Delta} V_i < P \quad (\theta \in \Theta_{00}) \]

Case III: \[ V_0 = \max_{i=0,...,N} V_i \quad \text{and} \quad \max_{i \in \Delta} V_i > P \quad (\theta \in \Theta_{0\delta}) \]

Case IV: \[ \max_{j \in \Omega} V_j > \max_{i=0,i \in \Delta} V_i \quad \text{and} \quad \max_{i \in \Delta} V_i < P \quad (\theta \in \Theta_{\omega \omega}) \]

Case V: \[ \max_{j \in \Omega} V_j > \max_{i=0,i \in \Delta} V_i \quad \text{and} \quad \max_{i \in \Delta} V_i > V_0 > P \quad (\theta \in \Theta_{\omega \delta}) \]

Case VI: \[ \max_{j \in \Omega} V_j > \max_{i=0,i \in \Delta} V_i \quad \text{and} \quad V_0 > \max_{i \in \Delta} V_i > P \quad (\theta \in \Theta_{\omega 0}) \]

The first three cases are virtually identical to those examined for liquidated damages, because the highest valued use is either with Firm 0 or a firm outside the CNC. In Case I, W will leave to go to Firm \( \delta \), the firm in \( \Delta = \{n+1, \ldots, N\} \) where her output would be valued most; in Case II, W will stay, and her wage will not be renegotiated since there would be no credible threat of departure; in Case III, W will threaten to leave to go to \( \delta \), and Firm 0 will renegotiate with her since her output value is maximized by staying put. Compared to the analysis under liquidated damages, note that now \( D = 0 \), and, for any \( \theta \), the best alternative outside of Firm 0 is chosen out of only those firms in \( \Delta \) rather than all \( i \). The size of the regions in Cases I-III will thus not be identical to the corresponding regions under liquidated damages.\(^{26}\)

The remaining categories of regions of \( \theta \) correspond to situations where it would be optimal for W to leave to work for Firm \( \omega \), the firm in \( \Omega = \{1, \ldots, n\} \) where her output would be valued most, yet where she is restricted from joining. When \( \theta \in \Theta_{\omega \omega} \) (Case IV), \( V_i < P \ \forall i \in \Delta \), and thus W can not credibly threaten to leave to go to any of the \( \Delta \) firms. Thus, Firm 0 negotiates with W and Firm \( \omega \) (either explicitly or implicitly) exactly in the manner in which we saw under specific performance.

In contrast, when \( \theta \in \Theta_{\omega \delta} \) or \( \theta \in \Theta_{\omega 0} \) (Cases V and VI), W would first threaten to join Firm \( \delta \), where \( \delta = \arg\max_{i \in \Delta} V_i, i \in \Delta \). This ensures that W earns at least \( V_\delta(I, \theta) \). The parties would then negotiate to allow W to move to Firm \( \omega \), where

\(^{26}\)The fact that \( D = 0 \) will tend to decrease \( \Theta_{00} \) and increase \( \Theta_{0\delta} \) relative to the corresponding liquidated damages regions when \( D > 0 \), but the restriction that \( i \in \Delta \) will have the opposite effect. Also, taken in aggregate, the \( \Theta_i \) regions in Case I will be a smaller subset of \( \theta \) values than were the \( \Theta_i \) regions in the case of liquidated damages due to the restriction that \( i \in \Delta \).
\(\omega = \text{argmax}_i V_i, i \in \Omega\). In Case V, where \(V_\delta(I, \theta) > V_0(I, \theta)\), Firm 0, Firm \(\omega\) and \(W\) will split the surplus \(V_\omega(I, \theta) - V_\delta(I, \theta)\) which is gained from \(W\) leaving to work for Firm \(\omega\) rather than for Firm \(\delta\) (we again assume that Firm 0 gets \(\pi_0^{SP}\) of the surplus when it shares with \(W\) and the new employer). In Case VI, where \(V_0(I, \theta) > V_\delta(I, \theta) > P\), Firm 0 would need to renegotiate to prevent \(W\) from moving to Firm \(\delta\). Firm 0 would obtain a payoff of \(\pi_0^{LD}(V_0(I, \theta) - V_\delta(I, \theta))\) by renegotiating with \(W\) to keep her at the firm, but could obtain an additional payoff of \(\pi_0^{SP}(V_\omega(I, \theta) - V_0(I, \theta))\) by allowing \(W\) to work for Firm \(\omega\).

The value of the contract to Firm 0 can thus be expressed as follows, where the first five lines of the expression correspond to each of the Cases II-VI, respectively (the payoff to Firm 0 in Case I is zero since \(W\) simply leaves to work for Firm \(\delta\)):

\[
V^{CNC}(I) = \int_{\theta \in \Theta_00} (V_0(I, \theta) - P) dF(\theta)
+ \sum_{\delta \in \Delta} \int_{\theta \in \Theta_{0\delta}} \pi_0^{LD}(V_0(I, \theta) - V_\delta(I, \theta)) dF(\theta)
+ \sum_{\omega \in \Omega} \int_{\theta \in \Theta_{\omega\omega}} (V_0(I, \theta) - P + \pi_0^{SP}(V_\omega(I, \theta) - V_0(I, \theta))) dF(\theta)
+ \sum_{\omega \in \Omega} \int_{\theta \in \Theta_{\omega\delta}} \pi_0^{SP}(V_\omega(I, \theta) - V_\delta(I, \theta)) dF(\theta)
+ \sum_{\omega \in \Omega} \int_{\theta \in \Theta_{\omega0}} (\pi_0^{LD}(V_0(I, \theta) - V_\delta(I, \theta)) + \pi_0^{SP}(V_\omega(I, \theta) - V_0(I, \theta))) dF(\theta)
- B^{CNC} - I
\]

(16)

For any given set of \(V_i\) functions, the CNC can be designed such that its scope, \(n\), can lead to first-best investment incentives. We now define \(V^{CNC}(I, n)\) explicitly as a function of the scope of the CNC. If \(n = 0\), \(V^{CNC}(I, 0) \equiv V^{LD}(I)\) (where \(D = 0\)) since there would be no restrictions preventing employment with other firms. If \(n = N\), \(V^{CNC}(I, N) \equiv V^{SP}(I)\), since \(W\) would need to renegotiate with Firm 0 should she choose to work for any other firm at \(T\).

**Proposition 3** If \(N\) is infinitely large, and \(V^{CNC}(I, n)\) is differentiable in both \(I\) and \(n\), there exists some \(0 < n^{FB} < N\) such that Firm 0’s optimal investment under such a CNC, \(I^{CNC}(n^{FB})\), will be equivalent to the first-best level, \(I^{FB}\).

While Proposition 3 is exact only for an infinite \(N\), for practical purposes it implies that the under- and over-investment problems associated with liquidated
damages and specific performance remedies, respectively, can effectively offset each other by carefully designing the “hybrid” CNC contract to have a scope that induces the appropriate mix between liquidated damages and specific performance. Increasing the scope of the CNC (i.e. the number of firms for which $W$ is restricted from working for) will create a stronger incentive to overinvest (by increasing the regions of $\theta$ in which Firm 0 will participate in the negotiation of a new employment contract), and a weaker incentive to underinvest (by decreasing the region where there will be a hold-up problem). Eventually, $n$ will be large enough such that these incentives will approximately (or precisely) balance each other off.

While the scope $n^{FB}$ leads to first-best investment, this is not necessarily the $n$ that maximizes Firm 0's value, or to be more precise, the joint value attributable to the contract between $W$ and Firm 0. As pointed out in Section 2, by increasing the number of firms that $W$ is restricted from working for, the contracting parties can extract more value from potential new employers. Firm 0 will be involved in more of the negotiations with new employers, thus reducing the share of the surplus going to these new firms. Furthermore, Firm 0 will have a larger incentive to increase investment, thus reducing the surplus that is shared with outside parties. This result is stated more formally in the following proposition, which is proved in the Appendix.

**Proposition 4** The initial contracting parties ($W$ and Firm 0) maximize the private value of their contract by agreeing to a CNC with scope $n^{CNC}$ that is larger than the first-best scope, $n^{FB}$, that induces socially optimal investment.

To summarize, a CNC partitions future states into two subsets. First, where the worker’s most valued use falls within the CNC, there is overinvestment of the type induced by specific performance. Second, where the worker’s most valued use lies outside the CNC, there is underinvestment of the type yielded by ordinary liquidated damages. There is at least one way of partitioning the states to balance ex ante the over- and underinvestment effects. However, the parties have the incentive to draft their CNC inefficiently broadly in order to extract more of the surplus from future entrants.

4 Legal Implications

CNCs are common in many industries and they bind a wide range of employees, from senior executives to rank-and-file workers. At the same time, the enforcement of these covenants remains checkered, both historically and across jurisdictions, as
law makers weigh the benefits from protecting valuable assets of employers against
the social costs of impeding labor mobility (Malsberger, 1996). There is no consensus
among courts and legislatures as to the optimal balance between these goals. For
example, while California legislation provides that CNCs are void (Gilson, 1999),
and Massachusetts has a statute that prohibits CNCs in the broadcasting industry
(Baker, 2001), the Texas legislature has enacted a statute to counter the historical
judicial hostility in that state against CNCs (Wood, 2000).

The most common approach is the “rule of reason” regulation of restraints on
trade, which is encapsulated in the Second Restatement of Contracts (American Law
Institute, 1979) and in several state statutes. Under this rule, the courts enforce
CNCs to protect legitimate interests of employers, particularly in trade secrets,
confidential information and customer relationships. However, the CNCs must
be reasonable by not imposing undue hardship on the employee or injury to the
public interest. Where either burden is disproportionately severe, the courts refuse
to enforce or they cut back the CNC coverage of activities, geography and time.

Our model provides the basis for a normative assessment of the legal enforcement
of CNCs. We found that, in each of the cases of no-renegotiation and costless rene-
gotiation, CNCs can yield performance and investment incentives that are superior
to those produced by the contract remedies of specific performance and liquidated
damages (including zero damages). We also showed that, if renegotiation is costless,
the parties have contracting incentives to draft CNCs with inefficiently broad scope
that causes overinvestment. Given the plausibility of the assumption that renegoti-
ation costs among workers and employers in the same industry are low, these two
results provide support for cautious enforcement of CNCs.

In the “rule of reason” states, CNCs are more likely to be enforced if they protect
trade secrets, confidential information and customer lists or relationships. These as-
sets are significant to the courts because they lose value when they fall into the
hands of competitors. Most states have laws designed specifically to protect these
assets from wrongful appropriation and these rules are frequently supplemented by
contractual terms in employment agreements. For example, most states have passed
the Uniform Trade Secrets Act and many employment contracts prohibit disclosure
of confidential information. However, these prohibitions are difficult to enforce be-
cause proving disclosure against a defendant is difficult and usually entails the public
revelation of the confidential information. A CNC may police trade secret theft more

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28 149 Mass. Gen. Laws Ann. Ch 149 Section 1 was passed after intensive lobbying effort by the
American Federation of Television and Radio Artists (AFTRA).
29 American Law Institute (1979), Restatement (Second) Contracts, section 188 ct. b,c,g.
effectively. The change in a worker’s employment is easy to verify and enforcement does not require the disclosure of confidential information. Moreover, by preventing the worker from moving to another employer, the firm can invoke internal sanctions to discipline disclosure or sale of sensitive information. The contribution of CNCs in this context is valuable, but for reasons external to this paper.

Under the rule of reason approach, courts occasionally recognize legitimate interests in protecting human capital investment, but much less often than interests in trade secrets, confidential information, goodwill and customer relations. Judges perceive a tradeoff between the protection of human capital investments and labor mobility. They are more reluctant to enforce restraints on the movement of workers to protect general investment than specialized training (Lester, 2001; Callahan, 1985; Decker, 1985, pp. 82-3). This normative concern with labor mobility is reflected in academic commentary that attributes the success of Silicon Valley partly to the California ban on CNCs (Hyde, 1998; Gilson, 1999).

The concern of law makers and academics is misplaced in this regard. Even when CNCs cannot be renegotiated, we have shown that they may yield a more efficient \textit{ex post} outcome than a contract that does not bind the worker. Thus, the courts should simply enforce CNCs as written when renegotiation is difficult. More significantly, the effect of CNCs on mobility is slight when the covenants can be easily renegotiated. Indeed, low-cost renegotiation is very plausible among workers and employers in the same industry and payments for the release of CNCs are common in practice. In this light, our model highlights a distinct concern with CNCs associated with their renegotiation. The parties have the incentive to draft them broadly in order to capture more of the rents from worker movement and these CNCs stimulate inefficiently high investment in relatively specific human capital. Therefore, the courts should scrutinize CNCs particularly closely when the investment of the initial employer is more specific than general.\footnote{In terms of our model, this corresponds to the case where $k_0$ is significantly higher than the $k_i$ of a prospective new employer, \textit{Firm} i.}

In contrast, the current judicial focus is on the degree of product competition between the employer and the firms covered by the CNC (Malsberger, 1996).

In California, where CNCs are banned, deferred compensation, including delayed vesting of stock options, is said to serve as a substitute and to discourage the movement of workers. Yet, deferred compensation has the same effect as liquidated damages in our analysis (because it is forfeited when a worker leaves the employer) and suffers from the same problem. Even if the deferral can be renegotiated, it is indiscriminate in externalizing investment costs to all alternative employers and consequently shares this disadvantage with conventional liquidated damages.
5 Conclusions and Extensions

The classic challenge in the economics of contracts is the dual optimization of *ex post* performance outcomes and *ex ante* investment incentives when significant actions and states of the world are not verifiable. As noted earlier, this challenge is further complicated in the case of human capital investments by employers because workers can carry human capital to new employers, workers are often judgment proof, and judges will not specifically enforce employment contracts. In light of these obstacles, this paper presents the CNC as a new and relatively simple solution to the dual optimization problem.

The model and insights presented in this paper can be extended in several directions. First, the significant defining feature of the CNC, in contrast to other remedies, is that it is typically enforced by injunction and is contingent on the worker’s choice among alternative employment opportunities. Similar hybrid remedies that condition on the worker’s post-breach behavior might be explored in future research. For example, a contract might combine a CNC that bars movement within its scope with positive liquidated damages that are applied when the worker moves outside. To the extent that deferred compensation such as unvested stock option grants represent a form of liquidated damages, the exploration of hybrid remedies may provide some insight into the co-existence of CNCs and deferred compensation schemes.

Second, the use of hybrid remedies such as the CNC may also be explored in contexts outside the employment relationship. CNCs are common in joint venture agreements, partnerships and franchise contracts and it is likely that the justification advanced in this paper extends to those relationships as well. In the case of the trade of goods or services, a contract might condition breach remedies on verifiable post-breach behavior and thereby reveal an alternative mechanism for achieving *ex post* and *ex ante* efficiency.

Third, we have followed convention in dividing our analysis between assumptions of no renegotiation and costless renegotiation. In practice, renegotiation is often possible, but at a cost that may or may not consume the surplus. Future research may examine the impact of renegotiation costs on the efficiency of CNCs and other hybrid remedies.
Appendix

Proof of Proposition 1

Based on (13) and (14), the difference between the marginal benefit of an additional unit of $I$ in the case of specific performance versus in the first-best case can be written as (suppressing $I$ in the value functions):

$$\frac{d(V_{SP} - V_{FB})}{dI} = \frac{d}{dI} \left[ \sum_{i=0}^{N} \int_{\theta \in \Theta_i} \left( (V_0(\theta) + \pi_{0}^{SP} (V_i(\theta) - V_0(\theta))) - V_i(\theta) \right) dF(\theta) - B^{SP} \right]$$

$$= \frac{d}{dI} \left[ \sum_{i=0}^{N} \int_{\theta \in \Theta_i} (1 - \pi_{0}^{SP}) (V_0(\theta) - V_i(\theta)) dF(\theta) - B^{SP} \right]$$

(17)

We denote the lower and upper boundaries of $\Theta_i$ by $\theta_i^-$ and $\theta_i^+$, respectively, Using Leibniz’s Theorem for differentiation of an integral, and suppressing $I$ and $\theta$ when expressing the value function $V_i(I, \theta)$ and its derivative with respect to $I$, $V_i'(I, \theta)$:

$$\frac{d(V_{SP} - V_{FB})}{dI} = (1 - \pi_{0}^{SP}) \left( \sum_{i=0}^{N} \int_{\theta \in \Theta_i} (V_i'(\theta^-) - V_i'(\theta^+)) dF(\theta) \right) +$$

$$(1 - \pi_{0}^{SP}) \left( \sum_{i=0}^{N} \left[ (V_0(\theta_i^+) - V_i(\theta_i^+)) f(\theta_i^+) \frac{d\theta_i^+}{dI} - (V_0(\theta_i^-) - V_i(\theta_i^-)) f(\theta_i^-) \frac{d\theta_i^-}{dI} \right] \right)$$

(18)

Recognizing that $\theta_i^- = \theta_h^+$ for some $h \neq i$ and $\theta_i^+ = \theta_j^-$ for some $j \neq i$, $V_0(\theta_i^-) = V_0(\theta_h^+)$ and $V_0(\theta_i^+) = V_0(\theta_j^-)$, and $V_i(\theta_i^-) = V_h(\theta_i^+)$ and $V_i(\theta_i^+) = V_j(\theta_i^-)$. As a result, the summation over all the $V_0$ and $V_i$ terms in the second line of equation (18) is equal to zero.\(^{32}\) Since $V_i' > V_i' \forall i$ by assumption, $\frac{dV_{SP}}{dI} > \frac{dV_{FB}}{dI}$ for any $I$, as long as $\pi_{0}^{SP} < 1$. For $I = I^{FB}$, $\frac{dV_{FB}}{dI} = 0$, and thus $\frac{dV_{SP}(I^{FB})}{dI} > 0$. Since $\frac{dV_{SP}(I^{SP})}{dI} = 0$ and $\frac{dV_{SP}(I^{SP})}{dI^2} < 0$, it follows that $I^{SP} > I^{FB}$.

Proof of Proposition 2

From (13) and (15), we can write the difference between the derivatives of $V^{LD}(I)$ and $V^{FB}(I)$ with respect to $I$ as (again suppressing $I$ in the value functions):

\(^{32}\)For some $i$, $\theta_i^- = \theta^-$, where $\theta^-$ is the lower bound of $\Theta$, and for another $i$, $\theta_i^+ = \theta^+$, where $\theta^+$ is the upper bound of $\Theta$. Since $\frac{d\theta^-}{dI} = \frac{d\theta^+}{dI} = 0$, the corresponding terms in the second line of the equation will also equal zero.
\[
\frac{d(V^{LD} - V^{FB})}{dI} = \frac{d}{dI} \left\{ \sum_{i=1}^{N} \int_{\theta_{i}} (P + D - V_i(\theta))dF(\theta) + \sum_{i=1}^{N} \int_{\theta_{0i}} (P + D + \pi_0^{LD}(V_0(\theta) - V_i(\theta)) - V_0(\theta))dF(\theta) - B^{LD} \right\}
\]

(19)

Using Leibniz’s Theorem for differentiation of an integral, and using negative and positive superscripts, as before, to denote the lower and upper boundaries of each region:

\[
\frac{d(V^{LD} - V^{FB})}{dI} = -\sum_{i=1}^{N} \int_{\theta_{i}} V'_i dF(\theta)
- \sum_{i=1}^{N} \int_{\theta_{0i}} (\pi_0^{LD}V'_i + (1 - \pi_0^{LD})V'_0) dF(\theta)
+ \sum_{i=1}^{N} \left( (P + D - V_i(\theta_i^+)) f(\theta_i^+) \frac{d\theta_i^+}{dI} - (P + D - V_i(\theta_i^-)) f(\theta_i^-) \frac{d\theta_i^-}{dI} \right)
- \sum_{i=1}^{N} (\pi_0^{LD}V_i(\theta_{0i}^+) + (1 - \pi_0^{LD})V_0(\theta_{0i}^+) - (P + D)) f(\theta_{0i}^+) \frac{d\theta_{0i}^+}{dI}
+ \sum_{i=1}^{N} (\pi_0^{LD}V_i(\theta_{0i}^-) + (1 - \pi_0^{LD})V_0(\theta_{0i}^-) - (P + D)) f(\theta_{0i}^-) \frac{d\theta_{0i}^-}{dI}
\]

(20)

Since the different subregions of \( \Theta \) are congruent, we know that \( \theta_i^+ \), the upper boundary of \( \Theta_i \) will be equal to one of: the upper boundary \( \theta^+ \) of \( \Theta \); the lower boundary \( \theta_{0i}^- \) of \( \Theta_{0i} \); or, the lower boundary \( \theta_j^- \) of \( \Theta_j \), for some \( j \neq 0 \). Similarly, \( \theta_i^- \) is equal to one of \( \theta^- \), \( \theta_{0i}^+ \), or \( \theta_h^+ \) for some \( h \neq 0 \). As a result, most of the terms in the last three lines of (20) cancel out given the shared boundary points of the various \( \theta \) regions. However, the terms at \( \theta^- \) and \( \theta^+ \), as well as those where \( \theta_{0i}^- = \theta_{00}^+ \) and \( \theta_{0i}^+ = \theta_{00}^- \) do not have offsetting terms. Thus, the last three lines of (20) can be reduced to:
\[+ (P + D - V_i(\theta^+)) f(\theta^+) \frac{d\theta^+}{dI} - (P + D - V_i(\theta^-)) f(\theta^-) \frac{d\theta^-}{dI}]

\[- (\pi_0 LD V_k(\theta_{00}) + (1 - \pi_0 LD) V_0(\theta_{00}) - (P + D)) f(\theta_{00}) \frac{d\theta_{00}}{dI} \]

\[+ (\pi_0 LD V_i(\theta_{00}^+) + (1 - \pi_0 LD) V_0(\theta_{00}) - (P + D)) f(\theta_{00}) \frac{d\theta_{00}^+}{dI} \quad (21)\]

The first two terms are equal to zero since \(\frac{d\theta^+}{dI} = \frac{d\theta^-}{dI} = 0\). Since \(V_0(\theta_{00}^-) > V_k(\theta_{00}) = P + D\) and \(\frac{d\theta_{00}^-}{dI} > 0\), the term on the second line of (21) is negative. Similarly, \(V_0(\theta_{00}^+) > V_i(\theta_{00}^+) = P + D\) and \(\frac{d\theta_{00}^+}{dI} < 0\), and thus the term on the third line of (21) is also negative.

Since \(V_i^i > 0 \forall i\), the terms in the first two lines of (20) are both negative, and thus \(\frac{dV}{dI} < \frac{dV_{FB}}{dI}\) for all \(I\). For \(I = I_{FB}\), \(\frac{dV_{FB}}{dI} = 0\), and thus \(\frac{dV_{LD}(I_{FB})}{dI} < 0\).

Since \(\frac{dV_{LD}}{dI} = 0\) and \(\frac{d^2V_{LD}}{dI^2} < 0\), \(I_{LD} < I_{FB}\).

**Proof of Proposition 3**

Given that \(V^{CNC}(I, N) \equiv V^{SP}(I)\) and \(V^{CNC}(I, 0) \equiv V^{LD}(I)\), from the proofs of Propositions 1 and 2, we know that:

\[
\frac{dV^{CNC}(I_{FB}, N)}{dI} = \frac{dV^{SP}(I_{FB})}{dI} > 0
\]

\[
\frac{dV^{CNC}(I_{FB}, 0)}{dI} = \frac{dV^{LD}(I_{FB})}{dI} < 0
\]

Under the assumptions in the proposition, the Intermediate Value Theorem implies that there exists some \(0 < n_{FB} < N\) such that \(\frac{dV^{CNC}(I_{FB}, n_{FB})}{dI} = 0\), and thus \(I_{CNC}(n_{FB}) = I_{FB}\).

**Proof of Proposition 4**

When contracting at \(t = 0\), Firm 0 and W agree on the scope of the CNC and a corresponding upfront bonus, \(B^{CNC}\), that is paid to W. The value associated with the contract is the sum of the present values of the payoffs to Firm 0, \(V^{CNC}(I)\), and to W, \(V^{W}(I)\). Given that renegotiation is permitted at \(T\), and thus first-best performance will occur, the value of the contract is also equivalent to the difference between the total value of W’s output over all \(\theta\) values minus the value that is captured by new employers. Consistent with our earlier notation, we denote new employers’ share of the surplus as \(\pi_3^{LD}\) when the new employer is outside the scope.
of the CNC and thus Firm δ negotiates directly with W, and πSPω when Firm 0 will be involved (directly or indirectly) in the negotiation with Firm ω since the CNC restriction is binding. We also suppress I and θ in V_i(I,θ) in the following expression.

\[
V^{CNC}(I) + V^W(I) = \sum_{\delta \in \Delta} \int_{\theta \in \Theta} (V_\delta - \pi^{LD}_\delta (V_\delta - V_0)) dF(\theta) + \\
\int_{\theta \in \Theta_0} V_0 dF(\theta) + \sum_{\delta \in \Delta} \int_{\theta \in \Theta_0} V_0 dF(\theta) + \\
\sum_{\omega \in \Omega} \int_{\theta \in \Theta_\omega} (V_\omega - \pi^{SP}_\omega (V_\omega - V_0)) dF(\theta) + \\
\sum_{\omega \in \Omega} \int_{\theta \in \Theta_\omega} (V_\omega - \pi^{SP}_\omega (V_\omega - V_\delta)) dF(\theta) + \\
\sum_{\omega \in \Omega} \int_{\theta \in \Theta_\omega} (V_\omega - \pi^{SP}_\omega (V_\omega - V_0)) dF(\theta) - I
\]  

(23)

The first six terms in (23) correspond to Cases I-VI for the CNC. Consider the changes in (23) if the scope of the CNC were broadened from nFB to include a restriction on W leaving to work for an additional firm, Firm nFB + 1, which we denote as n' to simplify the notation. θ values that were in Θ0 would now be part of Θ00 since W will no longer be able to threaten to leave to go to Firm n'. However, since the integrand is the same in both the second and third terms, this will simply represent a reallocation between W and Firm 0 that will not affect the total value of the contract in (23).

However, the other four of the first six terms of (23) will also be affected. Since n' is now part of Ω rather than Δ, the θ values where Vn' is the maximum output value across all firms (i.e. θ ∈ Θn') will now be distributed across Cases IV-VI rather than being part of Case I. The integrand will thus change from Vn' − π^{LD}_{n'} (Vn' − V_0) (Case I) to either Vn' − π^{SP}_{n'} (Vn' − V_0) (Case IV or VI), or Vn' − π^{SP}_{n'} (Vn' − V_δ) (Case V), where, with the removal of Firm n' from Δ (creating the new set Δ'), δ now represents the firm in Δ' that has the highest output value (greater than both P and V_0, otherwise the region Θn'δ is empty). Since V_δ > V_0 in Case V, and since it is reasonable to assume that π^{LD}_{n'} ≥ π^{SP}_{n'} (the former is based on a two-way negotiation between W and Firm n', while the latter on a three-way negotiation between W, Firm 0 and Firm n'), the contract value in (23) increases as n goes from nFB to n'.

It is worth noting here that, from the proof of Proposition 3, \( I^{CNC} > I^{FB} \) if
This increase in $I$ results from the fact that a larger $n$ reduces the hold-up problem from $W$ leaving to work for one of the $\Delta$ firms, and increases the incentive to reduce the expected profit of $\Omega$ firms as $\Omega$ grows to include more firms. In addition, there is a feedback effect that further increases $I^{CNC}$. As $I$ increases, the output value of $\Omega$ firms increases more than that of $\Delta$ firms. This results in an increase in the region $\Theta_\Omega$, and a corresponding reduction in the size of $\Theta_\Delta$, providing further incentive to increase $I$.

References


